Institute of Distance and Open Learning (IDOL)
University of Mumbai.

F.Y.B Com
MATHEMATICAL AND STATISTICAL TECHNIQUES

SEM- I

CONTENT

***

SYLLABUS

Unit I
Commission, Brokerage, Discount and Partnership:
Commission and Brokerage : Simple examples on calculation of commission and brokerage. Discounts : Trade Discount, Cash Discount. Profit and Loss. Sharing of profit in Partnership. Problems involving mixture of discount and profit are expected.

Unit II
Shares and Mutual Funds:
Concepts of shares, face value, market value, dividend, equity shares preferential shares, bonus shares, Simple examples. Mutual Funds, Simple problems on calculation of Net Income after considering entry load, dividend, change in Net Asset Value (N.A.V) and exit load. Averaging of price under the 'Systematic Investment Plan (S.I.P)'
Linear Programming Problems:
Sketching or graphs of (i) linear equation Ax + By + C = 0 (ii) linear inequalities. Mathematical Formulation of Linear Programming Problems upto 2 variables.

Unit III
Introduction
Meaning, Scope and Limitations of Statistics:
Basic Statistical Concepts: Population, Sample, Variable, attribute, parameter, statistic.
Collection of Data:
Primary and Secondary, Sample and Census, Survey (concept only), Tabulation of data upto 3 characteristics (Simple examples)
Diagrams and graphs:
Given a diagram, interpretation of it. Simple bar diagram, Multiple bar diagram, Percentage bar diagram, Pie diagram. Drawing of frequency curve, frequency polygon, Histogram (class – Intervals of equal lengths only) and ogives.

Unit IV
Measures of Central Tendency: Arithmetic mean, Weighted mean, Combined mean, Median , Mode-without grouping, Quartiles (No Example on missing frequency)
Measures of Dispersion: Range, Quartile deviation, Mean deviation from mean Standard deviation and their relative measures. (Concepts of shift of origin and change or scale are not to be done)

Unit V
Elementary Probability Theory:
Concept of Random experiment/trial and possible outcomes; Sample Space and Discrete Sample Space; Events and their types, Algebra of Events, Mutually Exclusive and Exhaustive Events, concept of.$^{n}C_{r}$. Classical definition of Probability, Addition theorem (without proof); Independence of Events: P (A $\cap$B) = P (A) P (B) Simple examples.

Random Variable: Probability distribution of a discrete random variable; Expectation and Variance; Simple examples. Concept of Normal distribution and Standard Normal Variate (SNV), simple examples.

UNIT I

1

COMMISSION, BROKERAGE, DISCOUNT AND PARTNERSHIP

OBJECTIVES

Money plays an important role in any transaction. We are going to study some Technical terms involved in business transactions.

1.1 COMMISSION

A producer or manufacturer of goods generally does not sell his goods directly to the ultimate consumer. There are agents who purchase the goods from the manufacturer and sell them to the consumer. In a sense, such agents bring the manufacturers and the consumers together for transaction. The remuneration which an agent gets for his services in the transaction is called *commission.* Most of the business transactions are made through intermediate persons.

1.2 RATE OF COMMISSION

The amount of commission that an agent gets in the transaction depends on the volume of work done or the services rendered by him. His commission is based on the value of the goods bought or sold and is generally fixed on a percentage basis. In some cases, he is paid a commission on the total sales brought by the agent or on different slabs. For example, it may be 5% on the first Rs. 10,000/-, 6% on the next Rs. 5,000/- and so on.

1.3 TYPES OF COMMISSION AGENTS

On account of the different fields of specialization in business activities agents can be differentiated as such as commission agents, brokers, del credere agents, factors, insurance agents, export agents, auctioneers, etc. Agents particularly salesmen are also paid regular salary in addition to commission they earn.

Commission Agent: A commission agent is that middle-man who buys or sells goods on behalf of some other person called as principal. He is usually employed by his principal and gets commission as some percentage of the sale value.

Broker : A broker is that middle-man who brings together a prospective seller and a prospective buyer and negotiates the sale between them. The commission that he gets is called brokerage, which may be charged from both the parties. Accordingly, there are stock-brokers, producer, brokers, bullion-brokers, bill-brokers, insurance-brokers depending upon the field of business in which they work.

Factor: A factor is an agent who takes possession of the goods and then sells them in his own name. A factor does the transaction without disclosing the name or identity of his principal. He receives payment in his own name and passes receipt for the same.

Del Credere Agent: A del credere agent sells goods and at the same time gives a guarantee of the collection of dues from the consumers. In return for this guarantee, he gets an additional commission known as del creders. This commission may be at a flat rate on all the sales or at a higher rate on credit sales only.

Auctioneer: A person who wants to sell his goods or property or machinery approaches such an agent. The agent then gives an advertisement in which he gives some description of the goods and the date of sale. The agent sells goods to the highest bidder, i.e. the person who offers to pay the highest amount in the auction. Such an agent is called an auctioneer. Generally, the auctioneer does not disclose the name of the principal.

Solved Examples:

Example 1

An agent sold Rs. 3,000 worth of articles on 4½ % commission basis. Find the commission of the agent.

Solution :

The agent sold Rs. 3,000 worth of articles on 4½ % commission basis

Agent's commission = 4½ of Rs. 3,000

$$= \text{Rs } \frac{9}{2} \times \frac{1}{100} \times 3000$$

= Rs. 135

Example 2

An agent was paid Rs. 1596 as commission at rate 12% on the sale of bicycles. The selling price of each bicycle was Rs. 950. Find the number of bicycles sold by the agents.

Solution:

Let χ be the number of bicycle sold by the agent.

Price of one bicycle = Rs. 950

Selling price of χ Bicycles = 950 x

@12% commission on 950 $x = \frac{12}{100} \times 950\,x$

$=114\,x$

Agent's commission $= 1596$

$114\,x = 1596$

$x = \frac{1596}{114} = 14$

Number of Bicycles sold = 14

Example 3

An insurance agent gets commission of 20% on first year premium, 6% on second and third year's premium and 4% on subsequent years premium on an insurance policy of Rs. 40,000. Annual rate of premium being Rs. 30 per thousand. Find the total earning of the agent for which 5 annual premiums have been paid.

Solution:

The rate of annual premium is Rs. 30 per thousand.

Annual premium $= \frac{30}{1000} \times 40000 = 1200$

$\therefore$ Commission for first year $= \frac{20}{100} \times 1200 = 240$

Commission for second and third year $= 2\left(\frac{6}{100} \times 1200\right) = 2\,(72) = 144$

Commission for fourth and fifth year $= 2\left(\frac{4}{100} \times 1200\right) = 2\,(48) = 96$

Total Earnings $= 240 + 144 + 96 = 480$

$=$ Rs. 480

Example 4

At 7% rate of commission, a sales girl, got Rs. 210 on the sale of combs. Find the value of the sale if the price of each comb is Rs. 15 per container. Find the number of containers sold by the sales girl.

Solution:

Since the rate of commission is 7% and she received Rs. 210,

total value of her sales= Rs. = Rs. 210 x $\frac{7}{100}$ = Rs. 3,000.

The price of each container is Rs. 15.

Total number of containers sold = $\frac{3000}{15}$ = **200**

Example 5

Find the commission on total sales worth Rs. 25,000, if the rate of commission is 3% on first Rs. 10,000 and 4½ on sales over Rs. 10,000.

Solution :

Total sales are worth Rs. 25,000.

Commission at 3% on first Rs. 10000 = Rs. 10,000 × 3 = Rs. 300

Commission at 4½ % on sales over Rs. 10,000 = Commission at rate 4½% on Rs.15000(25000-10000).

$$= \text{Rs. } \frac{9}{2} \times \frac{1}{100} \times 15{,}000 = \text{Rs. } 675$$

Total commission = Rs. (300 + 675) = Rs**.975**

Example 6

A salesman is allowed 5% commission on the total sales made by him plus a bonus of 1% on the excess of his sale over Rs. 20,000/-. If the total earning are Rs. 1,450/- on commission alone, find his total earnings?

Solution: Let the total sales be Rs. x.

Then commission at 5% on Rs. x $= \frac{5x}{100} = \frac{x}{20}$.

This is given to be Rs. 1450/-

$$\frac{x}{20} = 1450$$

$$x = 1450 \times 20$$

$$x = 29{,}000$$

the amount of total sales is = Rs. 29,000/-

excess of sales over Rs. 20,000= Rs. (29,000-20,000) = Rs. 9,000/-

Bonus at 1% on Rs. 9,000 = Rs. $\frac{1}{100} \times 9000$ = Rs. **90/-**

Example 7

A piece of land was sold for Rs. 19,00,000 through a broker who received 1.25% commission from the seller and 1.75% from the buyer. Find the amount paid by the buyer. Also find the amounts received by the seller and the broker.

Solution: Brokerage paid by the buyer at 1.75%

=1.75% of Rs. 19,00,000 = Rs. $\frac{1.75}{100} \times 19{,}00{,}000$ = Rs. 33,250/-

Total amount paid by the buyer = Cost of the land + Brokerage
= Rs. 19,00,000 + 33,250
= Rs. 19,33,250/-

Brokerage paid by the seller at 1.25%

1.25% of Rs. 19,00,000= Rs. $\frac{1.25}{100} \times 19{,}00{,}000$ = Rs. 23,750/-

Total amount received by the seller = Cost of the land - Brokerage
= 19,00,000 - 23,750

= Rs. 18,76,250/-

Total amount received by the broker = Brokerage from the buyer + Brokerage from the seller

= Rs. (33,250 + 23,750)

= Rs. 57,000/-

Example 8

A merchant instructs his agent to buy 1,000 micro tip pens and sell them at 15% above the purchase price. The agent charges 1% commission on the purchase and 3% commission on sales and earns Rs. 534/- as commission. Find the price at which the agent buys the pen.

Solution: Suppose the agent buys each pen for Rs. *x*.
Then cost price of 1000 pens = Rs. 1000 *x*

selling price of 1000 pens at 15% profit.

$$=\text{Rs. } \frac{115}{100} \times 1000\,x = \text{Rs. } 1150\,x$$

commission at 1% on the purchase.

$$=1\% \text{ of Rs. } 1000\,x = \text{Rs. } \frac{1}{100} \times 1000\,x = \text{Rs. } 10\,x \qquad \text{(i)}$$

commission at 3% on the sales.

$$=3\% \text{ of Rs. } 1150\,x = \text{Rs. } \frac{3}{100} \times 1150\,x = \text{Rs. } \frac{69}{2}\,x \qquad \text{(ii)}$$

From (i) & (ii)

Total commission of the agent

$$=\text{Rs. } 10\,x + \frac{69}{2}\,x = \frac{89}{2}\,x$$

This is given to be Rs. 534/-

$$\frac{89}{2}\,x = 534 \qquad x = \frac{532 \times 2}{89} = 12$$

The agent buys each pen for Rs. 12/-

Example 9

A del credere agent charges 3% commission on cash sales and 6% commission on credit sales. If his average commission is 4.3% , find the ratio of cash sales to credit sales.

Solution: Suppose the cash sales are Rs. x and the credit sales are Rs. y.

Then commission at 3% on cash sales of Rs. x is $= \text{Rs.} \frac{3x}{100}$ and

Commission at 6% on credit sales of Rs. y is $= \text{Rs.} \frac{6y}{100}$

$$\text{Total commission of the agent} = \text{Rs.} \frac{3x + 6y}{100} \qquad \text{(i)}$$

Average commission at 4.3% on total sales of Rs. $(x + y)$

$$= \text{Rs.} \frac{4.3(x + y)}{100} = \text{Rs.} \frac{43(x + y)}{1000} \qquad \text{(ii)}$$

From (i) & (ii)

$$\therefore \frac{43x + 43y}{1000} = \frac{3x + 6y}{100}$$

After simplification, $43x - 30x = 60y - 43y$

$$13x = 17y$$

$$\frac{x}{y} = \frac{17}{13}$$

The ratio of cash sales to credit sales is 17:13

Example10

A salesman is appointed on a fixed monthly salary of Rs. 1,500/- together with a commission at 5% on the sales over Rs. 10,000/- during a month. If his monthly income is Rs. 2,050/-, find his sales during that month.

Solution: Commission at 5% on the sales over Rs. 10,000/-

= monthly income - monthly salary
=Rs. 2050-1500
= Rs. 550/-

the sales over Rs. 10,000/-

$= \text{Rs. } 550 \times \frac{100}{5} \qquad = \text{Rs. } 11,000/-$

total sales during the month

= Rs. (10,000 + 11,000) = Rs. 21,000/-

Example 11

A salesman is paid a monthly salary plus a commission based as a percentage on sale. If on the sales of Rs. 20,000/- and Rs.25,000/- in two successive months he received Rs. 1600/- and Rs.1750/- respectively, find his monthly salary and the rate of commission paid on sales.

Solution: Let the monthly salary of the salesman be Rs. x and y % be the rate of commission.

commission in first month at y % on Rs. 20,000/-

$$=\text{Rs. } \frac{y}{100} \times 20000 = \text{Rs. } 200y$$

For the first month total earnings =salary + commission in the first month

$$1600 \quad = \text{Rs. } (x + 200y) \qquad \ldots\ldots (1)$$

Also commission in the second month at y % on Rs. 25,000/-

$$= \text{Rs. } \frac{y}{100} \times 25000 = \text{Rs. } 250y$$

For the second month total earnings =salary + commission in the second month

$$1{,}750 \quad = \text{Rs. } (x + 250y) \qquad \ldots\ldots (2)$$

Subtracting (1) from (2), we get, 50y = 150 $\therefore$ y=3

Substituting this value of y in (1), we get,

$$x + 200 \times 3 = 1600 \qquad \therefore \quad \boldsymbol{x = 1000}$$

The monthly salary of the salesman is Rs. 1000/- and the rate of commission is 3%.

Example 12

At what price should goods costing Rs. 18,000 be sold through an agent so that after paying her a commission at 4% on sales, a net profit of 20% on cost can be made?

Solution: Let the selling price be Rs. x.

After deducting 4% commission, the net receipt

$$=96\% \text{ of } x = \frac{96}{100}x$$

Now, profit at 20% of Rs. 18,000/-

$$= \frac{20}{100} \times 18{,}000 = \text{Rs. } 3{,}600/\text{-}$$

Net receipt = Cost + Profit

= Rs. (18,000 + 3,600)

= Rs. 21,600/-

$$\text{Thus } \frac{96}{100}x = 21{,}600$$

$$x = \frac{21600}{96} \times 100 = \text{Rs. } 22{,}500/\text{-}$$

Hence, the goods should be sold for Rs. 22,500/-

Example 13

A salesman gets 4% commission on the first Rs. 20,000, 7% on the next Rs. 20,000 and 12% on the excess. He also receives an incentive at the rate of 3% on total sales, if it exceeds Rs. 50,000. Find the total earning of two salesmen with total sales worth of Rs. 44,000 and Rs. 58,000.

Solution:

(i) For the first salesman

Sales is Rs. 44,000 = 20,000 + 20,000 + 4,000

So, commission = 4% of 20,000 + 7% of 20,000 + 12% of 4,000

= 800 + 1400 + 480 = Rs. 2680

As his sales of Rs. 44,000 is less than Rs. 50,000, he will not get any incentive amount.

So, total earning of the first salesman = Rs. 2680

(ii) For the second salesman

Sales is Rs. 58,000 = 20,000 + 20,000 + 18,000

So, commission = 4% of Rs. 20,000 + 7% of Rs. 20,000 + 12% of Rs. 18,000

= 800 + 1,400 + 2,160

= 4360

His sales is Rs. 58,000, which exceeds Rs. 50,000

So, incentive amount = 3% of 58,000 = 1,740

So, total earnings of second salesman = commission + incentive

= 4360 + 1740

= 6,100

Example 14

A merchant gained a net gain of 18.75% on his cost of Rs. 20,000 after giving the agent a commission at 5% on the sale price. Find the sale price.

Solution: Let x be the sale price

Commission = 5% on sale = $\frac{5}{100}x = 0.05x$

Gross Profit = Sale Price – Cost Price

= x – 20,000

Net Profit = Gross Profit – Commission

= (x – 20,000) – 0.05x

= 0.95x – 20,000

But Net Profit = 18.75% on Cost Price

$= \frac{18.75}{100} \times 20{,}000$

= 3,750

$\therefore$ 0.95x – 20,000 = 3,750

$\therefore$ 0.95x = 23,750

$\therefore$ $x = \frac{23{,}750}{0.95} 25{,}000$

$\therefore$ The sale price is Rs. 20,000

Example 15

A merchant employed an agent to buy and sell some article. The agent charged 3% on the purchase value and 2% on the sale value as commission. The purchased value was Rs. 40,000. The merchant after deducting the commission still received a net profit of 19.5% on cost. Find the sale price.

Solution: Let x be the sale price.

Commission on purchase = 3% on purchased price

$= \frac{3}{100} \times 40{,}000$

$= 1{,}200$

Commission on sales = 2% on sale price

$= 0.02x$

$\therefore$ Total Commission = Commission on purchase + Commission on Sale

$= 1{,}200 + \frac{2}{100}x$

Net Profit = 19.5% on Purchased Price

= 19.5% on 40,000 = Rs. 7,800

But Net Profit = Sale Price – Purchase Price – Total Commission

$7{,}800 = x - 40{,}000 - (1{,}200 + 0.02\,x)$

$7{,}800 = x - 40{,}000 - (1{,}200 - 0.02\,x)$

$7{,}800 = x - 0.98x - 41{,}200$

$49{,}000 = 0.98\,x$

$\therefore \quad \frac{49000}{0.98} = x$

$\therefore \quad 50{,}0000 = x$

$\therefore$ The sale price was Rs. 50,000

Example 16

At what price should goods costing Rs.16,000 be sold through an agent so that after paying her a commission at 4% on sales, a net profit of 20% on cost can be made?

Solution: Let gross Receipts be Rs. x

After deducting 4% commission, the net receipt

$= 96\% \text{ of } x = \frac{96}{100}x$

Now, profit at 20% of Rs. 16,000

$= \frac{20}{100} \times 16{,}000 = \text{Rs. } 3{,}200$

$\therefore$ Net receipt= Cost + Profit

= Rs. (16,000 + 3,200)

= Rs. 19,200

Thus $\frac{96}{100}x = 19{,}200$

$$x = 19{,}200 \times \frac{100}{96} = \text{Rs. } 20{,}000$$

Hence, the goods should be sold for Rs. 20,000

Check your progress

---

1) An agent earns $6\frac{2}{3}$ % commission on his total sales which are of Rs. 11,520. Find the amount of his commission. **Ans**. Rs. 768

2) An insurance agent gets commission of 25% on first year premium, 7% on second and third years premium and 5% on subsequent years premium on an insurance policy of Rs. 30,000, annual rate of premium being Rs. 40 per thousand. Find the total earning of the agent for which 5 annual premiums have been paid. **Ans**. Rs.588

3) An insurance agency pays 20% of annual premium as commission to its agent in the first years, 9% in the second and the third year and 7.5% in each subsequent year. A customer insured her car for Rs. 90,000 through an agent and her annul premium was fixed at Rs. 5,600. Find the agent' total commission amount if the customer has paid annul premium for six year up to now. **Ans**. Rs. 3,388

4) An insurance company pays its agent a commission at 20% on the first year's premium amount. The rate of commission reduces to 6% for the subsequent years. A customer purchased a policy premium at 5% of the policy amount. Find the agent's total earning for the premium for the period of seven years. **Ans**. Rs. 3,200

5) A merchant pays 10% commission on total sales and pays del credere at a rate of 3% on credit sales. If cash sales were Rs. 4,500 and credit sales were Rs. 7,000 find the total commission earned by the agent.
Ans. 1,360

6) A merchant instructs his agent to buy 500 Raincoats and to sell them 20% above the purchase price. The agent charges 1% commission on purchase and 2% commission on the sales and earns Rs. 1,360. Find the price at which the agent buys a Raincoat. **Ans**. 80

7) An agent was instructed to sell 5000 cotton shirts on 2% commission and invest the balance after dedcuting commission in purchasing kurtas. The commission paid on purchase is $3/4$% If the agent earns together Rs. 3550.50 in two transactions, find the price at which a shirt was sold
Ans. Rs. 26

8) A del credere agent charges 3% on cash sales and 6% on credit sales. If the agent earns at an average rate of 4.7% on total sales, find the ratio of cash sales to the credit sales. **Ans**. 13 : 17

9) A salesman is paid a fixed monthly salary plus commission at a certain rate on sales. The salesman received Rs. 1,130 and Rs. 1,360 as remuneration for two successive months and his sales were Rs. 17,100 and 21,700 respectively. Find the fixed monthly salary and the rate of commission. **Ans**. 275; 5%

10) A del credere agent charges 4% commission on cash sales and 7% commission on credit sales. If the agent earns at an over all rate of 5.4% of total sales, find the ratio of cash sales to credit sales.

Ans. 8 : 7

11) A salesman is paid fixed monthly salary together with commission at a certain rate on his total sales during the month. The salesman received Rs. 1,256 and Rs. 1,372 as remuneration for two successive months during which his total sales were Rs. 21,400 and Rs. 24,300 respectively. Find the fixed monthly salary and the rate of commission.

Ans. Rs. 400 and 4%

12) An agent was paid Rs. 21,708 as commission at a rate 9% on the sale of Refrigerators. The selling price of each Refrigerator was 13,400. Find the number of Refrigerators sold by agent.

Ans. 18 Refrigerators

13) A piece of land was sold for Rs. 19,00,000 through a broker who received 1¼ % commission from the seller and 1¾ % commission from the buyer. Find the amount paid the buyer. Find also the amount received by the seller and the broker

Ans. 19,33,250, Rs. 18,76,250, Rs. 57,000

14) A merchant instructs his agent to buy 600 pieces of an article and sell at 25% above the purchase price. The agent charges ½ % commission on the purchase and 3% commission on the sale and earns Rs. 1,020. Find the price at which the agent buys an article. **Ans**. Rs. 40

15) A del credere agent charges 4% on cash sales and 7% on credit sales. If the consolidated rate at which the agent earns is 5% of total sales find the ratio of cash sales to the credit sales. **Ans**. 2 : 1

16) A salesman is paid a fixed monthly salary together with commission at a certain rate on the sales. The salesman received Rs. 1,240 and Rs. 1,184 as remuneration for two successive months in which his total sales were Rs. 18,500 and Rs. 17,100 respectively. Find the rate of commission and the fixed monthly salary. **Ans**. 4% Rs. 500

17) A company fixed the rate of commission to its salesman as follows: 4% on the first Rs. 5,000, 5% on the next Rs. 8,000, 7% on the next Rs. 10,000 and 11% on the balance. Company has agreed to pay ¼ % of the total sales as bonus if the sales crossed Rs. 30,000. A salesman of the company secured sales worth Rs. 29,000. Calculate total earning of the salesman. **Ans**. Rs. 1,960

18) A merchant asked his agent to sell 250 hats at 2% commission and to invest the balance in purchasing ties. The agent charged $1\frac{1}{2}$% commissions on both the transactions. Find the price for which a hat was sold. **Ans**. Rs. 60

19) A merchant asked his agent to sell 200 caps at Rs. 15 per cap and to purchase 400 towels at Rs. 10 per towel. The agent charged 2%on the sales and 1% on the purchase. How much extra money did the merchant have to pay to the agent? **Ans**. Rs. 1,110

20) A merchant asked his agent to sell 40 music records at Rs. 25 each and use the money to buy some toys at Rs. 10 each. The agent quoted 15% commission on the sale and 6.25% on the purchase. The merchant instructed the agent to adjust the number of toys such that the merchant would not need to give him any extra money. How many toys would the agent buy? **Ans**. 800

21) The income of a salesman remains unchanged through the rate of commission is increased from 4% to 5%. Find the percentage reduction in his sales. **Ans**. 20%

22) At what price should goods costing Rs.48,250 be sold through an agent so that after paying him a commission at $3\frac{1}{2}$%, a net gain of 20% on the cost may be made? **Ans**. Rs. 60,000

1.5 DISCOUNT

During a festival season you must have seen advertisement banners displayed in front of each shops offering discount on the purchases made by the customer. Such advertisements are published in the newspapers also. Each shop owner proclaims to sell his goods at 20%, 15%, 30% discount etc.

Discount means reduction in price or less than marked price.

Marked price /printed price/Catalogue price/List price is the price quoted on the article for sale.

Trade Discount is allowed on the list price (L.P)
List Price - Trade Discount = Invoice Price
Invoice Price - Cash Discount = Net selling price
Net selling price – Profit = cost

Example 17

Sachin buys a kurta in a Khadi Bhandar marked at Rs. 400. If a discount of 18% is allowed, find the discount.

Solution:

Marked price = List price (L.P.) = Rs. 400
Rate of Discount = 18% on L.P.
Discount = 18% on L.P.
Discount = 18% on 400 $= \frac{18}{100} \times 400$ = Rs.72

Example 18

Priya purchased a table whose marked price is Rs.7,000. The dealer allowed 6% discount on it. Find (i) discount on Rs.7,000 (ii) amount paid by Priya for the table.

Solution:

(i) Discount = 6% on L.P.
= 6% on Rs. 7000 $= \frac{6}{100} \times 7000$
= Rs. 420

(ii) Amount paid by Priya for the table (or) S.P.
= L.P. – Discount = 7,000 – 420
= Rs. 6,580

Example 19

A furniture maker sells a wooden table listed Rs.2,500 for Rs.2,200. Find the rate of discount.

Solution:

Listed price = Rs. 2,500
S.P = Rs.2,200

$\therefore$ Discount = L.P. – S.P. = 2,500 – 2,200
= Rs. 300

Rate of discount $= \frac{Discount \times 100}{L.P.}$

$= \frac{300 \times 100}{2,500}$ = 12%

1.6 TRADE DISCOUNT and CASH DISCOUNT

When a manufacture or a trade offers a discount to another trader, the discount offered is usually termed as the trade discount. For example, when a wholesaler sells goods in bulk to a retailer, he / she gives a trade discount. It is also sometimes called as the bulk discount. We shall refers to this as the trade discount and write it as T.D.

The rate of Trade Discount is expressed as percentage of the list price. After deducting the trade discount, the remaining amount is what is put down in the invoice or the bill. Hence this is referred to as the invoice price or the reduced list price or the amount of the bill. We will refer to it as the invoice price and denote it as I .P.

Thus I.P. = L.P. – T.D. Amount.

Even after giving the above trade discount, the seller may offer an extra discount to those buyers who pay immediately by cash and do not ask for credit. This discount is called cash discount and we will denote it as C.D.

The rate of cash discount is expressed as a percentage of the invoice price. Note that the rate of cash discount is not expressed as a percentage of the list price.

After deducting the cash discount from the invoice price, we get the Net Selling Price (N.S.P) = I.P. - C.D.

This can be remembered in the following way:

L.P.

One discount is given
L.P. = Rs ( )
Discount = ( ) % on L.P.
SP = L.P. – discount

Two discounts given T.D, C.D.
L.P. = Rs. ( )
T.D. = ( ) % L.P.
I.P. = L.P. – T.D.
C.D. = ( ) % ON L.P.
S.P. = I.P. – C.D.

Example 20

A firm allows 20% discount on list price and a further 5% discount for cash payment. What is the net selling price of an article which is marked at Rs. 240

Solution: Given TD% = 20, CD% = 5 and L.P. = Rs. 240.

I.P. = L.P. – Trade Discount Amount
= L.P. – 20% of L.P.

$= 240 - 20\%$ of $240 = 240 - \frac{20}{100} \times 240 = 240 - 48$

I.P = Rs. 192.

N.S.P = I.P. – Cash Discount Amount = I.P. – 5% of I.P

$= 192 - 5\%$ of $192 = 192 - \frac{5}{100} \times 192 = 192 - 9.60$

N.S.P = Rs 182.40

Hence the net selling price is Rs. 182.40.

Example 21

A firm allows 25% of trade discount and a further discount for cash payment at 10% rate. Find the list price of an article with a net selling price of Rs.270.

Solution:

Given that T.D. % = 25, CD% = 10 and N.S.P. = Rs 270.

N.S.P. = I.P. – Cash Discount Amount

270 = I.P. – 10% of I.P = 90% of I.P. = $\frac{90}{100} \times I.P.$

$\frac{100}{90} \times 270 =$ I.P.

∴ Rs. 300 = I.P.

Now, I.P. = L.P. – Trade Discount Amount

300 = L.P. – 25% of L.P. = 75% of L.P. = $\frac{75}{100} \times L.P.$

$\frac{100}{75} \times 300$ = L.P.

∴ Rs. 400 = L.P.

Hence the list price is Rs. 400.

Example 22

An article was listed at Rs. 5,000. It was sold by the shop owner at a 30% discount and thus a profit of 75% on cost was earned. Find the cost price of the shop owner.

Solution:

Given that Discount% = 30, Profit% = 75 and L.P. = Rs. 5000

N.S.P. = L.P. – Discount Amount = L.P. – 30% of L.P

= 70% of L.P. = $\frac{70}{100} \times 5{,}000.$ = Rs. 3,500.

Now, N.S.P = C.P. + Profit

3500 = C.P. + 75% of C.P.

= 175% of C.P.

$\therefore$ 3500 = $\frac{175}{100} \times$ C.P.

$\therefore$ $\frac{100}{175} \times 3{,}500$ = C.P.

$\therefore$ Rs. 2,000 = C.P.

Hence the cost price is Rs. 2,000.

Example 23

An article was sold for a net selling price of Rs. 6,225 after giving a 17% discount on the list price and thus a 24.5% profit was gained on cost. Find the list price and the cost price.

Solution:

Given that N.S.P. = Rs 6,225, Discount = 17% and profit = 24.5%

N.S.P = L.P. – Discount Amount

6,225 = L.P. – 17% of L.P

= 83% of L.P. = $\frac{83}{100} \times L.P.$

$\frac{100}{83} \times 6{,}225 =$ L.P.

Rs. 7,500 = L.P.

Also, N.S.P = C.P. + Profit

6,225 = C.P. + 24.5% of C.P.

6,225 = 124.5% of C.P.= $\frac{124.5}{100} \times$ C.P.

$\frac{100}{124.5} \times 6{,}225 =$ C.P.

$\therefore$ Rs. 5,000 = C.P.

The list price is Rs. 7,500 and the cost price is Rs.5,000.

Example 24

A mall gave a 10% discount on a watch and still gained 62% profit. How much per cent above cost had the mall marked the watch for sale?

Solution:

Given that discount% = 10 and Profit % = 62. Since no price is supplied, we assume C.P. = 100

$\therefore$ N.S.P = C.P. + Profit = 100 + 62% of 100

= 100 + 62 = 162

Also, N.S.P = L.P. – Discount Amount

162 = L.P. –10% of L.P = 90% of L.P.

$$162 = \frac{90}{100} \times L.P.$$

$$\frac{100}{90} \times 162 = \text{L.P.}$$

$\therefore$ Rs. 180 = L.P.

Example 25

What percentage of profit is earned by marking goods 60% above cost and then giving a 8% discount?

Solution:

Since no price is given, we assume C.P. = 100

Now L.P. is 60% above C.P.

i.e. L.P = 160% of C.P. = C.P. + $\frac{60}{100} \times 100$ = 100+60 = 160

$\therefore$ Discount = 8% on L.P. = $\frac{8}{100} \times 160$ = 12.80

N.S.P = L.P. – Discount Amount

= 160 – 12.80 = 147.20

Profit = N.S.P. – C.P.= 147.20 – 100

= 47.20

Thus, on C.P = 100, the profit is 47.20. Hence the Profit is 47.20%

Example 26

A trader gives 35% discount and makes 30% profit on cost. Due to recession, the cost price reduces by 20%. By what per cent should he lower the list price, if he wished to keep the profit percentage and the discount percentage same as before?

Solution:

Given the Discount % = 35 and Profit% = 30.

Let the cost price be 100 i.e. C.P. = 100.

N.S.P = C.P. + Profit = C.P. + 30% of C.P.

$$= 100 + \frac{30}{100} \times 100 = 100 + 30$$

=130.

Now, N.S.P = L.P. – Discount Amount

130 = L.P. – 35% of L.P = 65% of L.P.= $\frac{65}{100} \times L.P.$

$\therefore$ $\frac{100}{65} \times 130$ = L.P.

$\therefore$ 200 = L.P.

Now due to recession, C.P. reduces from 100 by 20% of 100.

i.e. C.P. reduces from 100 by 20

i.e. C.P. reduces from 100 to 100 – 20 =80.

$\therefore$ New C.P. = 80.

The profit percentage is still 30 and the discount percentage is still 35.

$\therefore$ New S.P = New C.P. + New Profit = New C.P + 30% of New C.P.

New S.P. = $80+\frac{30}{100}\times100$ = 80 + 24 = 104.

Now, New N.S.P. = New L.P. – New Discount Amount

$\therefore$ 104 = New L.P. – 35% of New L.P. = 65% of New L.P.

$\therefore$ 104 = $\frac{65}{100}$ x L.P.

$\therefore$ $\frac{100}{65}\times104$ = New L.P

$\therefore$ 160 = New L.P.

$\therefore$ L.P. has reduced from 200 to 160

$\therefore$ L.P. has reduced by 40

We want to find the percentage reduction in L.P.

When L.P. = 200, the reduction = 40.

When L.P. = 100, the reduction = $\frac{100}{200}\times40=20$

$\therefore$ He should reduce his list price by 20%

Example 27

An article has a catalogue price of Rs. 400. The seller gave 20% trade discount and a further 5% discount for cash payment and still managed to get a 52% profit on her cost. What was the cost price?

Solution:

Given L.P. = Rs 400, T.D.% = 20, C.D. % = 5 and profit % = 52

L.P. = L.P. – T.D. Amount = 400 – 20% of 400

= 400 – $\frac{20}{100}\times400$ = 400 – 80 = Rs. 320

N.S.P. = I.P. – C.D. Amount = 320 – 5% of 320

= 320 – $\frac{5}{100}\times320$ = 320 – 16 = Rs. 304

Now, N.S.P = C.P. + Profit

304 = C.P. – 52% of C.P. = 152% of C.P.

$\therefore$ 304 $= \frac{152}{100}$ C.P.

$\therefore$ $\frac{100}{152} \times 304 =$ C.P.

$\therefore$ Rs. 200 = C.P.

The cost of the price is Rs. 200

Example 28

A trader allows a trade discount of 8% on the list price of his goods and a further discount of 2% for cash payment and still gains 12.7% profit on this cost. How much percent above cost has he marked his goods for sale?

Solution:

Given that T.D% = 8, C.D.% = 2 and profit % = 12.7

Since no price is given, we assume C.P. = 100

N.S.P. = C.P. + Profit = 100 + 12.7% on 100 = 100 + 12.7

= 112.7

Also, N.S.P. = I.P. – C.D. Amount

112.7 = I.P. – 2% of I.P

112.7 = 98% of I.P. = $\frac{98}{100}$ x I.P.

$\frac{100}{98} \times 112.7 =$ I.P.

115 = I.P.

Further, I.P. = L.P. – T.D. Amount

115 = L.P. – 8% of L.P. = 92% of L.P.= $\frac{92}{100}$ x LP

$\frac{100}{92} \times 115 =$ L.P.

125 = L.P.

The list price is 125 of goods whose cost is 100.

Thus, the goods are marked 25% above cost.

Example 29

An agent is instructed by the manufacture to allow the retailer a discount at the rate of 20% on the list price and received from the manufacture a commission at the rate of 8% on the net selling price. The Agent sold goods worth Rs. 20,000 as per the list price. Calculate the commission and the amount received by the manufacture.

Solution:

Discount Amount = 20% on L.P.= $\frac{20}{100} \times 20{,}000$ = Rs. 4,000.

∴ N.S.P. = L.P. – Discount Amount = 20,000 – 4,000 = Rs. 16,000.

Commission = 8% on N.S.P. = $\frac{8}{100} \times 16{,}000$ = Rs. 1,280.

∴ Amount received by the manufacturer = N.S.P. – Commission
= 16,000 – 1,280
= Rs. 14,720

∴ The commission is Rs. 1,280 and the amount received by the manufacturer is Rs. 14,720.

Example 30

A trader bought a computer for Rs. 50,000 and listed it for Rs.65,000. He sold it through an agent, giving 4% trade discount and a further 1% cash discount. The agent charged 10% commission on the net selling price. Find the trader's profit percentage.

Solution:

C.P. = Rs 50,000 and L.P. = Rs. 65,000.

T.D. % = 4, C.D.% = 1 and Commission% =10

Now, T.D. Amount = 4% on L.P. = $\frac{4}{100} \times 65{,}000$ = Rs. 2,600

L.P. = L.P. – T.D. amount = 65,000 – 2,600 = Rs. 62,400

C.P. Amount = 1% of L.P. = $\frac{1}{100} \times 62{,}400$ = Rs. 624.

N.S.P. = I.P. – C.D. Amount = 62,400 – 624 = Rs. 61,776

Commission = 10% of N.S.P. = $\frac{10}{100} \times 61{,}776$ = Rs. 6,177.60

Amount received by the trader = N.S.P. – Commission
= 61,776 – 6,177.60 = Rs. 55,598.40

Profit = Amount received by the trader – C.P. = 55,598.40 – 50,000
= Rs. 5,598.40

Profit% = $\frac{\text{profit}}{\text{C.P.}} \times 100 = \frac{5{,}598.40}{50{,}000} \times 100 = 11.1968. \approx 11.2$

∴ The profit Percentage is 11.2%

Example 31

An agent is instructed to give a 10% cash discount for any cash payment. The agent's commission is 2% on credit sales and 5% on cash sales on the net selling price. In a month, the agent sells goods worth Rs.

40,000 on credit and Rs. 50,000 for cash as per the list price. Find his total commission for the month.

Solution:

For credit Sales, there is no discount given, hence N.S.P. = L.P.

$\therefore$ Commission on credit sales = 2% on N.S.P.= 2% on L.P.

$$= \frac{2}{100} \times 40,000 = \text{Rs } 800$$

Now for the cash sales, Discount Amount =10% on L.P.

Discount Amount = $\frac{10}{100} \times 50,000 =$ Rs.5, 000.

$\therefore$ N.S.P. = L.P. – Discount Amount = 50,000 –5,000 = Rs. 45,000.

Commission on cash sales = 5% on N.S.P. = $\frac{5}{100} \times 45,000$

= Rs. 2,250.

Total Commission = Commission on Credit Sales + Commission on Cash Sales

= 800 + 2,250 = Rs. 3,050

Thus, the agent's total commission is Rs. 3,050.

Check your progress

---

1) The printed price of wrist watch is Rs. 880. Bur the factory supplied to a dealer at Rs 844.80 per watch. Find the rate of discount allowed on each wrist watch.

Ans. 4%

2) The Marked price of radio is Rs.700. The shopkeeper allows 12 ½ % discount on it. At what price does he sell the radio?

Ans. Rs. 612.5

3) A manufacture sold utensils for net Price of Rs. 9025 after giving 5% discount on the list price. What was the list price?

Ans. Rs. 9500

4) A firm allows a regular discount of 20% on the listed price and also a further 5% for cash payment. What is the selling price of the goods listed at Rs. 12,500?

Ans. Rs. 9,500

5) A firm allows a trade discount at 25% and a further discount of 10% on cash payment. Find the net price of an article which is listed for Rs. 400.

Ans. Rs. 270

6) An article is marked at Rs. 7,000. Trade discount of 25% is allowed and a further discount of 8% for cash payment is allowed. Find the net selling price of the article.

Ans. Rs. 4830

7) A firm allows a trade discount at 25% and a further discount of 10% for cash payment. Find the selling price of an article which is marked for Rs. 2,000.

Ans. Rs. 1350

8) Find the selling price of T.V. set which is marked for Rs. 10,000 after allowing at 20% trade discount and a 5.5% cash discount.

Ans. Rs. 7600

9) 3% discount is offered on T.V. set whose list price is Rs. 21,000 and a further 1% is offered on cash payment. Find what will be the selling price by cash payment.

Ans: Rs. 20166.30

10) A shopkeeper sold a T.V set for a net amount of Rs. 10,450 after allowing 20% discount on the list price and the further discount of 5% for cash payment. Find list price.

Ans.Rs.13750

11) An article is marked at Rs. 1500. A trader allows a discount of 7% and still gains 20% profit. What was the cost price of the article?

Ans. Rs.1162.50

12) A trader sold an article at 20% discount and still gets 25% profit on the cost. If the cost price was Rs. 1600, find the list price.

Ans. Rs.2500

13) A supermarket gave a 10% discount on an article and thus gained 44% profit. How much percent above cost had the super market marked the article for sale.

Ans. 60%

14) A trader marked an article 40% above his cost and even after giving a discount, gained 19% profit. What was the percentage of the discount given?

Ans. 15%

15) A wholesaler allows 25% trade discount on the list price and earns a profit of 20% on cost. If the cost increases by 10%, by what percent should he raise the list price so as to make the same rate of profit, after allowing the usual trade discount of 25%?

Ans. 100%

16) A trader bought an article for Rs. 80,000 and listed it for Rs. 1,00,000. His agent sold it by giving 14% discount o the list price and charging 2% commission on net selling price. Find traders profit percentage.

Ans.5.35%

1.7 PARTNERSHIP

A form of organization where two or more persons called partners carry on a business together and agree to share profit or losses at a specified proportion is called partnership.

Very often for starting a large business or for expanding an already existing business, a huge capital is required. Many a time a single person does not possess such large amount and in this case, some persons come together and provide necessary capital. Thus the business is run by more than one person by putting capital and sharing the profits and risks at agreed proportion is called *Partnership* and persons who provide capital are called *partners.*

The amount of profit or losses to be shared by partners depends on many factors such as (1) amount invested by partners, (2) the duration for which an amount is invested by a partner, (3) knowledge of a partner regarding the business, (4) effort made by different partner for the business etc.

Goodwill of a business organization is the difference between the market value and the net worth. The value of the goodwill is a quality that keeps changing. The valuation of goodwill is also required to be done in the event of death of a partner for settlement of his dues.

The calculation of goodwill at the time of entry of a new partner or exit of an existing partner is based on average profit of a certain number of previous years.

When a partnership is dissolved, its assets are sold and total liabilities which include the capital are paid. The balance if any is the goodwill; this is shared in the profit sharing ratio. If liabilities are more than the sale proceeds then the losses are also shared in the profit sharing ratio.

Solved Examples

Example 32

A, B, C start a business by investing Rs. 20,000, Rs. 35,000 and Rs. 45,000 respectively and share the profit of Rs. 10,000 at the of the year. Find the share in profit of each partner.

Solution:

Capitals are different but period is the same.

$\therefore$ The profit of Rs. 10,000 is to be shared in the proportion of capital.

The proportion of capital is 20,000 : 35,000 : 45,000 i.e. 4 : 7 : 9

Let A’s share of profit = Rs. $4x$,

B's share of profit = Rs.$7x$ and
C's share of profit = Rs. $9x$.

$4x + 7x + 9x = 10,000$

$20x = 10,000$

$x = 500$

$\therefore$ A's share of profit = Rs. 2,000

$\therefore$ B's share of profit = Rs. 3,500

$\therefore$ C s share of profit = Rs. 4,500

Example 33

Mr. X, Mr.Y and Mr. Z started a transport business by investing Rs. 1 lakh each. Mr.X left after 5 month from the commencement of business and MR. Y left 3 month later. At the end of year the business realized a profit of Rs. 37,500. Find the shared of profit of each partner.

Solution:

Capital is the same but the periods are different.

$\therefore$ Profit of Rs. 37,500 is shared in proportion of period of investment.

Proportion of period is 5 : 8 : 12

Let Mr X's share of profit = Rs. 5x

Mr. Y's share of profit = Rs. 8x

Mr.Z's share of profit = 12x

$\therefore$ 5x + 8x + 12x = 37,500

$\therefore$ 25x = 37,500

x = 1,500

Mr X's share of profit = Rs. 7,500

Mr. Y's share of profit = Rs. 12,000

Mr.Z's share of profit = Rs. 18,000.

Example34

A started a business by investing Rs. 40,000. 4 months after commencement of a business B joined as a partner investing Rs.60,000 and C joined the partnership one month after B by investing Rs. 60,000. At the end of the year the partnership earned a profit of Rs. 46,000. Find the share in profit of each partner.

Solution:

A's investment of Rs. 40,000 was for 12 months

B's investment of Rs. 60,000 was for 8 months

C's investment of Rs. 60,000 was for 7 months

$\therefore$ Total profit of Rs. 46,000 is shared in the proportion

$12 \times 40,000 : 8 \times 60,000 : 7\times 60,000$

i.e. 48 : 48:42 i.e. 8 :8 : 7

let A' share of profit be Rs. $8x$

B's share of profit be Rs. $8x$

C's share of profit be Rs. $7x$

∴ $8x + 8x + 7x = 46{,}000$
∴ $23x = 46{,}000$
∴ $x = 2{,}000$
∴ A's share of profit = Rs. 16,000
∴ B's share of profit = Rs. 16,000
∴ C s share of profit = Rs. 14,000

Example 35

Bala and Shiva are partners in a company and their capitals are their capitals are in the ratio 5 : 6. Bala and Shiva shared the yearly profit in the ratio 5 : 4 and Shiva had joined the business some months after Bala had started it. Find the period of investment of Y.

Solution:

Capital of Bala and Shiva are in the ratio 5 : 6
Let Capital of Bala = Rs. $5a$ and Capital of Y = Rs. $6a$
Period of investment by Bala = 12 months.
Let Period of investment by Shiva = b months.
∴ Bala and Shiva share the profit in the ratio $12 \times 5a : b \times 6a$ i.e. $10 : b$
Given : Bala and Shiva share the profit in the ratio 5 : 4
∴ $10 : b = 5 : 4$

$$\frac{10}{b} = \frac{5}{4}$$

∴ $b = 8$
∴ Period of Shiva's investment = 8 months.

Example 36

Rama and Krishna are partners having capital in the ratio 4 : 3. After 4 months Rama withdraws 25% of his capital but Krishna makes up for the lost capital by investing an amount equal to that withdrawn by Rama. At the end of the year Krishna received Rs. 30,000 as his share of yearly profit. Find the total profit.

Solution:

Capital of Rama and Krishna are in the ratio 4 : 3
Let Rama's Capital = Rs. $4x$ and Krishna's capital = Rs. $3x$
After 4 month Rama withdraws 25% of his capital i.e. Rs. x but Krishna makes up for the lost capital.
∴ Rama's investment is Rs. $4x$ for 3 month together with Rs. $3x$ for 9 months.
Krishna's investment is Rs. $3x$ for 3 month together with Rs. $4x$ for 9 months.
∴ The total profit is shared in the ratio
$(4x \times 3 + 3x \times 9) : (3x \times 3 + 4x \times 9)$
i.e. $39x : 45x$
i.e. 13 : 15

Let Rama's Share of Profit = Rs. 13y
& Krishna's share of yearly profit = 15y
Given : Krishna's share of yearly profit = Rs. 30,000
$\therefore$ 15y = 30,000
$\therefore$ y = 2,000
$\therefore$ Total Profit = Rs. 28y = Rs. 56,000

Example 37

Mr. Wadia and Mr. Jokhi started a business with a capital investment of Rs. 55,000 and Rs. 35,000 respectively. After 5 months, Mr. Wadia put in Rs. 10,000 more as capital while Mr. Jokhi withdrew Rs. 5,000 from his existing capital. At the end of the year, the profit was Rs. 11,150. Determine the proportionate distribution of the profit between the two partners.

Solution:

Mr. Wadia invested Rs. 55,000 for the first 5 months and Rs. 65,000 for the next 7 months. Mr. Jokhi invested Rs. 35,000 for the first 5 months and Rs. 30,000 for the next 7 months. Hence the profit distribution will be in the proportion
(55,000 × 5 + 65,000 × 7) : (35,000 × 5 + 30,000 × 7)
i.e. (55 × 5 + 65 × 7) : (35 × 5 + 30 × 7) … (dividing by 1,000)
i.e., in the proportion (275 + 455) : (175 + 210)
i.e., in the proportion 730 : 385
Now 730 + 385 = 1,115

$\therefore$ Mr. Wadia's share in the profit $= \frac{730}{1{,}115} \times 11{,}150 = Rs.7{,}300$

Mr. Jokhi's shars in the profit $= \frac{385}{1{,}115} \times 11{,}150 = Rs.3{,}850$

Example 38

A and B are partners in a business with their capital as with a capital as Rs. 2,00,000 and Rs. 3,00,000 respectively. C wishes to join the business with a capital of Rs. 3,00,000 at the beginning of a financial year. They agree that the goodwill will be taken as twice the average annual profit for the last three years. Last three years' profit are Rs. 90,000, Rs. 70,000 and Rs. 80,000 respectively. Find the goodwill amount that C would be required to pay A and B separately.

Solution:

Average of last three years' annual profits $= \frac{90000 + 70000 + 80000}{3}$

= Rs. 80,000

$\therefore$ Goodwill = 2 × 80,000 = Rs. 1,60,000.

Now, A, B, and C have capital of Rs. 2,00,000, Rs. 3,00,000 and Rs. 3,00,000 respectively.

$\therefore$ Their share in the profit as well as their share in the goodwill will be in the proportion 2,00,000 : 3,00,000 : 3,00,000, i.e., in the proportion 2 : 3 : 3.

∴ C's share of goodwill at the entry point

$$= \frac{2}{2+3+3} \times 1,60,000 = \frac{2}{8} \times 1,60,000 = \text{Rs.}40,000$$

∴ C must pay Rs. 40,000 as goodwill to A and B whose capitals are in the ratio 2:3

∴ A's share of goodwill = $\frac{2}{2+3} \times 40,000 = \frac{80,000}{5} = Rs.16,000.$

∴ B's share of goodwill = $\frac{2}{2+3} \times 40,000 = \frac{1,20,000}{5} = Rs.24,000.$

∴ C should pay Rs. 16,000 and Rs. 24,000 as goodwill to A and B respectively, on being admitted to the partnership.

Check your progress

---

1. Three friends A, B, C hired a small stall in partnership in an exhibition, by investing Rs. 2,100, Rs. 3,300 and Rs. 3,000 respectively. At the end of the day, their profit was Rs. 560. Find the share of each friend.

Ans. Rs. 140, Rs.220, Rs.200

2. Three partners opened a bicycle repairing shop with an investment of Rs. 4,000, Rs. 5,000 and Rs. 6,000 respectively. If their profit after one month is Rs. 4,500, find the share of each partner.

Ans. Rs. 2,000, Rs. 2,500, Rs. 3,000

3. Three partners floated a company with contribution of Rs. 10,000, Rs. 20,000 and Rs. 18,000 respectively. If the company made a profit of Rs. 4,800, find the share of each partner in the profit.

Ans. Rs. 1,000, Rs. 2,000, Rs. 1,800

4. Shikha launched an animation company with certain capital. After 2 months, Miraj joined the company and after 5 more months, Sagar too joined in as a partner. If the capital put in by all the partners in the company is the same and if the profit of the company at the end of the year was Rs. 81,000, find the share of each partner.

Ans. Rs. 36,000, Rs. 30,000, Rs. 15,000

5. Priya and Mythili are partners in the company and their capitals are in the ratio 5:6. Mythili has joined the business some months after Priya had started it and they shared the yearly profit in the ratio 5:4.Find the period of investment of Mythili.

Ans. 8 months.

6. A and B are partners in a business for some years. Their capital are Rs.3 lakhs and Rs. 2 lakhs respectively. C wants to join the business with a capital of Rs. 4 lakhs. They agree that the goodwill will be considered as three times the average of the last four years' profit which are Rs. 60,000, Rs. 75,000, Rs,65,000 and Rs. 70,000 respectively. What are the amounts to be paid by C to A and B as goodwill?

Ans. Rs. 54,000, Rs. 36,000

Formulae

1) Commission $= \frac{\text{rate of Commission}}{100} \times \text{salevalue}$

2) Gross profit = Sale Price – Purchase Price

3) Net Price = Gross profit – Commission

4) Discount % $= \frac{\text{Discount Amount}}{\text{List Price}} \times 100$

5) Trade discount(T.D) = $\frac{\text{Trade discount \%}}{100} \times \mathit{List}\ \mathrm{Pr}\,\mathit{ice}$

6) Invoice price (I.P) = List price – Trade discount

7) Cash discount (C.D) = $\frac{\text{cash discount \%}}{100} \times \mathit{Invoiceprice}$

8) Net selling price (N.S.P.) = Invoice price – cash discount

9) Profit % = $\frac{\text{Profit Amount}}{\text{Cost Price}} \times 100$

10) Net selling Price = Cost price + profit

11) Profit = N.S.P. – Commission – C.P.

12) Profit = L.P. – Discount – Commission – C.P.

13) Loss = Cost price – selling price

❖❖❖

UNIT II

2

SHARES AND MUTUAL FUNDS

OBJECTIVES:

After going through this chapter you will be able to:

- Define a share, face value, market value, dividend, equity shares preferential shares, bonus shares.
- Understand the concept of Mutual fund.
- Calculate Net Income after considering entry load, dividend, change in Net Asset Value (N.A.V) and exit load.
- Understand the Systematic Investment Plan (S.I.P).

2.1 INTRODUCTION

In day-to-day life we hear about shares, share market etc. Here we will see, exactly what these terms deal with .

When a group of persons plan to establish a company, they form a company under the companies Act 1956. Now this company is an established company. The people who establish this company are called promoters of the company. These promoters can now raise a certain amount of capital to start (run) the company. They divide this required amount into small parts called shares.

A share is the smallest unit of capital of a company. Stock is the American term for share. Usually a share is of value Rs. 100 /- or Rs.50/- or Rs. 10 /-or Rs. 5/- or Rs. 2/- or Rs./- 1 . This value is called the face value of the share. These shares are sold to the public. (usually face value is Rs. 10/- , unless otherwise specified). This sale is called the Initial Public Offer (IPO) of the company.

The company issues share certificates to the persons from whom it accepts the money to raise the capital. Persons who have paid money to form the capital are called share holders. Now-a- days the shares are not in the form of paper, but in the electronic dematerialised (Demat) form, hence the allotment of shares is done directly in the demat account,without a certificate.

Face value or nominal value or Par value is the value printed on the share certificate. Since shares exist in electronic demat form, we can say that the face value is the value stated in the I.P.O. subscription form.

The shareholders enjoy the profits (if any) of the company, after providing for the taxes and other reserve funds. This is called as **dividend.**

Types of shares ; Mainly the shares are of two types **i)** Preference shares and **ii)** Equity shares or common shares or ordinary shares .

i) Preference shares : These shares have a priority over the equity shares. From the profits made by a company, a dividend at a fixed rate is paid to them first, before distributing any profit amount to the equity shareholders. Also, if and when the company is closed down then while returning of the capital, these shareholders get a preference. Again, preference shares are mainly of two types:

a) Cumulative Preference shares: In case of loss or inadequate profit , The preference shareholders are not paid their fixed rate of dividend , then the dividend is accumulated in the subsequent years to these shareholders & is paid preferentially whenever possible .

b) Non-cumulative Preference shares: As in the case of cumulative preference shares , here the unpaid dividends do not accumulate.

ii) Equity shares : These are the shares for whom the dividend and the return of capital is paid after paying the preference shareholders. In case of equity shares , the rate of dividend is not fixed and it is decided by the Board of Directors .

Share Market

Shareholders are allowed to buy or sell shares like commodities. Selling or buying a share for a price higher than its face value is legal. The share prices are allowed to be subject to the market forces of demand and supply and thus the prices at which shares are traded can be above or below the face value.

The place at which the shares are bought and sold is called a share market or stock Exchange and the price at which a share is traded is called its Market Price (MP) or the Market value. If the market price of a share is same as its face value, then the share is said to be traded at Par.

If M.P. is greater than face value of a share , then the share is said to be available at a premium or above par and is called premium share or above par share.

If M.P. is lower than face value of a share , then the share is said to be available at a discount or below par & the share is called a discount share or below par share .

The purchase and sale of shares can take place through brokerage firms and depositary Participant (DP). e.g. Sharekhan.com, Kotak Securities ltd , ICICI direct.com etc. They charge a commission on the purchase and sale of shares, which is called as a brokerage. The brokerage is charged as a percentage of the M.P. of the share. Normally it is below 1%.

2.1.1 BONUS SHARES

Sometimes, when a company's free reserves are high, it may choose to capitalize a part of it by converting it into shares. This is done by issuing bonus shares to existing shareholders. These bonus shares are issued free of cost. The ratio of bonus shares to the existing shares is fixed.

Getting bonus shares increases the number of shares of shareholders. But since this applies to all the shareholders in a fixed ratio, hence the percentage of a shareholder's ownership of the company remains same as before.

Now, we will study some examples based on the above concepts:

Example 1

Mr. Prashant invested Rs. 75,375/- to purchase equity shares of a company at market price of Rs. 250 /- through a brokerage firm, charging 0.5% brokerage. The face value of a share is Rs. 10/-. How many shares did Mr. Prashant purchase?

Solution: Brokerage per share = $250 \times \frac{0.5}{100} = 1.25$

$\therefore$ cost of purchasing one share = 250+1.25 =251.25

$\therefore$ Number of shares purchased = $\frac{75375}{251.25} = 300$

Example 2

Mr. Sandeep received Rs. 4,30,272 /- after selling shares of a company at market price of Rs. 720 /- through Sharekhan Ltd., with brokerage 0.4%. Find the number of shares he sold.

Solution: Brokerage per share = $720 \times \frac{0.4}{100} = 2.88$

selling price of a share = 720 - 2.88 = 717.12

$\therefore$ Number of shares sold = $\frac{430272}{717.12} = 600$

Example 3

Ashus Beauty World ' has issued 60,000 shares of par value of Rs. 10/- each. The company declared a total dividend of Rs. 72,000 /- . Find the rate of dividend paid by the company.

Solution: Face value of 60,000 shares = 60,000 x 10 = 6,00,000

$$\text{Rate of Dividend} = \frac{\text{Total Dividend}}{\text{Face value of 60,000 share}} \times 100$$

$$= \frac{72000}{600000} \times 100 = 12$$

$\therefore$ The rate of dividend paid by the company is 12%

Example 4

The capital of ABC Company consists of Rs. 15 lakhs in 6 % cumulative preference shares of Rs. 100 each and Rs. 30 lakhs in equity shares of Rs.10/- each. The dividends on cumulative preference shares for earlier year was not paid . This year , the company has to distribute profit of Rs . 3 lakh after keeping 20 % as reserve fund. Find the percentage rate of dividend paid to the equity shareholders.

Solution: Reserve fund = $\frac{20}{100}$ x 300000 = Rs. 60,000/-

Profit to be distributed = 3,00,000 - 60,000 = 2,40,000

Annual dividend for 6 % cumulative preference shareholders

$$= \frac{6}{100} \times 1500000 = 90,000$$

This needs to be paid for 2 years (last year & current year) as the preference shares are cumulative & last year's dividend was not paid .

∴ Total Dividend paid to Preference shareholders

= 2 x 90000= 1,80,000/-

Now , dividend to be distributed to the equity shareholders

= 2,40,000 - 1,80,000= Rs. 60,000/-

∴ Rate of dividend = $\frac{60,000}{30,00,000} \times 100 = 2$

∴ The rate of dividend to the equity shareholders is 2 %

Example 5

Mr. Dinesh bought some shares of a company which had a face value of Rs.100 /-. The company declared a dividend of 15 % but Mr. Dinesh's rate of return on investment was only 12% . At what market price did he purchase the shares ? There was no brokerage involved .

Solution:

Dividend on one share = $\frac{\text{Rate of Dividend}}{100}$ x face value of one share

= 15 x 100 = Rs. 15/-

∴ Rate of Return on investment = $\frac{\text{Dividend on one share}}{\text{purchase price of 1 share}} \times 100$

∴ 12 = $\frac{15}{\text{purchase of 1 share}} \times 100$

∴ purchase price of 1 share = $\frac{15}{12} \times 100 = 125$

Example 6: Comparison of two stocks

Mr. Subu invested Rs. 20,000 /- in Rs. 100/- shares of company A at the rate of Rs. 125/- per share . He received 10 % dividend on these shares. Mr. Subu also invested Rs. 24,000/- in Rs. 10/- shares of company B at Rs.12/- per share and he received 15 % dividend. Which investment is more beneficial?

Solution : **<u>Company A</u>**

Rate of return = $\dfrac{\text{Dividend on one share}}{\text{purchase price of 1 share}}$ x 100 = $\dfrac{10}{125}$ x 100 = 8%

125

<u>Company B</u>

Rate of return = $\dfrac{1.5}{12}$ x 100 = 12.5 %

Investment in company B is more profitable .

Example 7

Ms. Ashma Mehta bought 300 shares of a company of face value Rs. 100 /- each at a market price of Rs. 240 /- each . After receiving a dividend at 8 % , she sold the shares at Rs . 256 /- each . Find her rate of return on investment. There was no brokerage involved .

Solution: Difference in the market price = 256-240= 16

Dividend on 1 share = $\dfrac{\text{Rate of dividend}}{100}$ x face value of 1 share

= $\dfrac{8}{100}$ x 100= 8

Rate of Return on Investment

= $\dfrac{(\text{Price change}) + (\text{Dividend on 1 share})}{\text{purchase price of 1 share}} = \dfrac{16+8}{240}$ x 100

= $\dfrac{2400}{240}$ = 10

$\therefore$ The rate of return on investment was 10 % .

2.1.2 Splitting of shares:

Sometimes companies split the face value of a share & break it up into smaller units . For e.g. a Rs. 100 /- share can be split into 10 shares each of face value Rs. 10 /- or a Rs. 10/- share can be split into two shares of face value Rs. 5/- each . Usually this does not affect a shareholder's wealth . However , it can make selling of a part of the holdings easier.

Example 8

Mr. Joshi purchased 30 shares of Rs. 10/- each of Medi computers Ltd. on 20th Jan. 2007, at Rs. 36/- per share . On 3rd April 2007 , the company decided t split their shares from the face value of Rs. 10/- per share to Rs. 2/- per share . On 4th April 2007 , the market value of each share was Rs. 8/- per share . Find Mr. Joshi's gain or loss , if he was to sell the shares on 4th April 2007? (No brokerage was involved in the transaction).

Solution: On 20th Jan 2007

purchase cost of 30 shares = 30 x 36 = 1080/-

On 3rd April 2007, each Rs. 10/- share became 5 shares of Rs. 2/- each.

$\therefore$ No. of shares = 30 x 5 =150

On 4th April 2007, market value of 150 shares was @ Rs. 8 each

= 150 x 8 = 1200

$\therefore$ His gain = 1200-1080 = 120/-

Example 9

Rahul purchased 500 shares of Rs. 100 of company A at Rs. 700 /-. After 2 months , he received a dividend of 25 % . After 6 months, he also got one bonus share for every 4 shares held . After 5 months , he sold all his shares at Rs. 610/- each. The brokerage was 2% on both, purchases & sales . Find his percentage return on the investment.

Solution: For purchase:

Face value = Rs. 100 /- , No. of shares = 500, market price = Rs.700/-

Dividend = 25% , brokerage = 2%

Purchase price of one share = $700 + \frac{2}{100}$x700= 714

$\therefore$ Total purchase = 500 x 714 = Rs.3,57,000/-

Dividend = $\frac{25}{100}$ of 100 i.e. Rs. 25 /- per share

$\therefore$ Total dividend = 500 x25 = Rs. 12,500 /-

Now, bonus shares are 1 for every 4 shares .

$\therefore$ No. of bonus shares = $\frac{1}{4}$ x 500 = 125

$\therefore$Total No. of shares = 500 +125 =625

For sales,

No. of shares = 625, market price = 610 , Brokerage 2%

Sale price of one share = 610 - 2% of 610 = 597.8

$\therefore$ Total sale value = sale price of one share x No. of shares

= 597.8 x 625 = Rs. 3, 73,625/-

Net profit = sale value + Dividend - purchase value

= 3,73,625 + 12500 - 3,57,000

= Rs . 29,125/- .

$\therefore$ % gain = $\frac{29,125}{3,57,000}$ x 100 = 8.16

= 8.16

EXERCISE

1) Mr. Amar invested Rs 1,20,480/- to buy equity shares of a company at market price of Rs . 480 /- at 0.4 % brokerage . Find the No. of shares he purchased . **Ans**: 250

2) Aditi invested Rs. 19,890 /- to purchase shares of a company with face value of Rs.10/- each , at market price of Rs. 130/- . She received dividend of 20 % as well Afterwards , she sold these shares at market price of Rs. 180/- . She had to pay brokerage of 2 % for both purchase and sales of shares. Find her net profit.

Ans: No. of shares =150 , sales = 26460 , Dividend = 300 , purchase = 19,890, profit= 6870

3) Amol wants to invest some amount in company A or company B , by purchasing equity shares of face value of Rs. 10 /- each , with market price of R. 360/- and Rs. 470/- respectively . The companies are expected to declare dividends at 20 % and 45% respectively . Advise him on the choice of shares of company.
Ans: company B is a better choice .

4) Find the percentage gain or loss if 200 shares of face value Rs. 10/- were purchased at Rs . 350/- each and sold later at Rs. 352 /- , the brokerage being 0.5 % on each of the transaction .

Ans: -0.43 % i.e. a loss of 43 %

5) Find the number of shares if the total dividend at 8% on the shares with face value Rs.10/- was Rs. 240. **Ans** :- 300

2.3 MUTUAL FUNDS

In the previous unit shares, we have studied how one can transact in shares. Now, we will study what are the mutual funds and how they function.

An investor can invest money directly in shares or he can invest his money through mutual funds. Mutual funds are managed by large financial services with a professional team of fund Managers & research experts.

Mutual fund is a pool of money, drawn from investors .The amount collected is invested in different portfolios of securities, by the fund managers and the profits (returns) , proportional to the investment, are passed back to the investors.

At a given time, the total value is divided by the total number of units to get the value of a single unit a given time. This is called Net Asset Value (NAV).

$$\therefore \text{NAV} = \frac{\text{Net Assets of the scheme}}{\text{Total No. of units outstanding}}$$

$$\text{or} \quad \text{NAV} = \frac{\text{Total Assets- liabilities}}{\text{Total No. of units outstanding}}$$

There are mixed or hybrid funds which invest in both debt and equity. The offer documents give the guidelines / constraints under which the fund managers would operate. e.g. investment in equity 80 % to 100 %, investment in money markets 0 % to 20 % etc.

In India , the mutual funds are governed by SEBI (Securities and Exchange Board of India) .There are different companies , called the ' Fund Houses ' (like SBI or Reliance or HDFC) which float different mutual funds. Each such fund is called a 'scheme' , e .g. HDFC has a scheme ' HDFC Tax saver ' etc .

Like IPO of a company's share , a mutual fund scheme starts by having a N.F.O. (New Fund Offer) . Investors can invest by purchasing Units of the mutual funds .Usually a unit is of Rs. 10/- . A share is the smallest unit of a company's capital , whereas in mutual funds , even a fraction of a unit can be purchased after the N.F.O .

Let us study the following example to understand this concept:

Example 10

A mutual fund 's scheme shows the following on 01/01/2007

Total value of securities (Equity , Bonds etc.)	Rs. 1500 crores
Cash	Rs . 100 crores
Liabilities	Rs . 200 crores
Total No. of units outstanding	Rs. 100 crores

$$\therefore \quad \text{NAV} = \frac{\text{Rs. 1500 crores + Rs. 100 crores - Rs. 200 crores}}{\text{100 crores}}$$

$$= \text{Rs. } \frac{\text{1400 crores}}{\text{100 crores}} = \text{Rs . 14 per unit .}$$

The NAV of a mutual Fund scheme is calculated and disclosed to the publc for evey working day . The NAV changes daily. Investors try to invest when NAV is low and sell the units and get profits when the NAV is high .

Most mutual fund schemes are not traded at stock market. Thus, investor purchases as well as sells the units to AMC i.e. Asset management company, This sale is called redemption of units.

Basically funds are of two types :-

1) close ended funds 2) open ended funds .

1) Close ended mutual funds :- These are offered with a fixed date of maturity and can be purchased from mutual fund companies during a specific period . The investor can get the amount after expiry date of the fund . If an investor wants to exit before the maturity date , he can sell the units on the stock exchange at a discount or through a buy-back option by the fund .

2) Open ended funds : These have no fixed date of maturity and the units can be sold or repurchased at any time .The no. of units & its capital changes daily .

Entry load & Exit load : Some mutual fund schemes collect a charge when investors purchase or redeem units . These are usually percentage of NAV . The charge levied while purchasing a unit is called the entry load & the charge collected on redemption is called exit load .

Usually , the debt funds have not loads . When there are no charges while purchasing or selling of units , these funds are called No Load Funds .

Mutual Funds can be broadly categorised into two types : 'Dividend ' funds which offer a dividend and 'Growth ' funds which do not offer a dividend .

In mutual unds , the dividend given has no direct relation to the profit earned . The mutual fund invests the money in different shares that may or may not give a dividend at different times & different rates . The fund manager may at any arbitrary point , decide to give a part of the units' value back to the investors . This is called dividend .

For a growth fund , the NAV does not come down due to dividends . It moves up or down purely on the basis of the gains or losses of the securities that the fund has invested in.

For a growth fund , the gains per unit are purely from the difference between the redemption price and the purchase price i.e. the total gain is purely the capital gain . For a dividend fund , the total gain is the addition of the capital gain & the dividend .

Capital gain = Amount received after redemption - Amount invested .

$$\text{Rate of Return} = \frac{\text{Change in NAV + Dividend}}{\text{NAV at the beginning of the period}} \times 100$$

(This is for a given period) .

$$\text{Annualised rate of Return} = \text{Rate of Return} \times \frac{365}{n}$$

where n is the number of days .

Some important Terms :

i) Assets :- It refers to market value of investment of M.F. in government securities , bonds etc. , its receivables , accrued income & other assets .

ii) liabilities :- It includes all expenses like accrued expenses , payables and other liabilities for the M.F. scheme .

iii) Net Assets :- Total Assets - liabilities

iv) The valuation Date is the date on which NAV is calculated .

Example 11

Mr. Deore invested Rs. 25,000/- to purchase 2,500 uits of ICICI MF - B plan on 4th April 2007 . He decided to sell the units on 14th Nov. 2007 at NAV of Rs. 16.4 /-. The exit load was 2.5 % . Find his profit (Calculations are upto 2 decimal points)

Solution :

No. of units =2500 , purchase cost = Rs. 25,000/-

NAV on the date of sale = RS. 16.4/- , exit load =2.5%= of 16.4 = 0.41

$\therefore$ selling price of 1 unit = 16.4- 0.41 = 15.99

$\therefore$ sale value = 2500 x 15.99

= Rs. 39,975/-

$\therefore$ Profit = 39,975 - 25 ,000
= Rs. 14,975 .

Example 12

Ragini invested Rs. 94,070/- in mutual Fund when NAV was Rs. 460 /- with entry load of 2.25 % . She received a dividend of Rs. 5/- per unit . She, later sold all units of fund with an exit load of 0.5 % . If her gain was Rs. 1654/-, find NAV at which she sold the units .
(Calculations are upto 2 decimal points)

Solution : purchase price of one unit = 460 + 2.25% of 460
= 460 +10.35 = 470.35

$$\text{No. of units purchased} = \frac{94{,}070}{470.35} = 200$$

Total dividend = 200 x5 = 1000

Gain = Profit + Dividend

$\therefore$ 1654 = Profit + 1000

$\therefore$ Profit = 1654- 1000= 654

While selling let NAV of one unit be y

$\therefore$ sale price of one unit = NAV - exit load
= y- 0.5% of y
= 0.995 y

$\therefore$ sale price of 200 units = 200 x 0.995 y= 199 y
Also , profit = Total sale - Total purchase
654 = 199y - 94,070
$\therefore$ 199y = 654 + 94,070
$\therefore$ 199y = 94724
$\therefore$ y= 476
$\therefore$ NAV at which she sold units = Rs. 476/- .

Example 13

If a mutual fund had NAV of Rs. 28 /- at the beginning of the year and Rs. 38/- at the end of the year , find the absolute change and the percentage change in NAV during the year .

Solution : NAV at the beginning = Rs. 28/-
NAV at the end = Rs. 38/-
$\therefore$ Absolute change in NAV = in 38-28 = Rs. 10/-

$$\% \text{ change} = \frac{\text{Absolute change}}{\text{NAV at the beginning}} \times 100 = \frac{10}{28} \times 100 = 35.71\ \%$$

Example 14

If NAV was Rs. 72/- at the end of the year , with 12.5 % increase during the year , find NAV at the beginning of the year .

Solution : Let 'x' be the NAV at the beginning of the year .

$\therefore$ Absolute change in NAV = 12.5 % of x = $\frac{12.5}{100}$ x x = 0.125 x

$\therefore$ NAV at the end of the year = x + 0.125 x = 1.125 x

$\therefore$ 1.125 x = 72

$\therefore$ x = $\frac{72}{1.125}$

= 64

$\therefore$ NAV's initial value was Rs. 64 /- .

Example 15

Rohit purchased some units in open end equity fund at Rs. 16/- . The fund distributed interim dividend of Rs. 5/- per unit , and the NAV of the fund at the end of the year was Rs. 25/- . Find the total percentage return . (Calculations are upto 2 decimal points)

Solution : Total gain = change in NAV + Dividend

= (25-16) + 5

= 9+5

= 14

$\therefore$ Total percentage gain = $\frac{\text{Total gain}}{\text{NAV at the beginning}}$ x 100

= $\frac{14}{16}$ x 100 = 87.5 %

Example 16

Mr. Hosur purchased some units in open- end fund at Rs. 30/- and its NAV after 18 months was Rs. 45/- . Find the annualised change in NAV as a percentage .

Solution : change in NAV for 18 months = 45-30 = Rs. 15/-

$\therefore$ annualised change = $\frac{\text{change in NAV}}{\text{NAV at beginning}}$ x $\frac{12}{\text{No. of months}}$ x 100

= $\frac{15}{30}$ x $\frac{12}{18}$ x 100

= 33.33 %

Check your progress

1) Mr. Kamble purchased 586.909 units of Kotak cash plus retail Growth on 1st June 2007 when the NAV was RS. 20.4461. Its NAV as on 3rd December, 2007 was Rs. 21.1960/- . The fund has neither entry load nor an exit load. Find the amount invested on 1st June 2007 and the value of Mr. Kamble's investment on 3rd December 2007 .

Ans . 12,000 , 12440.12 .

2) Ms . Kannan purchased 113.151 units of 'FT India Prima Plus' on 9th April 2007 and redeemed all the units on 7th Aug 2007 when the NAV was Rs. 35.5573 . The entry load was 2.25 % and the exit load was 1 % .

If she gained Rs. 483.11 , find the NAV on 9th April 2007 . (Calculations are upto 2 decimal points) **Ans** . 30.2514

3) Mr. Pandit invested Rs. 10,000/- in Birla Sunlife Equity Fund- Divjdend plan ' on 10/07/2007 , when the NAV was Rs. 78.04 ,and redeemed all the units on 12/11/2007 when the NAV was Rs. 84.54 . In the meanwhile , on 31/08/2007 , she had received a dividend @ Rs. 10 per unit . Find her total gain and the rate of return considering loads as follows:
The entry load was 2.25 % and the exit load was 0.5 % The number of units were calculated correct upto 3 decimal places.

Ans . Total gain = Rs . 1794.46, Rate of return = 17.94%

4) Given the following information , calculate NAV of the mutual fund :-
No. of units =15000
Market value of investments in Govt . securities = Rs. 20 lakhs
Market value of investments in corporate Bonds = Rs. 25 lakhs
Other Assets of the fund = Rs. 15 lakhs
Liabilities of the fund = 6 lakhs

Ans . Rs. 360/- .

5) Mumtaz purchased 1200 units of TATA BIG Bond- G Rs. 12,000 /- on 14th April 2007 . She sold her units on 9th Dec 2007 at NAV of Rs. 15.36/- . The short term gain tax (STGT) was 10% of the profit . Find her net profit . (Calculations upto 2 decimal points)
Ans . profit = 6432 , STGT = 643.2 , Net profit = 5788.8
(profit- STGT) .

2.4 SYSTEMATIC INVESTMENT PLAN (SIP)

In SIP an investor invests a fixed amount (e.g. say 1000/-) every month on a fixed date (e.g. 1st of every month) . In general the minimum amount is Rs. 1000/- per month , in diversified equity schemes . It can be even Rs. 500/- as well in ELSS schemes . If this is done for many months , then each time units are purchased at a different NAV . Over a period of few months, an investor gets the benefit of a phenomenon called 'Rupee cost Averaging' .

Rupee-cost- averaging :- If NAV increases , the no. of units decreases & if NAV decreases , the no. of units purchased increases . Thus on the whole , it lowers the average cost of units because indirectly ,the investor buys more units when NAV prices are low & he buys less units , when NAV prices are high . It is called Rupee-cost- Averaging .

Consider the following example :-
Mr. Shaikh keeps Rs. 5000/- on 3rd of every month for 4 months as follows :-(Calculations are correct to 2 points of decimal)

Month	Amount (in Rs.)	NAV	No. of units he gets
1	5000	109.48	5000/109.48=45.67
2	5000	112.36	5000/112.36=44.50
3	5000	108.14	5000/108.14 =46.24
4	5000	105.62	5000/105.62=47.34
Total	20,000		183.75

∴ Avg price of units = 20,000 / 183.75 = 108.84

If Mr. Shaikh would have invested the entire amount of Rs. 20,000/- 0n 3rd of first month only , with NAV Rs. 109.48/- , the no. of units purchased would have been 20,000/ 109.48 = 182.68

Thus he gained more units and average price of units also was Rs.108.84 instead of Rs.109.48 which was NAV on 3rd of the first month

If SIP is followed for a long period of time , it can create wealth to meet a person's future needs like housing , higher education etc .

Now , we will study the following examples to understand SIP .

Example 17
Mr. Patil invested in a SIP of a M.F. , a fixed sum of Rs. 10,000/- on 5th of every month , for 4 months . The NAV on these dates were Rs. 34.26 , 46.12 , 39.34 and 41.85 . The entry load was 2.25 % through out the period . Find the average price , including the entry load , using the Rupee-cost-Averaging method .How does it compare with the Arithmetic mean of the prices ? (Calculations are correct to 4 digits decimal)

Solution :

Month	NAV	Entry load = 2.25%	Total price	No.ofunits=1000/ Total price
1	34.2600	0.7708	35.0308	285.4627
2	46.1200	1.0377	47.1577	212.0544
3	39.3400	0.8851	40.2252	248.6006
4	41.8500	0.9141	42.7916	233.6906
TOTAL			165.2053	979.8083

By using Rupee-cost-Averaging method :-

$$\text{Avg Price} = \frac{\text{Total amount}}{\text{Total No. of units}}$$

$$= \frac{40,000}{979.8083} = 40\ .8243$$

A.M. of price $= \frac{\text{Total price}}{4} = \frac{165.2053}{4} = 41.3013$

∴ Avg. price using Rupee-cost- Averaging method is less than A.M. of prices .

Example 18

Mr. Desai invested Rs. 5000/- on 1st of every month for 5 months in a SIP of a M.F. with NAV's as 48.15 , 52.83 ,41.28,35.44 & 32.65 respectively .There was no entry load charged . Find the average price , Mr. Desai paid using the Rupee-cost-Averaging method . After 6 months ,he sold all his units , when NAV was Rs. 51.64 with contingent deferred sales charge (CDSC) as 2.25 % . Find his net gain. (Calculations are correct to 2 digits decimal)

Solution : consider the following table :-

Month	Amount (in Rs.)	NAV	No. of units
1	5000	48.15	5000/48.15=103.84
2	5000	52.83	5000/52.83=94.64
3	5000	41.28	5000/41.28=121.12
4	5000	35.44	5000/35.44=141.08
5	5000	32.65	5000/32.65=153.14
TOTAL	25000		613.82

Avg . price of units $= \frac{25000}{613.820} = 40.73$

For selling :

selling price of one unit = 51.64 - 2.25% of 51.64 = 50.48

Total sales = 50.48 x 613.82= 30,991.77

∴ Net gain = 30,991.77 -25,000 = Rs. 5991.77 .

Check your progress

1) Mr. Thomas started a SIP in 'HDFC long term advantage Fund ' . On the 10th July , Aug and Sept 2007 he invested Rs. 1000/- each at the NAVs Rs. 44.100, Rs. 43.761/-, s. 45.455 respectively . The entry load was 2.25% . Find his average acquisition cost per unit upto 3 decimal places . (Calculations are up to 3 decimal points).

Ans. Rs. 45.427/- .

2) Maneeshad Rs. 20,000/- on 2nd of every month for 5 onths in a SIP of a mutual fund , with NAVs as Rs. 53.12 , Rs. 56.26 , Rs. 48.86 ,Rs.50.44 and Rs. 54.62 respectively . The entry load was 2.25 % throughout this period .Find average price , including the entry load , using the Rupee-cost -Averaging method and compare it with Arithmetic mean of prices . (Calculate up to 2 decimal points)

Ans . 53.70 , 53.84 .

❖❖❖

UNIT II

3

LINEAR PROGRAMMING PROBLEMS

OBJECTIVE

From this chapter students should learn Introduction, meaning of linear programming problem, formulation of linear programming problem, some examples on formulation, Sketching of graphs of linear equation and linear inequalities, and solving of linear programming problem graphically.

3.1 INTRODUCTION

Planning is the heart and the soul of any project, be it a business empire or a simple task required to be done by an individual. However, we will be discussing planning w.r.t. production houses here.

Every organization, uses labour, machine, money, materials, time etc. These are called resources. As one cannot have an unlimited supply of resources, there is always an upper limit on these resources. Therefore, the management has to plan carefully and systematically to use these resources, so as to get the maximum profit at a minimum cost. This is Basic Principle of running any business successfully. Such a planning is called "Programming the strategies". This is done by writing a management problem as a mathematical model and then solve it scientifically.

Thus, a programming problem consists of Business problems where one faces several limitations causing restriction. One has to remain within the frame – work of these restriction and optimize (maximize or minimize, as the case may be) his goals. The strategies of doing it successfully, is called solving a programming problem.

We will be learning Linear Programming in the chapter, which is the most widely used technique in Production Planning.

3.2 MEANING OF LINEAR PROGRAMMING PROBLEM

As the name suggest, a Linear Programming problem is the problem of maximizing or minimizing a linear function, subject to linear constraints.

Consider a general Programming Problem with a certain goal. Obviously, there are restrictions also.

i. If the restrictions, when expressed mathematically, are in the form of Linear inequalities, the programming problem is called a Linear Programming problem (L.P.P)
The restrictions are also called constraints.
ii. If the constraints do not have more than two variables, the L.P.P. can be solved graphically.
iii. The goals when written mathematically are called objective function.

We will be solving only those linear programming problems having only one objective to be achieved at a time. We will be required to optimize (i.e. maximize or minimize) this objective function.

For example: if the variable is the profit, then we would like to maximize it, but if the variable is the cost or expenditure, we would naturally wish to minimize it. Hence, solving a Linear Programming Problem means, optimizing the given objective function within the given constraints.

Our first job will be to transform the management problems into appropriate mathematical modules.

3.3 FORMULATION OF LINEAR PROGRAMMING PROBLEM

The easiest way to learn this concept is with the help of examples. We begin with the following:

EXAMPLE 1:

A manufacturer produces two types of toys for children, Flutes and drums, each of which must be processed through two machines A and B. The maximum availability of machine A and B per day are 12 and 18 hours respectively. The manufacturing a Flutes requires 4 hours in machine A and 3 hours in machine B, whereas a drum require 2 hours of machine A and 6 hours of machine B, if the profit for Flute is Rs. 20 and per drum is Rs. 50, formulate the problem to maximize the profit.

Solution: Let us suppose that, the manufacture produces x Flutes and y drums per day. Then tabulating the given data as fallows, we observe that, if x Flutes and y drums are manufactured per day, he will require 4x + 2y hours of machine A and 3x + 6y hours of machine B.

Machine	Flutes (x)	Drum (y)	Maximum available
A	4	2	12
B	3	6	18

Since the availability of the machine A and B are not more than 12 and 18 hours respectively, we must have $4x + 2y \leq 12$ and $3x + 6y \leq 18$.

Note that he may not be able to complete a flute or a drum in a day therefore, x or y need not be integers, but x and y can never be negative, since nobody can manufacture a negative number of production. Hence, the condition $x \geq 0, y \geq 0$ will have to be taken

Thus, the problem will be written mathematically as

$4x + 2y \leq 12$

$3x + 6y \leq 18$ and

$x \geq 0, y \geq 0$

These inequalities are the constrains on the problem.

Now to find the objective function.

Since the profit per Flute and drum is Rs. 20 and Rs. 50 respectively, the objective function would be

Profit $z = 20x + 50y$, which is to be maximized

Thus Maxmise $z = 20x + 50y$

Subject to $4x + 2y \leq 12$

$3x + 6y \leq 18$

$x \geq 0, y \leq 0$

EXAMPLE 2:

Three different kinds of food A,B and C are to be considered to form a weekly diet. The minimum weekly requirements for fats, carbohydrates and proteins are 12, 30 and 20 units respectively. One Kg. of food A has 2, 16 and 4 units respectively of these ingredients and one Kg. of food B has 6, 4 and 3 units respectively whereas one Kg. of food C has 1, 5and 7 kgs of these ingradients. If the cost per kg. of food A is Rs. 75, per kg. and that of food B is Rs. 80 and per kg. of food C is Rs. 60, construct the problem to minimize the cost.

Solution: If x kg. of food A, y kg. of food B and z kg. of food C are to be considered for weekly diet, then the data can be represented by the following tabular form.

	A (x)	B (y)	C (z)	Minimum requirements
Fats	2	6	1	12
Carbohydrates	16	4	5	30
Proteins	4	3	7	20

The constraints can be written as

$2x + 6y + z \geq 12$

$16x + 4y + 5z \geq 30$

$4x + 3y + 7z \geq 20,$

$x \geq 0, y \geq 0$

Since the cost per kg. of the food A, B and C are given to be Rs 75, Rs. 80 Rs. 60 respectively, the objective function would be:

Cost C = 75x + 80y + 60z, which is to be minimized under the given constraints.

Thus Minimize C = 75x + 80y + 60z

Subject to $2x + 6y + z \geq 12$

$16x + 4y + 5z \geq 30$

$4x + 3y + 7z \geq 20,$

$x \geq 0, y \geq 0$

EXAMPLE 3:

Two types of food packets A and B are available. Each contain vitamins N_1 and N_2. A person need 4 decigrams of N_1 and 12 decigrams of N_2 per day. Food packet A contain 2 decigram of vitamin N_1 and 4 decigram of vitamin N_2. Food packet B contain 1 decigrams of vitamin N_1 and 4 decigrams of vitamin N_2. Food packed A and B cost Rs. 15 and Rs. 10 respectively. Formulate L.P.P. which will minimize the cost.

Solution: Let x = no. of packet of food A. y = no. of packet of food B.

Food packet A cost Rs. 15 and food packet B cost Rs. 10.

$\therefore$ Objective function i.e. cost function

Z = 15x + 10y subject to

Side constraints

	Food packed A	Food packet B	Requirement (≥)
Vitamin N_1	2	1	4
Vitamin N_2	4	4	12

x and y are no. of packet of food A and B respectively.

$\therefore$ $x \geq 0, y \geq 0$

Mathematical form

Min Z = 15x + 10y

Subject to $2x + y \geq 4$

$4x + 4y \geq 12$

$x \geq 0, y \geq 0$

EXAMPLE 4:

A machine is used for producing two products A and B. Product A is produced by using 4 units of chemical salt and 2 units of chemical mixture. Product B is produced by using 2 unit chemical salt and 3 units of chemical mixture. Only 100 units of chemical salt and 1500 units of chemical mixture are available. The profit on product A is Rs. 30 and on B is Rs. 20 per unit. Formulate this L.P.P. to maximize the profit.

Solution: Let x = no. of unit of product A be produce.

y = no. of unit of product B be produce.

Profit on product A is Rs. 30 per unit of chemical mixture are available.

	Product A	Product B	Availability
Chemical salt	4	2	1000
Chemical mixture	2	3	1500

x and y are numbers of units ∴ $x \geq 0, y \geq 0$.
Mathematical form

$$\text{Max } z = 30x + 20y$$

Subject to $4x + 2y \leq 1000$ (chemical salt)
$2x + 3y \leq 1500$ (chemical mixture)
$x \geq 0, y \geq 0$.

3.4 SKETCHING OF GRAPHS:

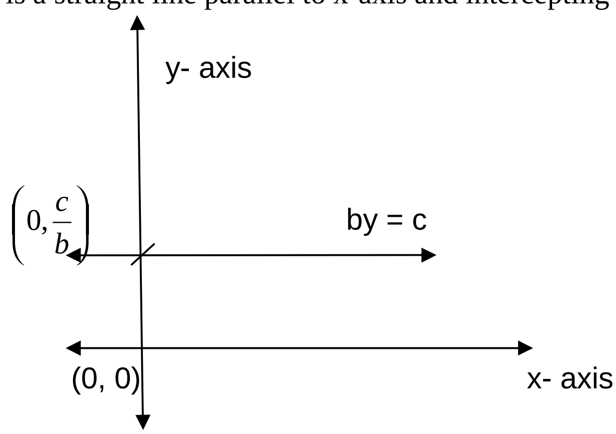
Before we begin to solve the problem of L.P.P. graphically, we shall first see how to sketch the graph of

1. Linear equation
2. Linear inequality

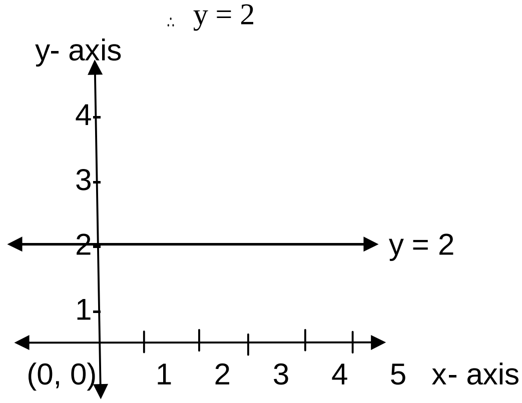
3.4.1.Graph of a linear equation:

A linear equation ax + by = c in two variables x and y where a, b, and c are constants. Here a and b are constant coefficient of x and y respectively not all zero simultaneously, the graph of the linear equation ax + by = c represents a straight line in xy plane, intercepting the x-axis and y-axis. There are four possibilities.

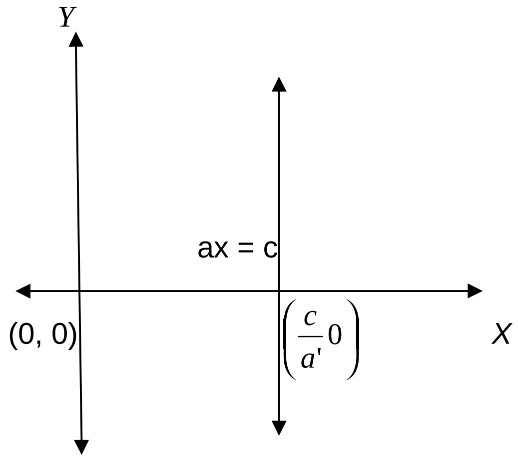
Case (i): If a = 0, the linear equation reduces to $by = c$. The graph of this equation is a straight line parallel to x-axis and intercepting y-axis at c/b.



Particularly,
Sketching of Graph of 3y = 6
∴ y = 2

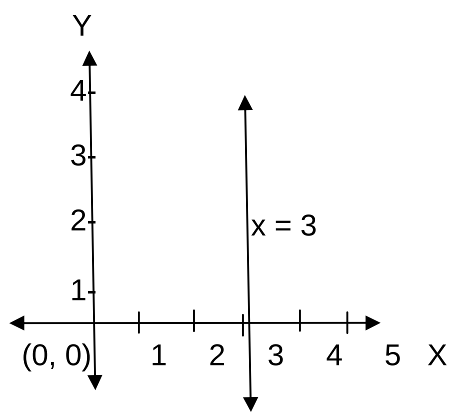


Case (ii): If a ≠ 0, b = 0 and c ≠0. Linear equation reduces to $ax = c$. The graph of this equation is a straight line parallel to y-axis intercepting x-axis at c/a

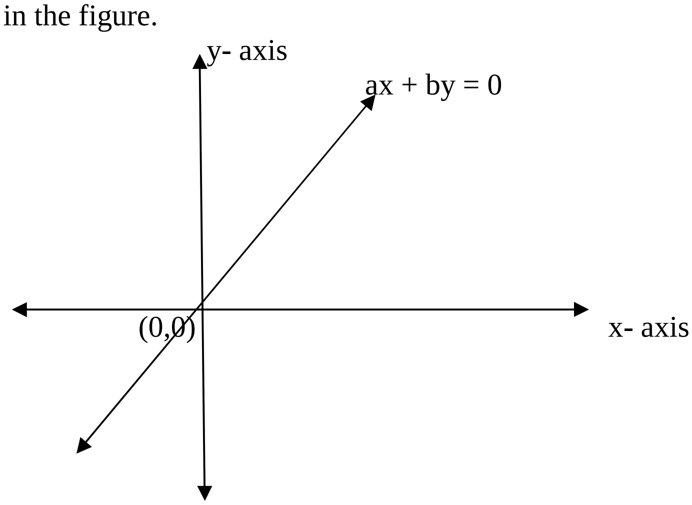


Particularly, Sketching of Graph of 9x = 27

$\therefore$ x = 3

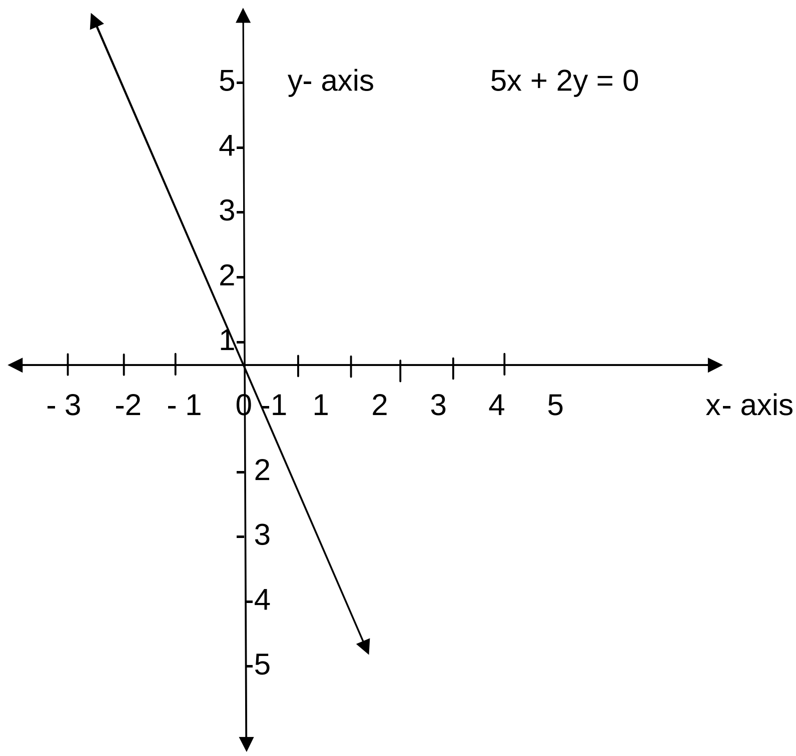


Case (iii): If a = 0, b = 0, c = 0, the equation reduces to ax + by = 0. The graph of this equation is a straight line passing through the origin as shown in the figure.



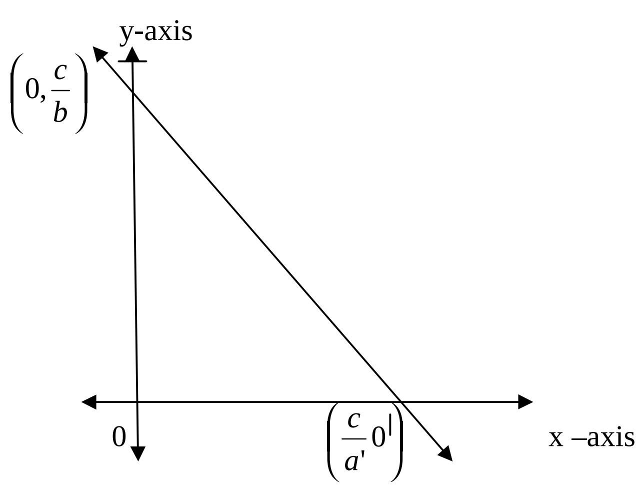
Particularly, Sketching of Graph of 5x + 2y = 0.
x- intercept and y – intercept are '0'

∴ Graph of equation 5x + 2y = 0 is a straight line passing through the origin.



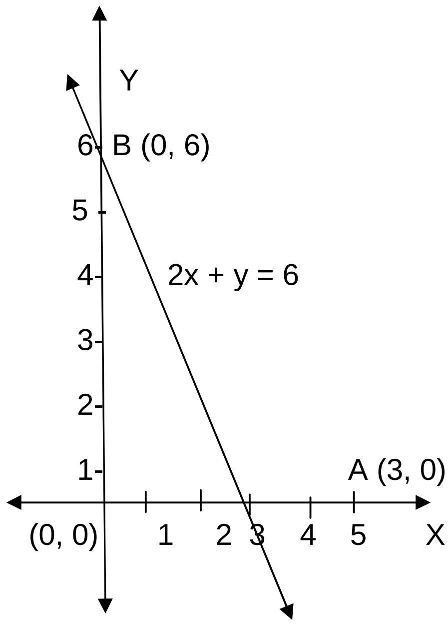
x	-2	2
y	5	-5

Case (iv): If a ≠ 0, b ≠ 0, c ≠ 0, the equation reduces to ax + by = c. The graph of this equation is shown below.



Particularly, Sketching of Graph of 2x + y = 6

x	y	points
0	6	(0,6)
3	0	(3,0)



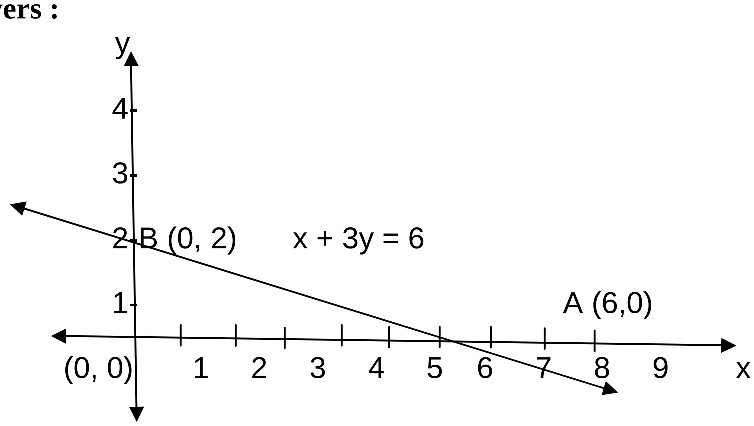
Exercise

Draw the graph of the following linear equations:

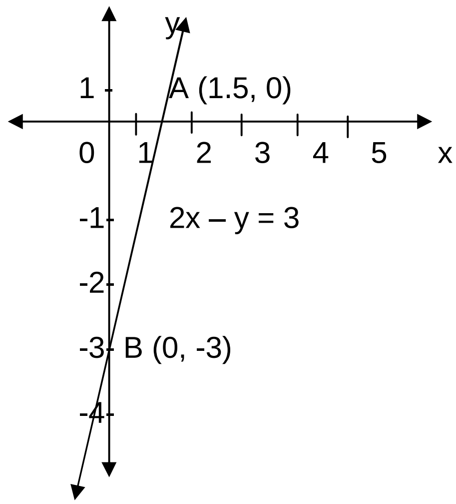
1. x + 3y = 6;
2. 2x – y = 3;
3. 3x – 5y = 8;
4. x = 5;
5. x = -3;
6. y = -2.
7.

Answers :

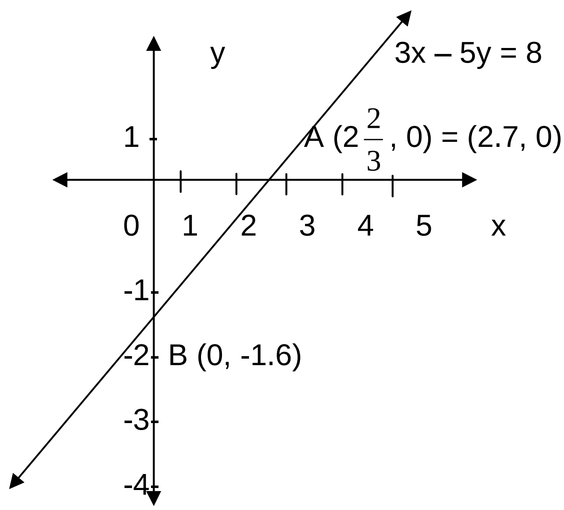
1



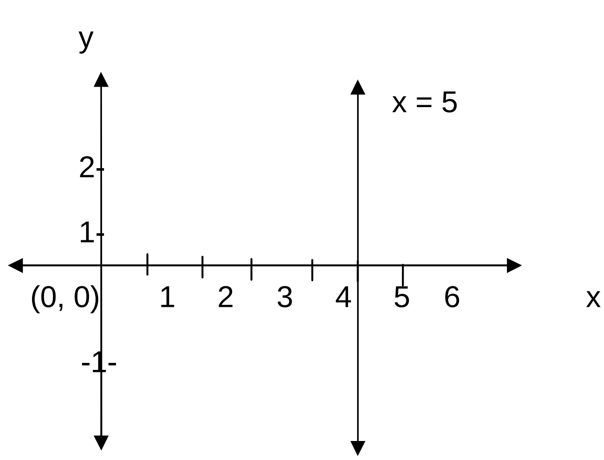
2


y
1
A (1.5, 0)
0 1 2 3 4 5 x
-1
2x – y = 3
-2
-3 B (0, -3)
-4

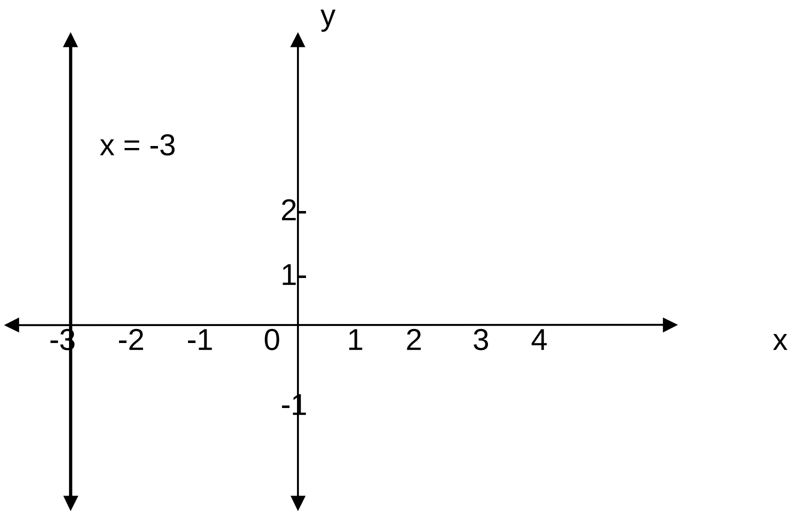
3.


y
3x – 5y = 8
1
A (2 2/3, 0) = (2.7, 0)
0 1 2 3 4 5 x
-1
-2 B (0, -1.6)
-3
-4

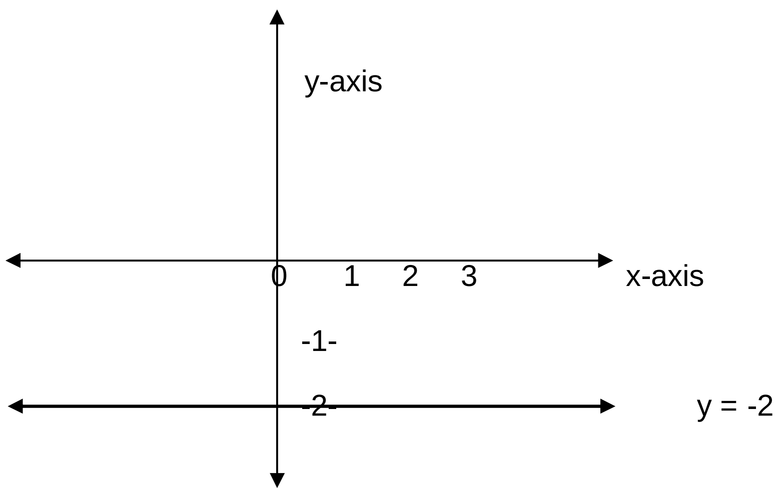
4.


y
x = 5
2
1
(0, 0) 1 2 3 4 5 6 x
-1

5



6.



3.4.2. Graphs of Linear Inequalities:

ax + by + c is called a linear expression. We have seen that if ax + by + c = 0, then it is a linear equation and it represents a straight line, if a, b, c are real numbers and a,b are both not zero.

Linear inequalities are of four types:

1. $ax + by + c \leq 0$,
2. $ax + by + c \leq 0$,
3. $ax + by + c \geq 0$,
4. $ax + by + c \geq 0$

These inequalities represent region of the plane, if a, b, c are real numbers and a, b are not both zero. We are interested in knowing what these regions are.

First draw the line ax + by + c = 0 on the graph paper. We know that the plane is now divided into three mutually disjoint set viz. the line itself and the two half planes one on each side of the line. Let us denote by P_1, the plane which contain the origin and by P_2 the half plane which does not contain the origin.

Now consider the origin (0, 0). Putting $x = 0$ and $y = 0$ in the linear expression $ax + by + c$, we get c. We consider two cases.

i. $c < 0$ and
ii. $c > 0$.
iii. $c = 0$
iv.

Case (I): if $c < 0$, then the half plane P_1 which contain the origin represents the inequality $ax + by + cb < 0$. Naturally, the half plane P_2 which does not contain the origin represents the inequality $ax + by + c > 0$

Further, the inequality $ax + by + c \leq 0$ is represented graphically by the union of the half plane P_1 and the line $ax + by + c = 0$. Whereas, the inequality $ax + by + c \geq 0$ is represented graphically by the union of the half plane P_2 and the line $ax + by + c= 0$.

Case (II): If $c > 0$, then the half plane P_1 which contain the origin represent the in inequality $ax + by + c > 0$. Naturally, the half plane P_2 which does not contain the origin represents the inequality

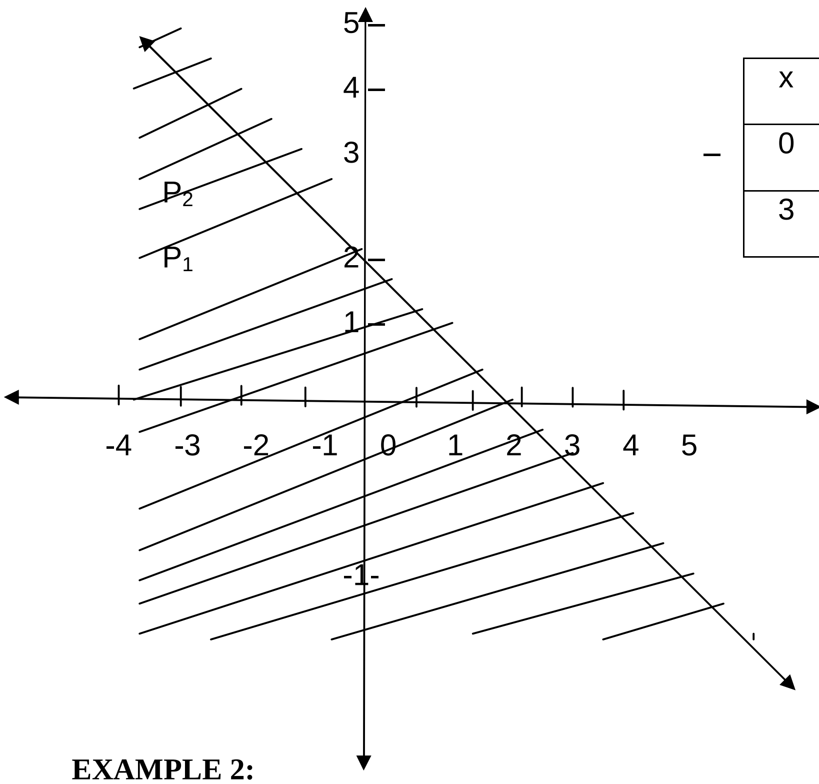
$$ax + by + c < 0$$

Further, the inequality $ax + by + c \leq 0$ is represented by the union of the half plane P_1 and the line $ax + by + c = 0$. Whereas, the inequality $ax + by + c \leq 0$ is represented graphically by the union of the half plane P_2 and the line $ax + by + c = 0$.

EXAMPLE 1: Represent the inequality $2x + 3y \leq 6$ graphically.

Solution: Draw the line $2x + 3y = 6$ on the graph paper. Consider the origin (0, 0). Putting $x = 0$ and $y = 0$ in the inequality, left hand side we get $0 < 6$. Thus (0, 0) satisfies the inequality $2x + 3y \leq 6$.

Hence shaded area including the line will be $2x + 3y \leq 6$



x	y	points
0	2	(0,2)
3	0	(3,0)

EXAMPLE 2:

Represent the inequality $2x + y \leq 5$ graphically.

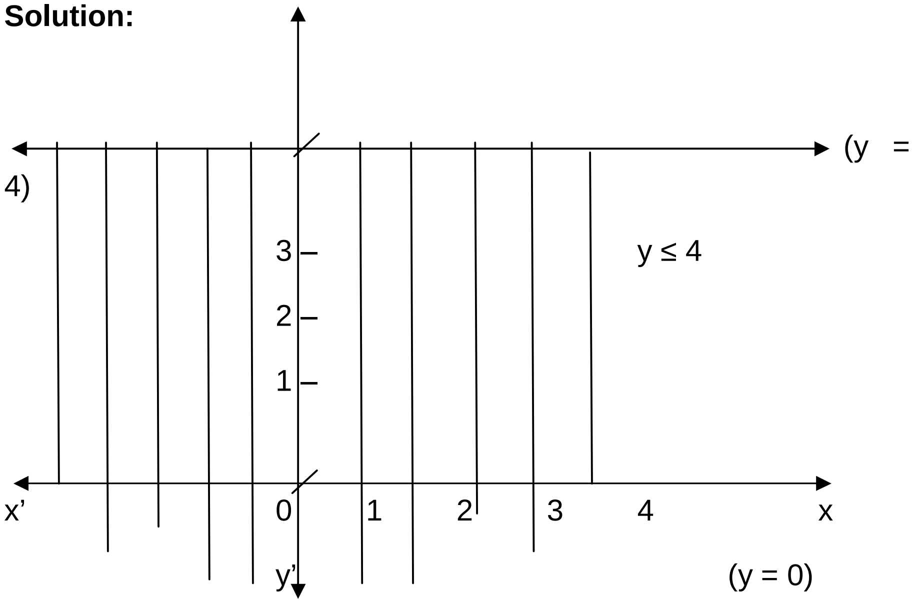
Solution:

Draw the line $2x + y = 5$ on the graph paper. Consider the origin O (0, 0). Putting x = 0 and y = 0 in the left hand side we get $0 > 5$. But we want $2x + y \geq 5$.

Hence the shaded area will be away from origin above the line.

EXAMPLE 3. Represent the inequality $y \leq 4$ graphically.

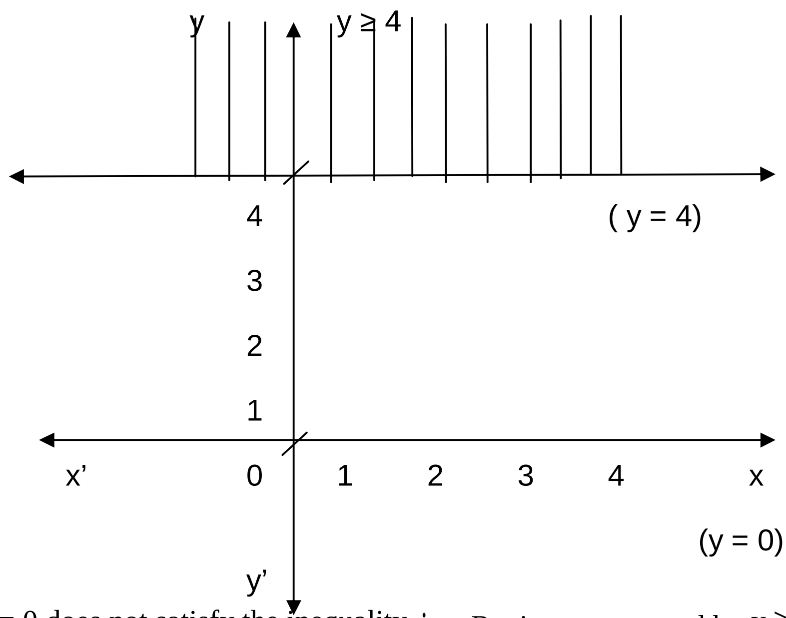
Solution:



(y = 0) satisfies the inequality.

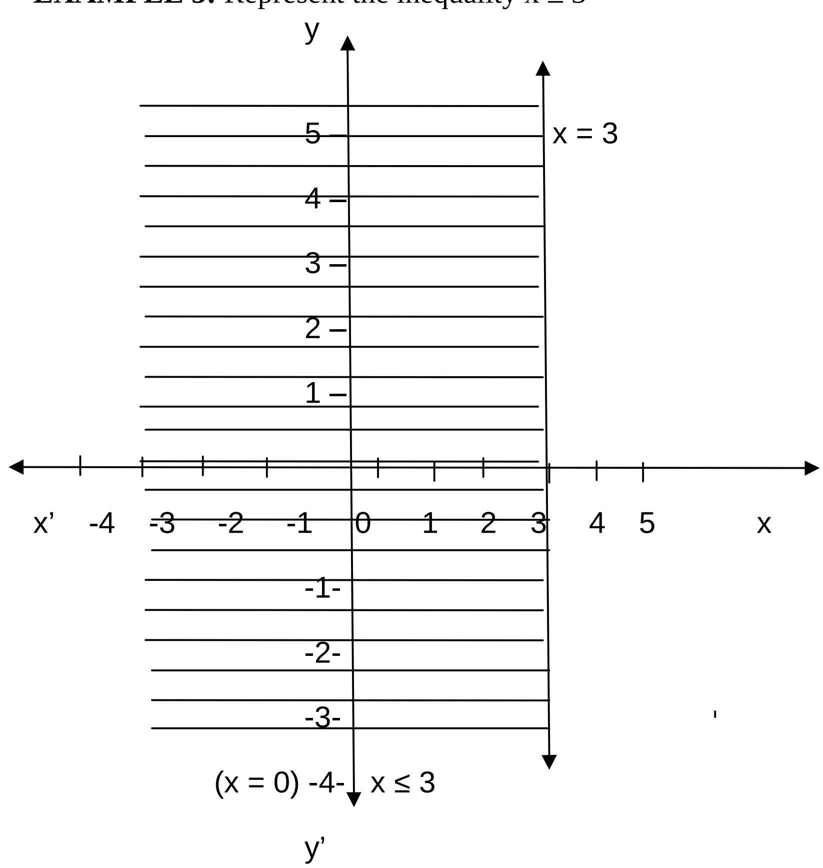
∴ Region represented by y ≤ 4 is towards origin. Below the line y = 4.

EXAMPLE 4: Represent the inequality y ≥ 4 graphically.



y = 0 does not satisfy the inequality.∴ Region represented by y ≥ 4 is above the line y = 4

EXAMPLE 5: Represent the inequality x ≤ 3

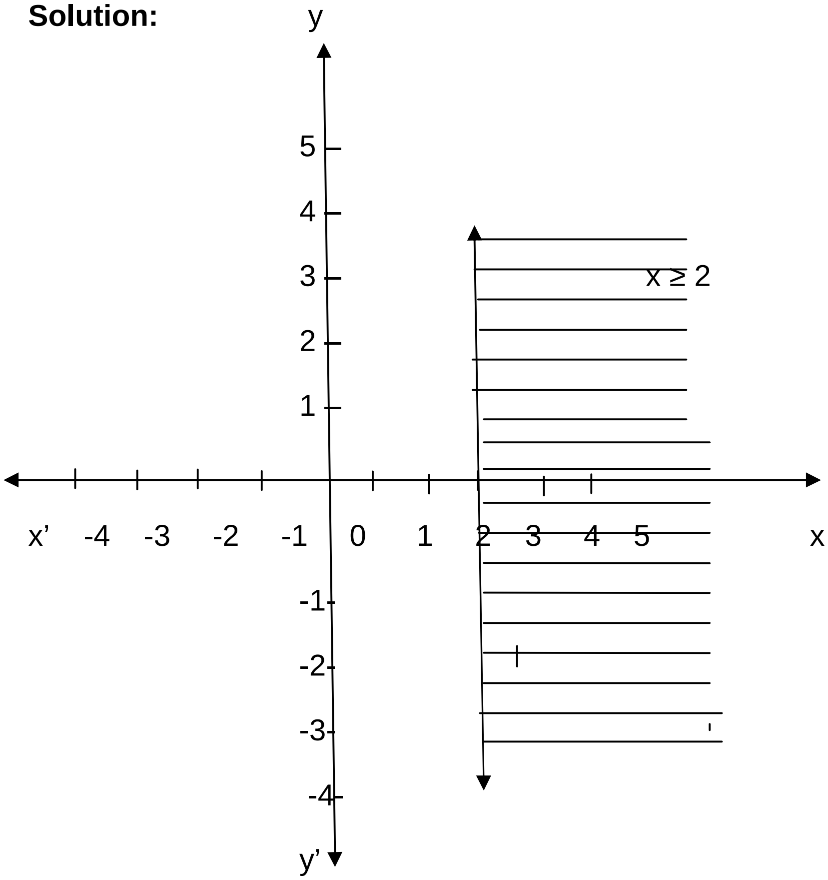


O (0, 0), x = 0 satisfies the inequality

∴ Region is to be left side of the line x = 3

EXAMPLE 5: Represent the inequality $x \geq 2$ graphically

Solution:

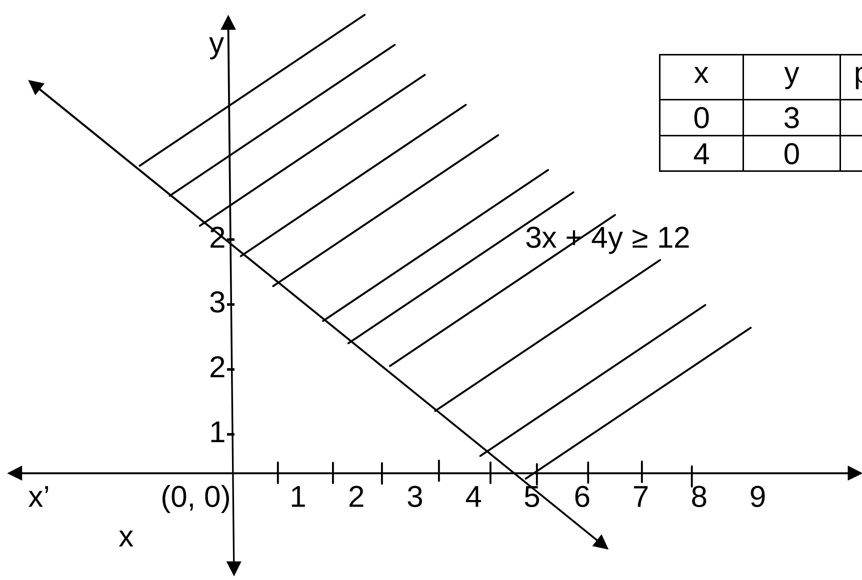


x = 0 does not satisfy the inequality $x \geq 2$

∴ Region is to be right side of the line x = 2

EXAMPLE 6: Represent the inequality $3x + 4y \geq 12$ graphically.

Solution:



x	y	points
0	3	(0,3)
4	0	(4,0)

First draw graph of equation $3x + 4y = 12$.

Origin O(0, 0) does not satisfy the inequality $3x + 4y \geq 2$.

∴ Region represented by the inequality $3x + 4y \geq$ does not have origin as a point.

Exercise

Sketch the graph of the following linear inequalities.

1) $x + 3y \leq 6$
2) $x + 3y \leq 6$
3) $2x + 5y \geq 10$
4) $2x + 5y \geq 10$
5) $x \leq 2$
6) $y \geq 1$
7) $x \geq 0$
8) $y \geq 0$

3.5 SOLUTION OF L.P.P. BY GRAPHICAL METHOD

In the earlier section we studied how to formulate L.P.P. However, we need to find solution for the L.P.P. i.e. we have to find the values of the variable which will optimize (Maximize or minimize) the objective function and satisfy all the inequality constraints as well as non – negativity restrictions.

Let us revise some concept we studied in the earlier section in linear inequalities. These concepts will be useful to find solution of L.P.P. by graphical method.

We studied how to find the points in two dimensional co – ordinate geometry, which will satisfy the given linear inequality. If this feasible region is bounded, it is in the form of polyhedron. The set of points of feasible region, which is polyhedron is called polyhedral set. A polyhedral set is called a convex set if a line joining any two points from the set lies entirely in the set. Hence, the feasible region is the convex set if it is a polyhedral set and the line joining any two points from polyhedral set entirely lie in the polyhedral set (feasible region). In the terminology of L.P.P. we can say that feasible region is the set of points, which satifies the inequality constraints as well as non – negative restriction. Such as set of points is called feasible solution of L.P.P. To find optimal solution of L.P.P. the following result is extremely useful.

Result:

If the convex set of feasible solution is a convex polyhedron set, then at least one of the extreme points giving an optimal solution.

To find solution to L.P.P. graphically by using these results these are the steps in the method.

Following steps are involved in solving L.P.P. graphically.

The steps involved in this method are:

1) Draw the graph for the inequality restriction.
2) Indicate the area of feasible solution (feasible region)
3) Determine the co-ordinates of all points at the corners (points) of all the feasible region.
4) Find the value of objective function corresponding to all the solution points determined in (3).
5) Determine the feasible solution which optimizes the value of the objective function.

Example1: Solve graphically the L.P. Problem.

Maximize $z = 9x + 12y$,

subject to $2x + 3y \leq 18$,

$2x + y \leq 10$,

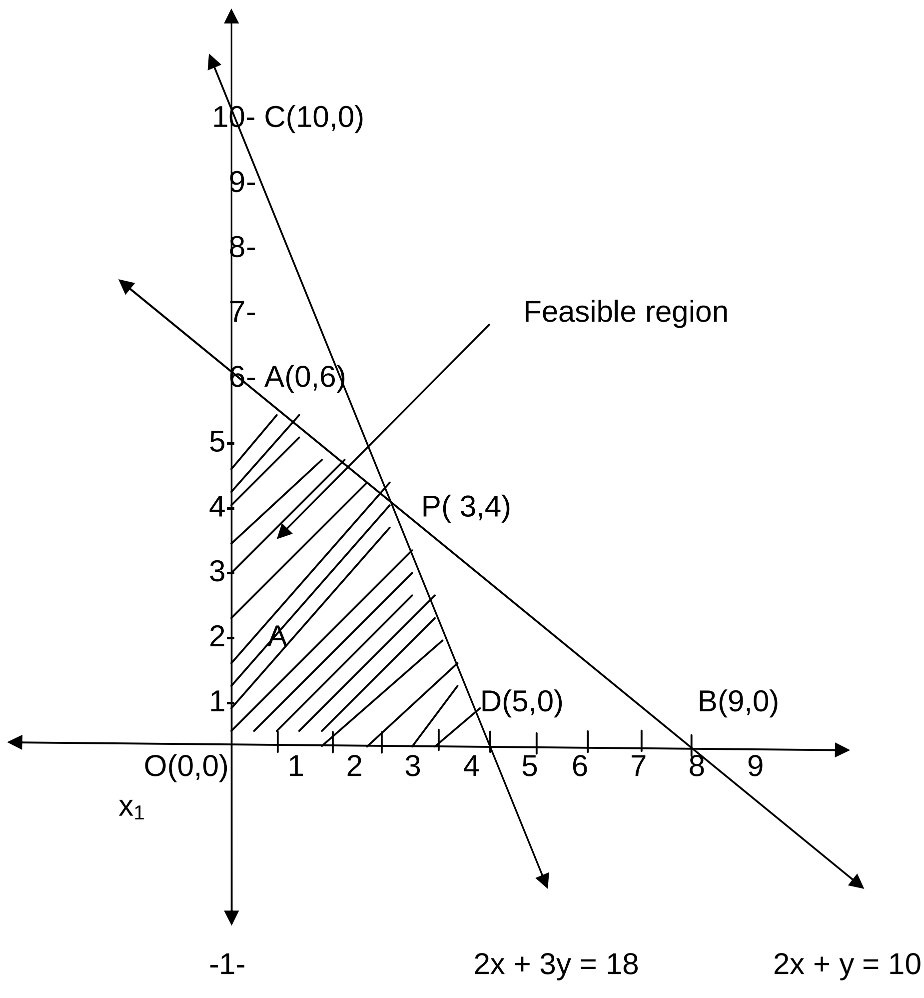
$x \geq 0, y \geq 0$

Solution: For the line $2x + 3y \leq 18$

x	y	Points
0	6	(0,6)
9	0	(9,0)

For the line $2x + y \leq 10$

x	y	Points
0	10	(0,10)
5	0	(5,0)



We first find the region, each point of which satisfies all the constraints by drawing graphs of all the constraints. This is called the region of feasible solution. The region of feasible solution is the quadrilateral OAPD. The corner point of this region are O, A, P and D.

We find the co-ordinates of P by solving the following equations simultaneously.

$$2x + 3y = 18,$$
$$2x + y = 10,$$
$$2y = 8$$
$$y = 4$$

Substituting value of y in $2x + y = 10$, we get $2x + 4 = 10$. $\therefore$ $x = 3$

So the point of intersecting = P (3, 4)

Objective function is $Z = 9x + 12y$,

To check the value of Z at end points of shaded feasible region O, A, P and D

At O (0, 0), $Z = 9(0) + 12(0) = 0$

At A(0, 6), $Z = 9(0) + 12(6) = 72$

At P(3, 4), $Z = 9(3) + 12(4) = 75$

At D (5, 0), $Z = 9(5) + 12(0) = 45$

So, at P (3, 4), Z is Maximized and the maximum value is 75 for x = 3 and y = 4.

Example2: Solve graphically the L.P. Problem.

Maximize $Z = 5x_1 + 3x_2$

subject to the constraints $3x_1 + 5x_2 \leq 15$,

$5x_1 + 2x_2 \leq 10$,

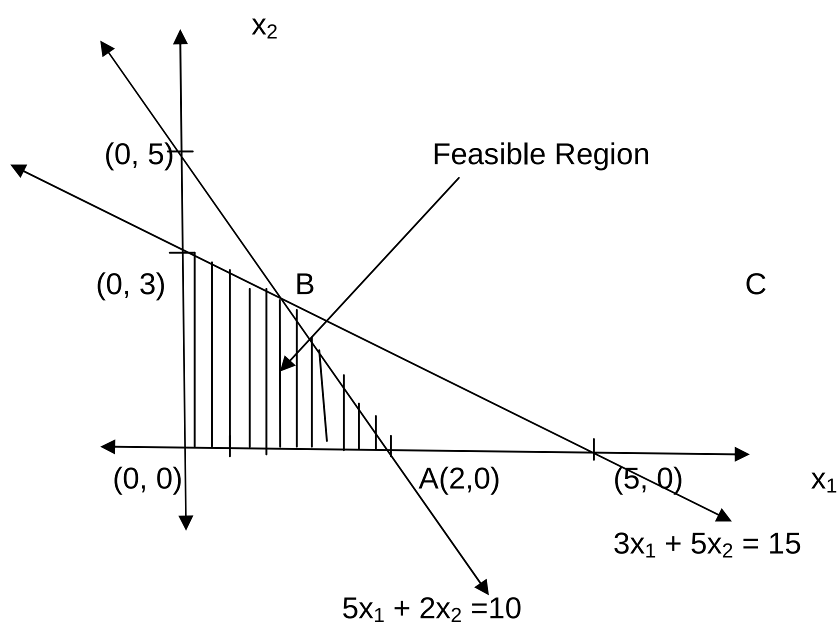
$x_1 \geq 0, x_2 \geq 0$.

Solution: For the line $3x_1 + 5x_2 \leq 15$

x_1	x_2	Points
0	3	(3,0)
5	0	(5,0)

For the line $5x_1 + 2x_2 \leq 10$

x_1	x_2	Points
0	5	(0,5)
2	0	(2,0)



We first find the region, each point of which satisfies all the constraints by drawing graphs of all the constraints. This is called the region of feasible solution. The region of feasible solution is the quadrilateral OACB. The corner point of this region are O, A, B and C.

We find the co-ordinates of C by solving the following equations simultaneously.

$3x_1 + 5x_2 = 15$ ……….(1) $\times 5$

and $5x_1 + 2x_2 = 10$ ………..(2) $\times 3$

$$
\begin{array}{r}
15x_1 + 25x_2 = 75 \\
15x_1 + 6x_2 = 30 \\
- \quad - \qquad - \\
\hline
19x_2 = 45
\end{array}
$$

$$x_2 = \frac{45}{19}$$

substituting x_2 value in any one of the equation, we get $x_1 = \frac{20}{19}$

The objective function is $Z = 5x_1 + 3x_2$.

We find the value of Z at O, A, B, C.

At O (0,0), $Z = 5 \times 0 + 3 \times 0 = 0$

At A (2,0), $Z = 5 \times 2 + 3 \times 0 = 10$

At C $\left(\frac{20}{19}, \frac{45}{19}\right)$, $Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{235}{19}$

Z has maximum value $\frac{235}{19}$ at C $\left(\frac{20}{19}, \frac{45}{19}\right)$.

i.e. $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$

Example3:

Maximize $Z = 4x_1 - 2x_2$.

Subject to $x_1 + 3x_2 \leq 6$

$x_1 + x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0$.

Solve the above L.P. problem graphically.

Solution:

Consider the given condition as equality

$x_1 + 3x_2 = 6$ (I)

$x_1 + x_2 = 2$ (II)

For point of intersection, solve the equations simultaneously

$$
\begin{array}{rcl}
x_1 + 3x_2 & = & 6 \\
x_1 + x_2 & = & 2 \\
- \quad + & - & \\
\hline
4x_2 & = & 4
\end{array}
$$

$\therefore x_2 = 1$

Substituting $x_2 = 1$ in (ii), $x_1 = 3$

So the point of intersecting = P (3, 1)

To draw the straight line representing (i), (ii) consider

x_1	x_2	Points
0	2	(0,2)
	0	(2,0)

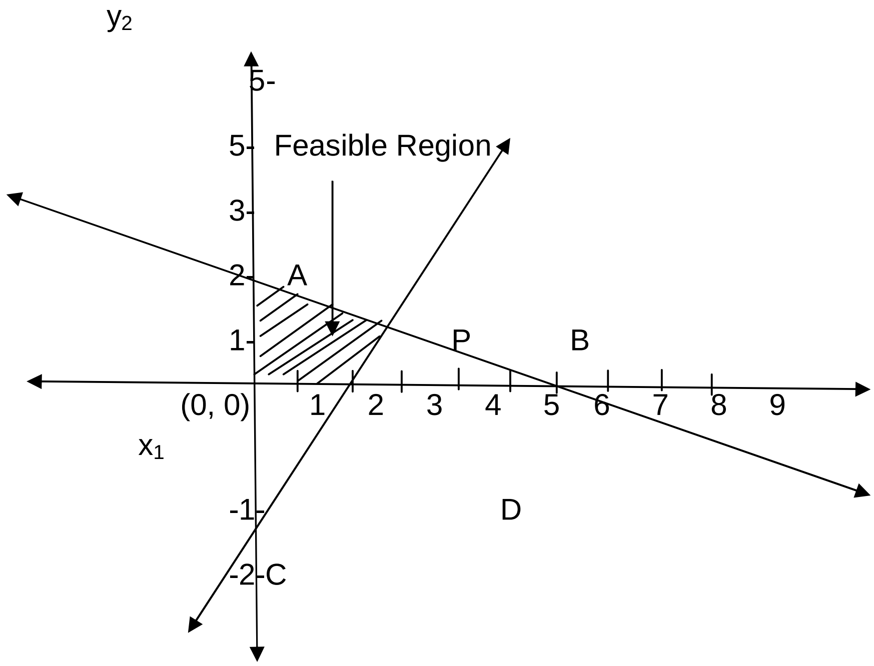
(i)

x_1	x_2	Points
0	-2	(0,-2)
2	0	(2,0)

(ii)

By plotting the point A (0,2), B(6, 0), C(0, -2) and D(2, 0) and joining we get straight line AB and CD representing equation (i) and (ii) and the feasible region is shaded as

$3x + 4y \geq 12$



Note : To get P, the line CD is extended.

Too check the value of Z at end points of shaded feasible region A, P and D

At o (0, 0), Z = 4(0) – 2(0) = 0

At (0, 2), $Z = 4x_1 - 2x_2 = 4(0) - 2(2) = -4$

At P(3, 1), Z = 4 (3) – 2 (1) = 10

At D (2, 0), Z = 4(2) – 2(0) = 8

So, at P (3, 1), Z is Maximized and the maximum value is 10 for $x_1 = 3$ and $x_2 = 1$

Example 4:

Maximize z = 800x + 100y,

subject to $4x + 6y \leq 120$,

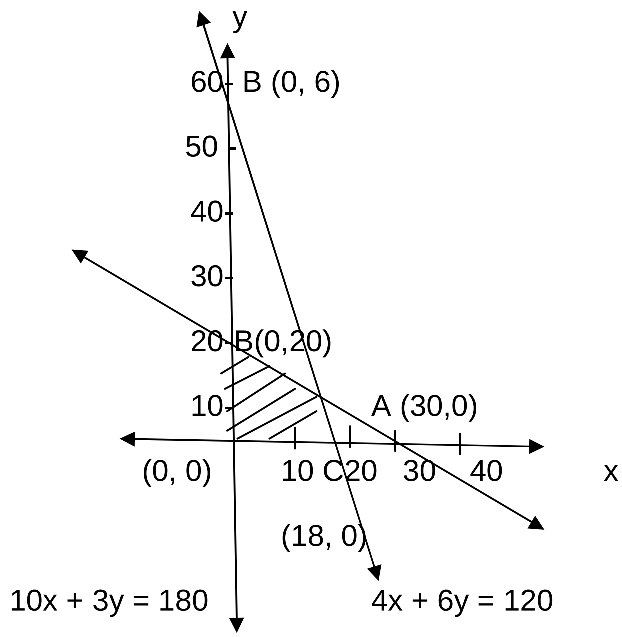
$10x + 3y \leq 0, y \geq 0$

Solution : Now we solve this problem by the graphical method as follows:

First we draw the graph of the solution set of the linear inequalities.

$4x + 6y \leq 120, 10x + 180, x \geq 0, y \geq 0$

At each point of this region, we can find the values of the objective function z = 800x + 100y.



Hence this region is called a feasible region. We want to find that point at which the objective function z = 800x + 100y has a maximum value. For this purpose , we use the following result.

If the feasible region is a polygonal one, then the maximum and minimum value of the objective function lie at some vertices of the region.

Now we draw the line AB and CD whose equations are 4x + 6y = 120 and 10x + 3y = 180 respectively. For this purpose, we find their point of intersection with the co-ordinate axes. It is convenient to from a table for their co-ordinates as follows:

For the line
4x + 6y = 120

x	y	points
30	0	A(30,0)
0	20	B(0,20)

For the line
10x + 3y = 180

x	y	points
18	0	C(18,0)
0	60	D(0,60)

Shade the feasible region OCPBO by horizontal lines. Its vertices are O (0, 0), C (18, 0), B (0, 20) and P which is the point of intersection of the line AB and CD. Hence to get the co-ordinates of P, we solve their equation:

$4x + 6y = 120$ …..(1)
$10x + 3y = 180$ …..(2)
Multiplying equation (2) by 2 and subtracting equation (1) from it,

$20x + 6y = 360$
$4x + 6y = 120$
$- \quad - \quad\quad -$
$16x = 240$

x = 15

∴ from (2)

150 + 3y = 180

∴ 3y = 30 ∴ y = 10

∴ P is (15, 10).

We find the values of the objective function at these vertices:

z (O) = 800 × 0 + 100 × 0 = 0

z(P) = 800 × 15 + 100 × 10 = 13000

z (C) = 800 (18) + 100 × 0 = 14400 z

(B) = 800 × 0 + 100 × 20 = 2000

∴ z has the maximum value 14400 at the point C (18, 0).

∴ the manufacturer should produce 18 scooters and 0 bicycles, in order to have maximum profit of Rs. 14400.

Example5:

Minimize z = 40x + 37y

Subject to constraints: 10x + 3y ≥ 180

2x + 3y ≥ 60

x ≥ 0, y ≥ 0

Solution:

We first draw the line AB and CD whose equation are 10x + 3y = 180 and 2x + 3y = 60.

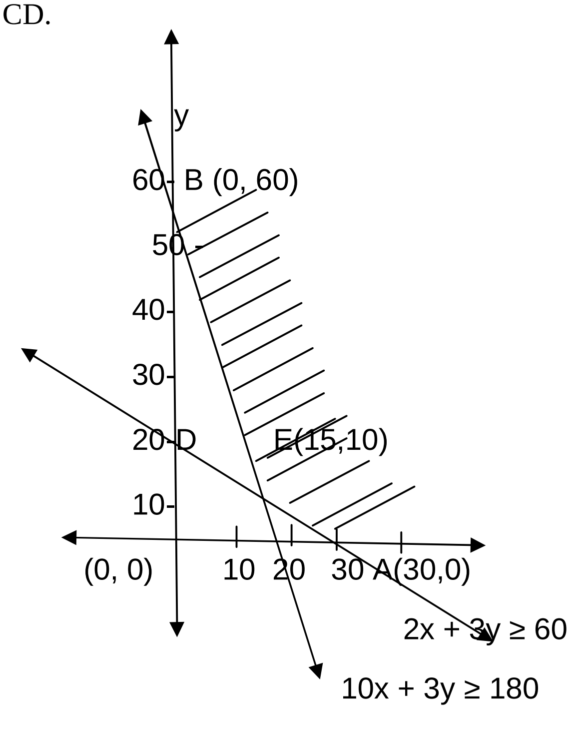
For the line 10x + 3y = 180

x	y	Points
18	0	(18,0)
0	60	(0,60)

For the line 2x + 3y = 60

x	y	Points
30	0	(30,0)
0	20	(0,20)

The feasible region is shaded in the figure. Its vertices are C (30, 0), B (0, 60) and E, where E is the point of intersection of the lines AB and CD.



For the point E, we solve the two equations simultaneously.

$$\begin{aligned} 10x + 3y &= 180 \\ 2x + 3y &= 60 \\ -\quad - &\quad - \\ \hline 8x &= 120 \\ x &= 15 \end{aligned}$$

$\therefore$ 2x + 3y = 60 gives 30 + 3y = 60,

i.e., 3y = 30, i.e. y = 10

$\therefore$ E is (15, 10)

The values of the objective function z = 40x + 37y at these vertices are:

Vertex (x, y)	z = 40x + 37y
C (30, 0)	z (C) = 40 × 30 + 37 × 0 = 1200
B (0, 60)	z (B) = 40 × 0 + 37 × 60 = 220
E (15, 10)	z (E) = 40 × 15 + 37 × 10 = 600 + 370 = 970.

$\therefore$ z has minimum value 970 at the point E (15, 10), where x = 15 and y = 10.

Example6: Minimize z = 4x + 2y

Subject to constraints: x + 3y ≥ 3, 2x + y ≥ 2, x ≥ 0, y ≥ 0

Solution:

We first draw the line AB and CD .

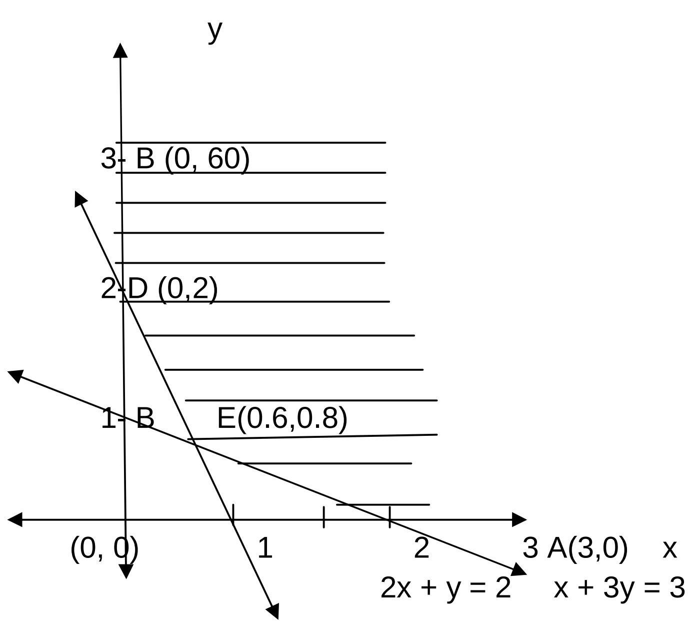
For the line x + 3y = 3

x	y	Point
3	0	A(3,0)
0	1	B(0,1)

For the line 2x + y = 2

x	y	Point
1	0	C(1,0)
0	2	D(0,2)

The feasible region is shaded in the figure. Its vertices are A (3, 0), D (0, 2) and E, where E is the point of intersection of the lines AB and CD.



To find the point E, we solve the two equations(1) and (2) simultaneously.

$$\begin{aligned} 2x + 6y &= 6 \\ 2x + y &= 2 \\ - \quad - \quad & \quad - \\ \hline 5y &= 4 \\ x &= \frac{4}{5} = 0.8 \end{aligned}$$

Substituting y = 0.8 in (1), we get,

$$\therefore x + 3 \times 0.8 = 3$$

$$\therefore x + 2.4 = 3$$

$$\therefore x = 3 - 2.4 = 0.6$$

Thus E is (0.6, 0.8)

The values of the objective function z = 4x + 2y at the vertices are calculated below:

Vertex (x, y)	z = 4x+ 2y
C (3, 0)	z (A) = 4× 3 + 2× 0 = 12
E (0.6, 0.8)	z (E) = 4 × 0.6 + 2 × 0.8 = 2.4 + 1.6 = 4
D(0, 2)	z (D) = 4 × 0+ 2 × 2= 4

We can see the minimum value of z is 4, at two vertices E(0.6, 0.8) and D(0, 2). Thus z is minimum at any point on the line segment ED.

Exercise

Maximize:

1) z = 5x + 10y, subject to
   5x + 8y ≤ 40, 3x + y ≤ 12, x ≥ 0, y ≥ 0
2) z = 7x + 6y, subject to
   2x + 3y ≤ 13, x + y ≤ 5, x ≥ 0, y ≥ 0
3) z = 5x + 3y, subject to
   2x + y ≤ 27, 3x + 2y ≤ 48, x ≥ 0, y ≥ 0
4) z = 20x + 30y, subject to
   x + y ≤ 6, 3x + y ≤ 12, x ≥ 0, y ≥ 0
5) z = 6x + 7y, subject to
   2x + 3y ≤ 12, 2x + y ≤ 8, x ≥ 0, y ≥ 0
6) z = 30x + 20y, subject to
   2x + y ≤ 20, x + 3y ≤ 15, x ≥ 0, y ≥ 0
7) z = 20x + 25y, subject to
   5x + 2y ≤ 50, x + y ≤ 12, x ≥ 0, y ≥ 0
8) z = 90x + 130y, subject to
   2x + 3y ≤ 18, 2x + y ≤ 12, x ≥ 0, y ≥ 0

Minimize:

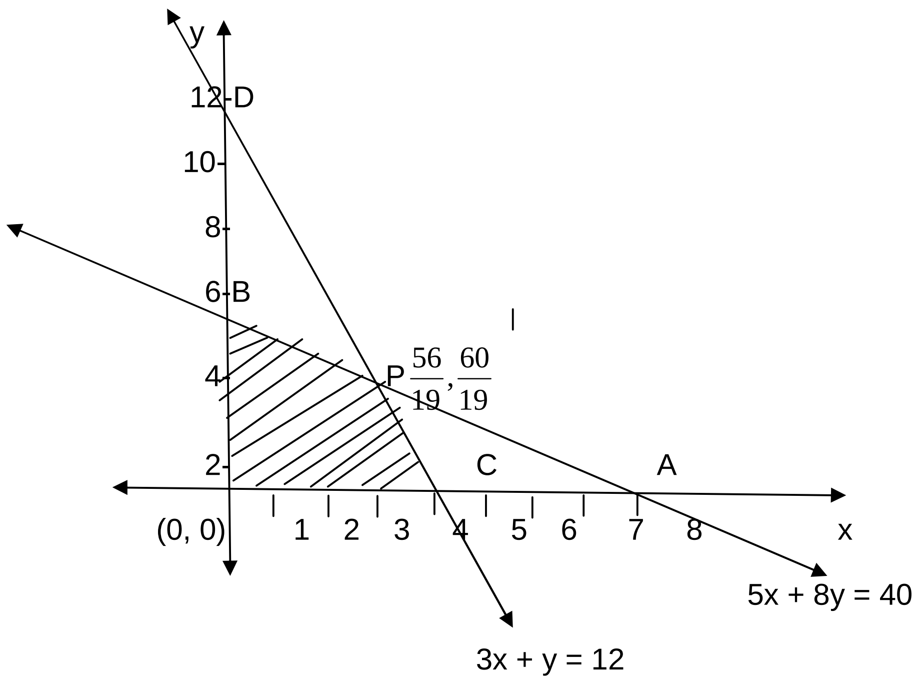
1) z = x + 4y, subject to
   x + 3y ≥ 3, 2x + y ≥ 2, x ≥ 0, y ≥ 0
2) z = 5x + 2y, subject to
   10x + 2y ≥ 20, 5x +5 y ≥ 30, x ≥ 0, y ≥0

3) z = 9x + 10y, subject to
   x + 2y ≥ 30, 3x + y ≥ 30, x ≥ 0, y ≥ 0
4) z = 50x + 55y, subject to
   x + 3y ≥ 30, 2x + y ≥ 20, x ≥ 0, y ≥ 0
5) z = 80x + 90y, subject to
   6x + 5y ≥ 300, 2x +3y ≥ 120, x ≥ 0, y ≥ 0
6) z = 13x + 15y, subject to
   3x + 4y ≥ 360, 2x +y ≥ 100, x ≥ 0, y ≥ 0
7) z = 12x + 20y, subject to
   x + y ≥ 7, 5x +2y ≥ 20, x ≥ 0, y ≥ 0

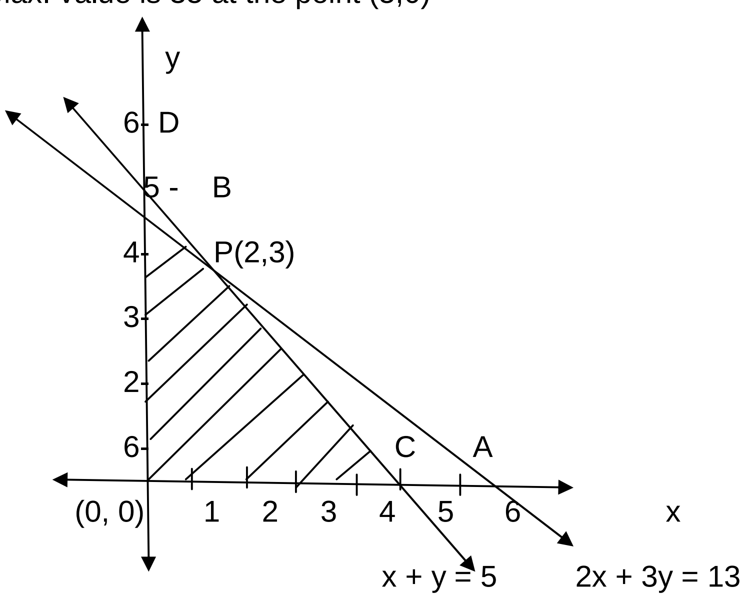
Answer:

Maximize:

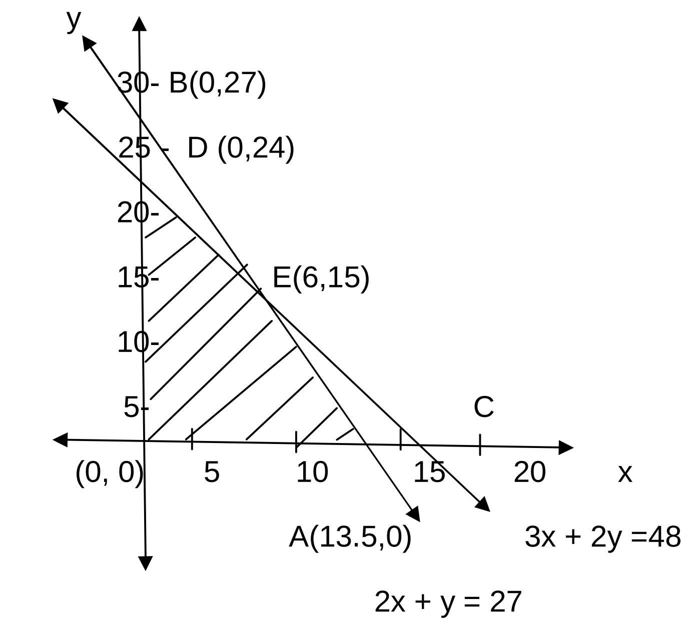
1) Max. value is 50 at the point (0,5)



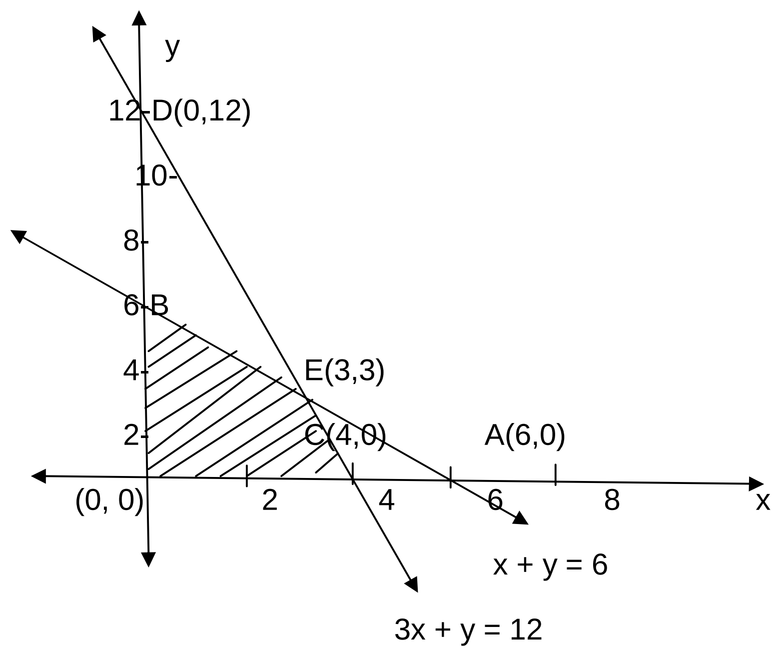
2) Max. value is 35 at the point (5,0)



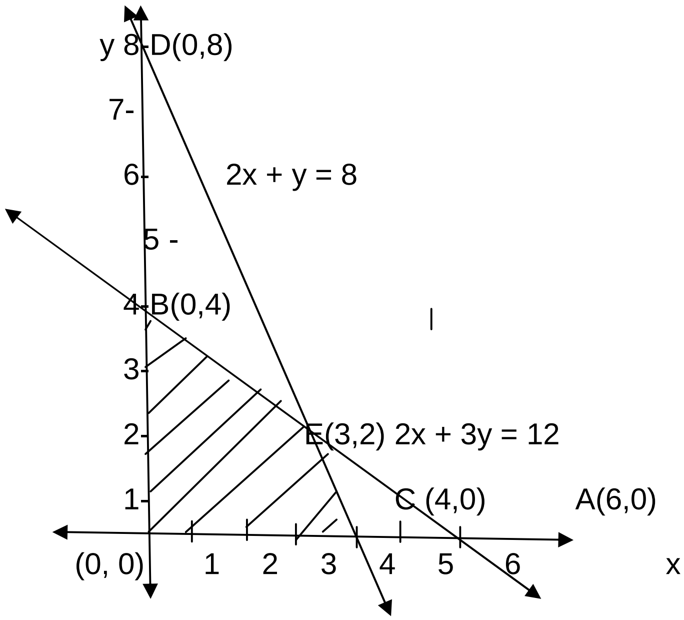
3) Max. value is 75 at the point. (6,15)



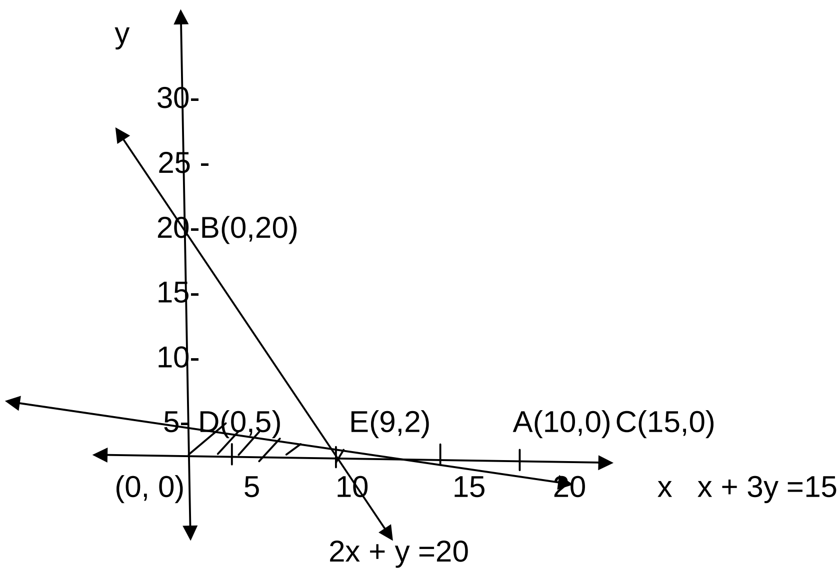
4) Max. value is 180 at the point (0,6)



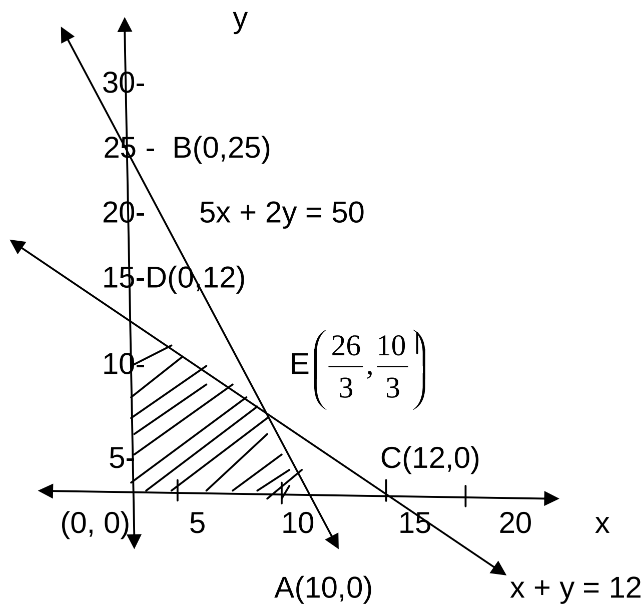
5) Max. value is 32 at the point (3,2)



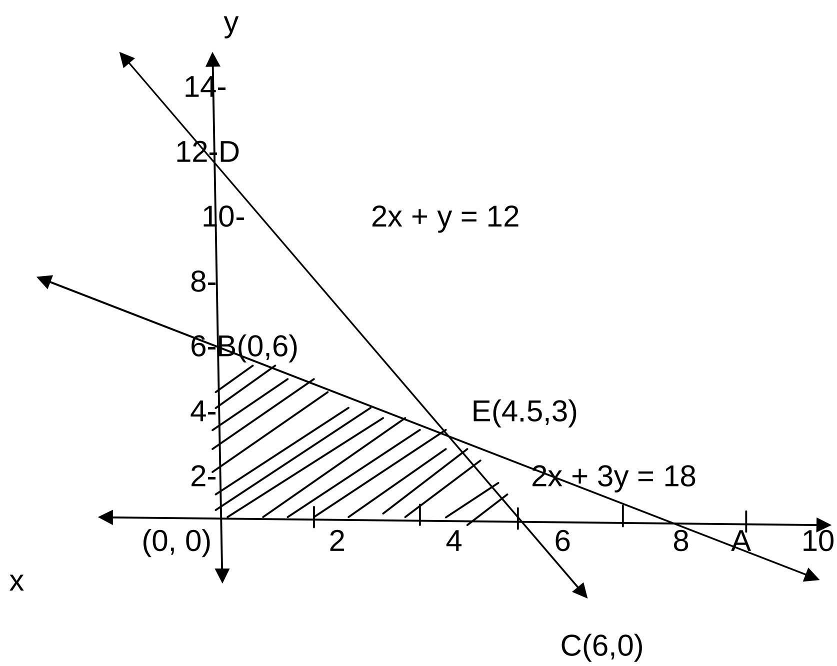
6) Max. value is 310 at the point (9,2)



7) Max. value is 300 at the point (0,12)

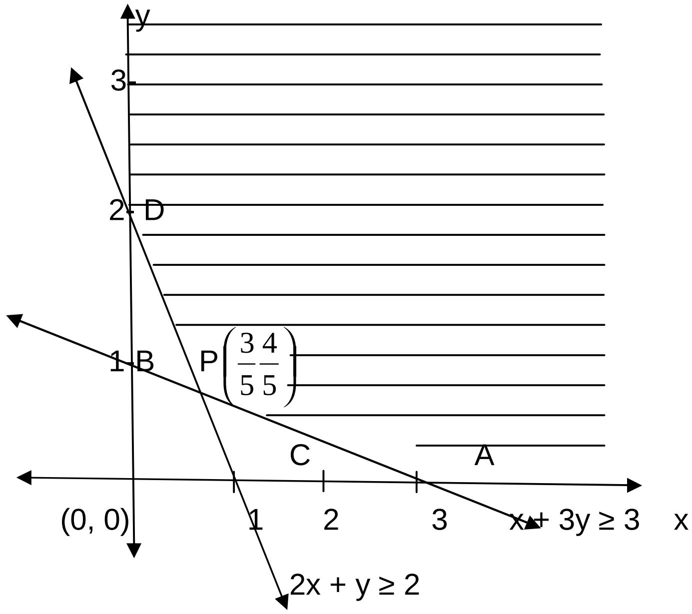


8) Max. value is 795 the point (4.5, 3)

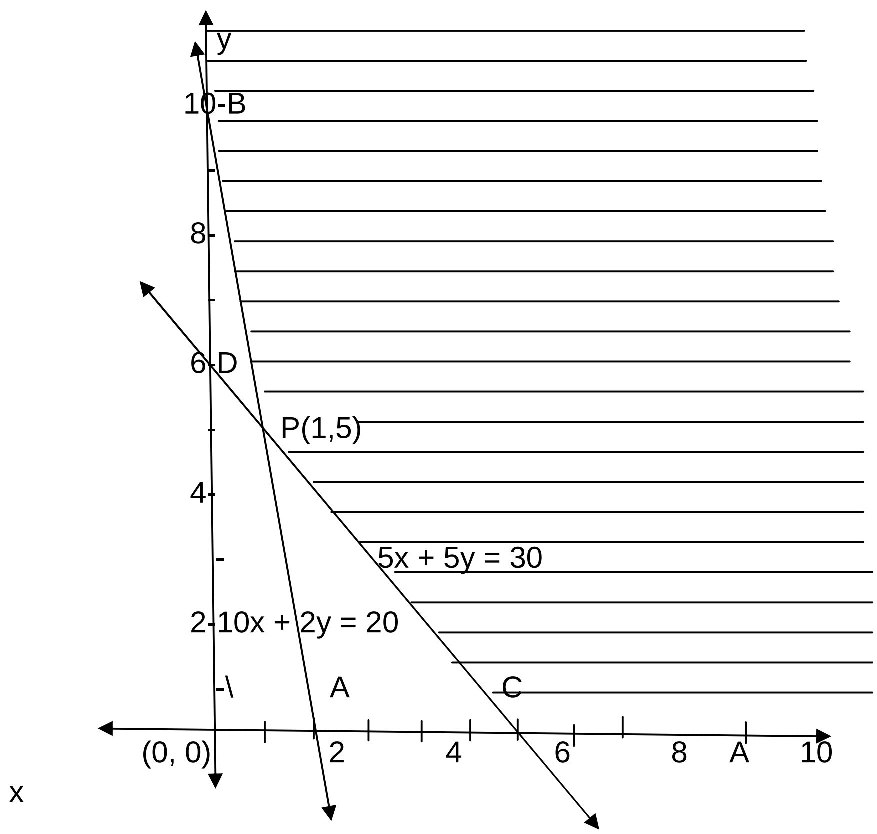


Minimize:

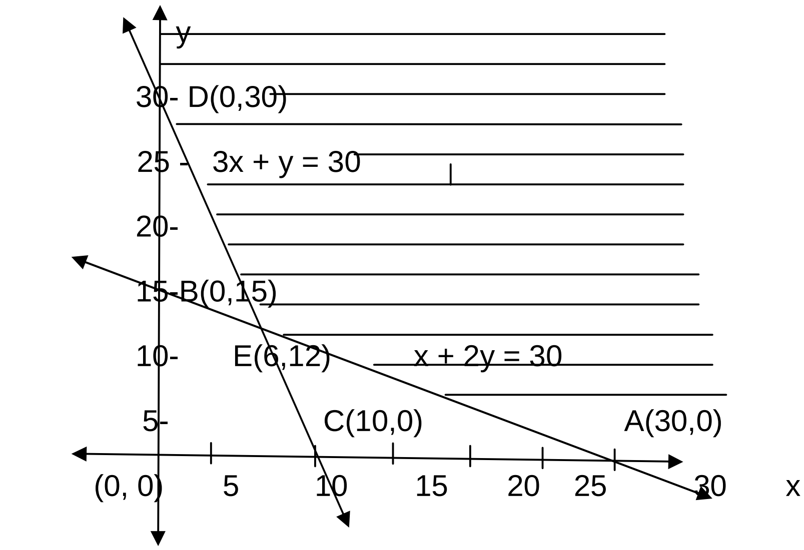
1) Min. value is 3 at the point (3,0)



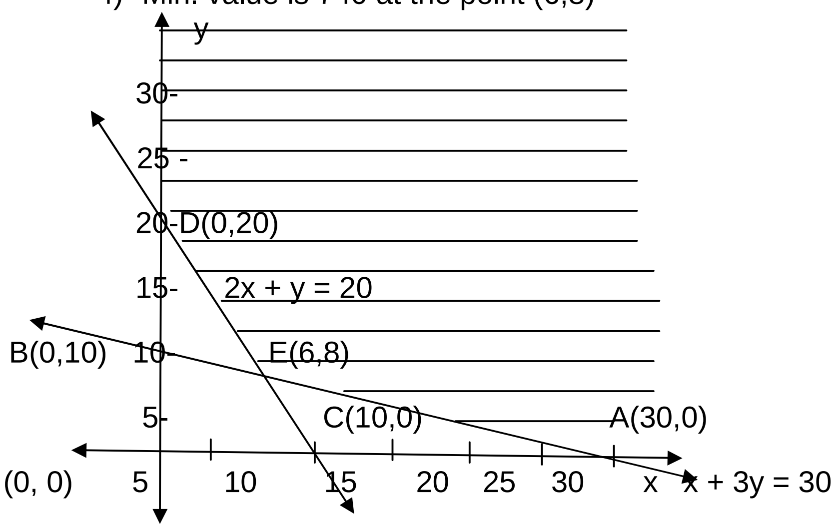
2) Min. value is 15 at the point (1,5)



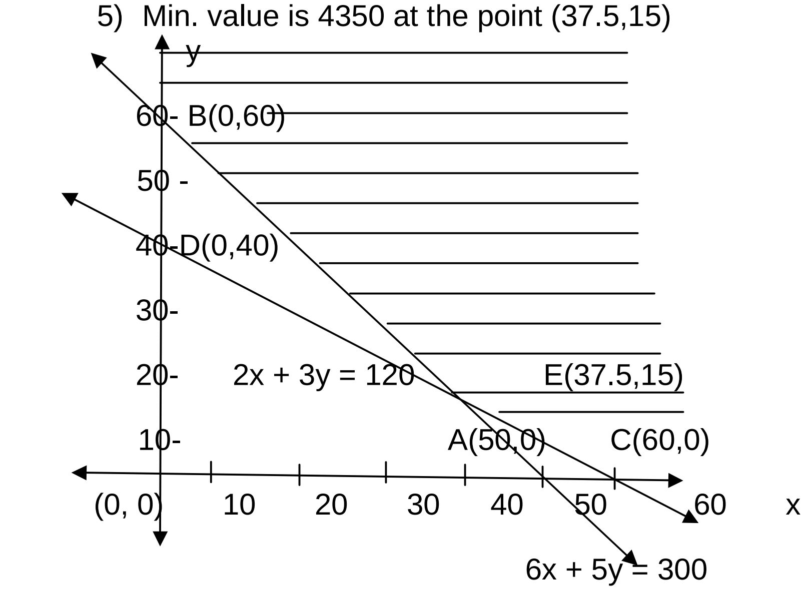
3) Min. value is 174 at the point (6,12)



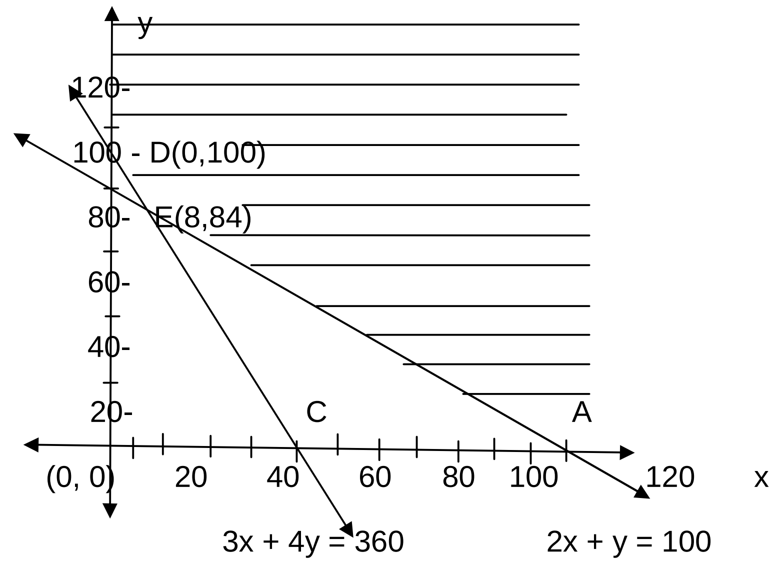
4) Min. value is 740 at the point (6,8)



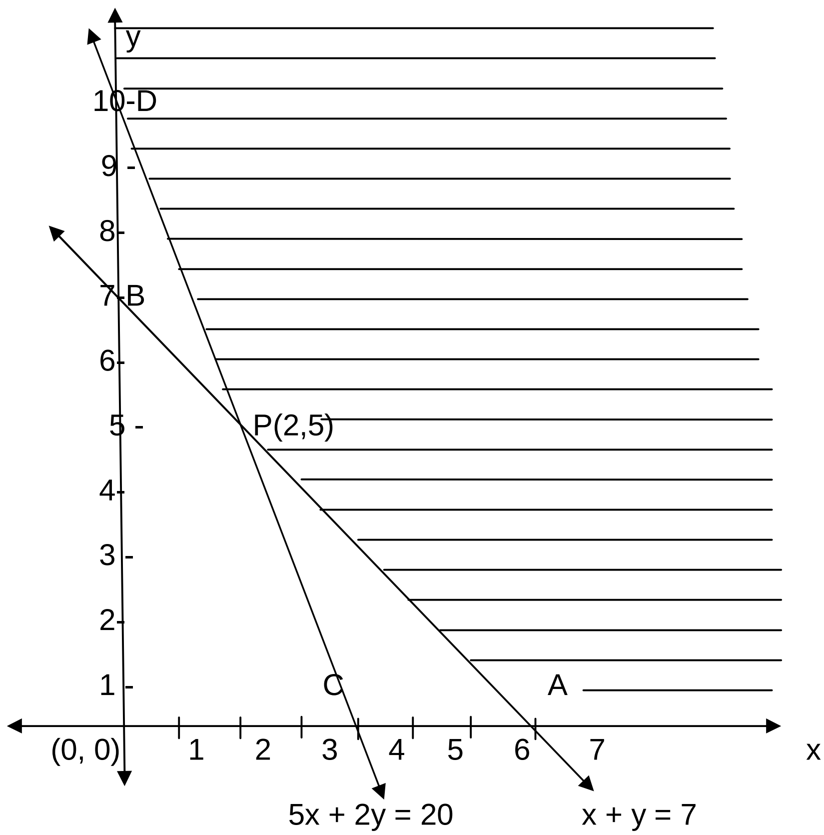
5) Min. value is 4350 at the point (37.5,15)



6) Min. value is 1364 at the point (8,84)



7) Min. value is 84 at the point (7,0)



❖❖❖

UNIT III

4

INTRODUCTION TO STATISTICS AND DATA COLLECTION

OBJECTIVES:

The objectives of this chapter are to give an overview of:

1. What is statistics and why one should learn it?
2. How did it originate?
3. Scope and limitations.
4. Definition of some basic terms that are used in the subject.
5. Types of data and how they are collected.
6. How should the data be arranged?

4.1 INTRODUCTION

In almost all areas of our daily living in which we make simple statements we are actually doing what one may call statistical thinking. What may seem like a simple "I walk on an average 4 kms per day" is a statement which involves statistics. 'There are 60 percent chances that a particular political party will get reelected at the next election", "This year it was much hotter than the same month previous year", are all examples of the common man doing statistics without realizing it. Due to the simple capacity to observe people, things and event in our environment we notice similarities and differences and look for patterns and regularities as part of a survival mechanism to sense danger or opportunity in an effort to grow wiser and understand our environment better. The simple act of observation results in us getting data. The word *data* means fact, and is plural of the word *datum*. This in turn makes us look for connections among the data that we have noticed. While most of us make connections and make conclusions which affects us and others around us the question that we must ask is 'How mature or informed our conclusions are?", "How can we be sure that our conclusions are factually correct?" These questions also lead us to ask "Is there a scientific, rational way of determining whether our conclusions are valid? Can we use structured, methodical, time tested ways to make these conclusions?" The remaining part of this chapter is devoted to answer these questions.

4.1.1 What is statistics?

In its simplest sense *statistics* refers to the science of collecting data, assortment of data and analyzing them to reach a definite conclusion. The methods by which the data are analysed are called as *Statistical Methods*.

The word statistics seems to have originated from the German word “statistik’, the Italian word ‘statistica’ or the Latin word ‘status’ all meaning ‘state’ in the political sense of the word. This in turn seems to have come because statistics was widely used initially to collect data for the state (Government) so that officials could use the data for better planning in the future. Statistical thinking however has a long history as from very early times kings and rulers have been collecting data about their populations and resources.

4.1.2 Functions of Statistics

Following are the important functions of statistics

1. *It represents data in a definite form*

One of the important function of statistics is that it enables one to make statements which are precise and in quantitative terms. To say that India is a overpopulated country, or that a country has high level of poverty are all general statements but do not convey any precise meaning. Words such as “high”, “low”, “good”, “bad”, are very vague and subject to interpretation by different people in different manner. Statements of facts made in exact quantitative terms are more convincing.

2. *It simplifies complex data or a mass of figures*.

Statistics not only helps one to express data in a concrete definite form but also reduces the data to a few significant figures which give the essence of the issue under consideration.

3. *It facilitates comparison.*

Generally plain facts by themselves have little value unless they are seen in the correct context in which they occur. *This requires that we place the facts in relation to facts of similar type but in other situations*. For example the fact that the population of a country is 12 million by itself would have a restricted meaning but in the context of how big the geographical area is and what are the populations of other similarly placed countries would throw light on some very important aspects of population density and other issues such as crowding and pollution.

4. *It helps in forecasting.*

Statistics helps in predicting what may happen in future on the basis of past data and analysis. It is useful for anybody in business to have an idea of what might be the possibility of selling goods or services so that the person can plan for any future changes in demands. Production Planning, inventories are all based on statistical modes which help by predicting excess or shortfalls in demands.

5. *It helps in formulating policies*

Due to its potential to do forecasting statistics helps policy makers in drafting policies which help in better governance. The annual finance budget relies heavily on statistical methods to decide issues of direct and indirect taxation and budgetary allocations based on past data and its analysis.

4.1.3 Scope of Statistics

There has been hardly any area whether from the exact sciences or social sciences where statistics has not been applied, whether it be trade, industry, commerce, economics, life sciences, education where statistical methods have not been applied. However certain fields have used statistics very frequently and effectively. We list some of the important fields.

1. **State** : As we have stated earlier from very early times statistics have been used by governments in framing policies on the basis of data about population, military, crimes, education etc The present day governments have special departments which maintain a variety of data of significance to the state. The extensive proliferation of technologies such as the internet has made huge amount of data available making statistical tools absolutely indispensible.

2. **Business :** With the advent of globalization and the extensive range of operations Business has grown almost exponentially in many areas. Business now extends across geographical and political boundaries. The large amount of data that it generates is used for analysis and forecasting of consumption patterns and has given rise to advance techniques such as Data Mining.

4.1.4 Limitations of statistics

Although there are many areas where statistics is useful there are certain limitations also as there are areas where it is not applicable and there are possibilities of its misuse. We indicate some of them as under.

1. **Statistics does not deal with individual measurement.**
   Statistical measurements are generally mass measurements and it deals with a collection of data and not any particular measurement. In fact a particular measurement can be very different form an average.

2. **Statistics deals with quantitative characteristics.**
   Statistics are numerical statement of facts. There are many aspects of life which cannot be quantified and are therefore not within the purview of statistics. For e.g. The honesty or integrity of person, whether a person is affectionate or not, whether or not a person is intelligent are some qualities which are not easily quantifiable and not available for statistical analysis.

3. **Statistical results are true on an average.**
   Unlike exact sciences in which there is a clear cause effect relationship in the phenomena which is being studied, in statistics the results are true only on an average. Any individual element in the data collected may not have any resemblance with the average.

4. **Statistics is one of the many methods of studying phenomena**.
   A statistical study is not complete in itself. There are many ways in which phenomena can be studied and one of it is statistics. Generally

statistics complements other methods to substantiate the understanding of phenomena.

5. **Statistics can be misused**.
   Perhaps the greatest limitations of statistics are that it can be misused. Many people use statistics to prove a point to which they have already subscribed. For e.g if a person wants to prove that malnutrition deaths have been high in an area then the researcher can deliberately collect data that will favour that conclusion and ignore the data that will not be favorable. Although there are ways in which this bias can be detected an intelligent researcher can easily mislead people. One of the reasons why one should learn statistics to understand how one can be mislead by such people and be cautious.

4.2 BASIC STATISTICAL CONCEPTS.

In using statistics it is necessary that we have a clear understanding of the words that are used. We give some definitions of some of the very basic concepts in statistics.

4.2.1 Data

The word **data** is plural of the word *datum* which in Greek means fact. It is a collection of observations expressed in numerical quantities. Data is always used in the collective sense and not in singular.

4.2.2 Population

The word population in statistics means the *totality of the set of objects under study*. It should not be understood in the limited sense in which it is generally used to mean people in a certain city or country.

4.2.3 Sample

A sample is a selected number of entities or individuals which form a part of the population under study. The study of a sample is more practical and economical in most situations where the population is large and is used to make conclusions about the entire population.

4.2.4 Characteristic

The word characteristic means an aspect possessed by an individual entity. We may study the rainfall of a certain region, or the marks scored by students in a certain school. These are referred to as characteristics.

4.2.5 Variables and attributes.

In statistics characteristics are of two types. Measurable and non-measurable. Measurable characteristics are those that can be quantified as expressed in numerical terms. The measurable characteristics are known as ***variables***. A non-measurable characteristic is qualitative in nature and cannot be quantified. Such a characteristic e.g nationality, religion, etc are called as ***attributes***.

4.3.1 Collection of Data

To apply statistical methods or to study a problem we must collect data. Collection of data is very important as it forms the basis of the analysis. It is important to understand the techniques of collection of data because if the data is not collected properly its reliability itself becomes questionable and our entire analysis will be on weak foundations.

Based on whether the investigator collects data himself/herself or uses data collected by some other person or agency data is classified into two types. Primary data and Secondary data.

1. **Primary Data**

   Primary data are those which are collected directly from the field of enquiry for a specific purpose. This is raw data original in nature and directly collected from the population. The collection of the data can be made through two methods. a) **Complete enumeration** or ***census method*** or b) **Sampling survey** methods.

   The complete enumeration or census method is a study of the entire population and data are collected about each individual of the population. Generally this is a laborious, time consuming and expensive method. Large organizations and Government semi - government organizations, Research institutions and Public sector bodies such as the RBI (Reserve Bank Of India) can afford such methods of collecting data.

2. **Secondary Data**

   Secondary data are such information which has already been collected by some agency for a specific purpose and is subsequently compiled by the investigator from that source for application in a different area. Data used by any other person or agency other than the one which collected it constitutes secondary data. The same data is primary when collected by the source agency and becomes secondary when used by any other agency. Data after analysis are also termed as secondary data.

4.3.2 Collection of primary data

Primary data can be collected by the investigator in following ways.

i) By direct personal observation

The investigator may collect data by direct observation. This can be done by meeting and interrogating people who may supply the desired information. Such data is directly obtained and can be very reliable but it is a time consuming and costly process.

ii) By indirect oral investigation

Here the information is collected not by questioning the concerned people but by asking people connected with the concerned people. These people can be called as witnesses who have knowledge about the persons concerned or situation involved. Here the investigator has the added responsibility of ensuring that the witnesses are not biased and therefore the reliability of data cannot be questioned. Sometimes this can be the only way to get the desired information as the people themselves are either not accessible or not willing to give the information.

iii) By sending questionnaires by mail or email

A questionnaire is a proforma containing a set of questions. A collection of questions relevant to the area of study is created and sent by post or email to selected people with a request to fill up and returned by post or email. This method can cover a large population economically but does not necessarily result in good response. The questionnaire has also to be carefully drafted so as to not have leading questions which direct the person to an answer desired by the investigator.

iv) By sending schedules through paid investigation

This method is used quite widely particularly by market researchers. Here schedules are prepared and the investigators are trained to meet people concerned with the schedules. A **schedule** is a form where information is to be noted by an enumerator who questions people. The investigators are to fill up the schedules on the basis of answers given by the respondents. The success of this method largely depends on how efficient the investigators are and how tactfully they collect the needed data.

4.3.3 Collection of Secondary Data

Secondary data are those which are collected by some other agency and are used for further investigation. The sources of secondary data can be classified into two:

a) **Published sources** b) **Unpublished sources**

Published Sources

Some of the published sources providing secondary data are:

a**. Government Publications**: Government, semi-government and private organizations collect data related to business, trade, prices, consumption; production, industries, income, health, etc. These publications are very powerful source of secondary data. Central Statistical Organization (C.S.O.), National Sample Survey Organization (N.S.S.O.), office of the Registrar, and Census Commissioner of India, Directorate of Economics and Statistics and Labour Bureau-Ministry of labour are a few government publications

b. **International Publications** : Various governments in the world and international agencies regularly publish reports on data collected by them on various aspects. For example, U.N.O.'s Statistical Year book, Demography Year Book.

c. **Semi-official Publications** : Local bodies like District Boards, Municipal Corporations publish periodicals providing information about vital factors like health, births, deaths etc.

d. **Reports of Committees and Commissions** : At times state and central governments appoint committees and commissions with a specific reference to study a phenomenon. The reports of these committees and commissions provide important secondary data. For example, Kothari commission report on education reforms, Report of National Agricultural Commission.

e. **Private Publications** : The following private publications may also be enlisted as the source of secondary data :

Journals and Newspapers such as the Financial Express and Economic times **Research Publications** of well known agencies such as Indian Statistical Institute (I.S.I.) Calcutta and Delhi, I.C.A.R., N.C.E.R.T., I.C.M.R., Publications of Business and Financial Institutions such as the Stock Exchanges, Chambers of Commerce, Newspaper Articles are also sources of secondary data.

Although secondary data are easier and cheaper to procure they have certain limitations which one must be aware of. The limitations include

1. The possibility that proper procedure might not have been followed in their collection.
2. These may not be relevant in the present contest.
3. These may not be free from personal bias and prejudices.
4. These may not have the needed accuracy or reliability.
5. These may not be adequate.

4.4 CLASSIFICATION AND TABULATION OF DATA

In the previous unit we learned how to collect data. After data collection comes the important task how to present that data so that it is meaningful and can be used effectively. The collected data is known as raw data and these are in an unorganized form and need to be organized and presented in a meaningful and understandable form in order to carry out subsequent statistical analysis. The collected data in following ways:

1. Classification and Tabulation
2. Diagrammatic Presentation
3. Graphical Presentation

Our emphasis will be mainly on Classification and Tabulation.

4.4.1 What is Classification of Data?

Classification is the method by which things are arranged in groups according to their similarity. For example, students in a class may be grouped according to their sex, age, etc. One may classify documents in an organization such as purchase, sales, tax related, etc

4.4.2 Why Do We Need Classification of Data?

The following are the main objectives of classifying the data:

1. It condenses the mass of data in an easily understandable form.
2. It removes unnecessary details.
3. It helps comparison and emphasizes the important elements of data.
4. It enables one to get a good idea of the information and helps in making conclusions.
5. It helps in the statistical treatment of the information collected.

4.4.3 What is Tabulation?

Tabulation is the process of summarizing classified or grouped data in the form of a table so that it is easily understood and an investigator is able to locate the desired information promptly. A **Table** is a systematic arrangement of classified data in columns and rows. A statistical table makes it possible for the investigator to present voluminous data in a detailed and orderly form. It helps comparison and reveals certain patterns in data, which are otherwise not obvious. Classification and Tabulation, as a matter of fact; are not two distinct, processes. Actually they go together. Before tabulation data are classified and then displayed under different columns and rows of a table.

4.4.4 Advantages of Tabulation

Statistical data arranged in a tabular form serve following objectives:

1. It simplifies complex data and the data presented are easily understood,
2. It helps in comparison of related facts.
3. It helps in computation of various statistical measures like averages, dispersion, correlation etc.
4. It presents facts in least amount of space and avoids repetitions and explanations.. The needed information can be easily located.
5. Tabulated data are good for references and they make it easier to present the information in the form of graphs and diagrams.

4.4.5 Preparing a Table

The making of a compact table is a desirable skill. It should contain all the information needed within the minimum space. What the purpose of tabulation is and how the tabulated information is to be used are the main points to be kept in mind while preparing for a statistical table. An ideal table should consist of the following main parts:

1. Table number.
2. Title of the table
3. Headnote
4. Captions or column headings.
5. Stubs or row designations
6. Body of the table.
7. Footnotes.
8. Sources of data.

Table Number

A table should be numbered for easy reference and identification. This number, if possible, should be written in the center at the top of the table. Sometimes it is also written just before the title of the table.

Title

A good table must have a clearly worded, brief but clear title explaining the nature of data contained in the table. It should also state arrangement of data and the period covered. The title should be placed centrally on the top of a table just below the table number (or just after table number in the same line).

Headnote

If figure are expressed in percentage or any other unit it should be written in the head-note.

Captions or Column Headings

Captions in a table stand for brief and self-explanatory headings of vertical columns. Captions may involve headings and subheadings as well. The unit of data contained should also be given for each column. Usually, a relatively less important and shorter classification should be tabulated in the columns.

Stubs or Row Designations

Stubs stands for brief and self explanatory headings of horizontal rows. Normally, a relatively more important classification is given in rows. Also a variable with a large number of classes is usually represented in rows. For example, rows may stand for score classes and columns for data related to sex of students. In the process, there will be many rows for score classes but only two columns for male and female students.

Body

The body of the table contains the numerical information of frequency of observations in the different cells. This arrangement of data is according to the description of captions and stubs.

Footnotes

Footnotes are given at the foot of the table for explanation of any fact or information included in the table, which needs some explanation. Thus, they are meant for explaining or providing further details about the data that have not been covered in title, captions and stubs. e.g. Some figures are projected and not collected.

Sources of Data

Lastly one should also mention the source of information from which data are taken. This may preferably include the name of the author, volume, page and the year of publication. This should also state whether the data contained in the table is of primary or secondary nature.

A model structure of a table is given below

Model Structure of a Table
Table Number
Title of the Table
Headnote

<table>
<tr><td rowspan="2">Stub Heading</td><td colspan="3">Caption Headings</td><td rowspan="2">Total</td></tr>
<tr><td colspan="3">Caption sub headings</td></tr>
<tr><td>Sub Stub Headings</td><td></td><td rowspan="2">BODY</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Totals</td><td></td><td></td><td></td><td></td></tr>
</table>

Foot notes: 1- 2-
Sources Note: 1- 2-

4.4.6 Requirements of a Good Table

A good statistical table is not just a careless grouping of columns and rows but should be such that it summarizes the total information in an easily accessible and understandable form in least possible space. While preparing a table, one must have a clear idea of the information to be presented, the facts to be compared and the points to be stressed. The following points should be kept in mind:

1. A table should be formed in keeping with the objects of a statistical enquiry.
2. A table should be prepared meticulously so that it is easily understandable.
3. A table should be formed so as to suit the size of the paper. But such an adjustment should not be at the cost of legibility.
4. If the figures in the table are large, they should be rounded or approximated. The method of approximation and units of measurements too should be specified.
5. Rows and columns in a table should be numbered and certain figures to be stressed may be put in ‘box’ or ‘circle’ or in bold letters.
6. The arrangement of rows and columns should be in a logical and systematic order. This arrangement may be an alphabetical, chronological or according to magnitude.
7. The rows and columns are separated by single, double or thick lines to represent various classes and sub-classes used.
   The corresponding proportions or percentages should be given in adjoining rows and columns to allow easy comparison. A vertical expansion of the table is generally more desirable than the horizontal one.

8. The averages or totals of different rows should be given at the right of the table and that of columns at the bottom of the table. Totals for every sub-class too should be mentioned.
9. In case it is not possible to accommodate all the information in a single table, it is better to have two or more related tables.

Type of Tables

Tables can be classified according to their purpose, number of characteristics used, stage of enquiry and other parameters. On the basis of the number of characteristics, tables may be
classified as follows:

1. Simple or one-way table.
2. Two-way table.
3. Manifold table.

Simple or One-Way Table

A simple or one-way table is the simplest table, which contains data of one characteristic only. A simple table is easy-to construct and simple to follow. For example the table below may be used to show the number of students in different faculties of a college.

Faculties	**No of students**
Commerce	
Arts	
Science	

If we consider the strength of the students in various academic years for the different faculties we get a **two-way table** as we are now considering two characteristics year and faculties.

Year	**Commerce**	**Arts**	**Science**
2001-02			
2002-03			
2003-04			
2004-05			
2005-06			
Totals			

If we consider finding out how many males /female students exist in each faculty for each of the academic year and their totals we have a **three-way** or **manifold** table

Year	**Commerce**			**Arts**			**Science**			**Total**		
	M	F	T	M	F	T	M	F	T	M	F	T
2001-02												
2002-03												
2003-04												
2004-05												
2005-06												
Totals												

Example 1:

The number of students in AVS College in the year 1990 was 500, of which 200 were rural students. In 1991, the number of students increased by 150 and the number of urban students increased by 75. In 1992, the number rural students increased by 20%, while the total number of students increased by 50%. Tabulate the above data.

Solution: As we can observe that there are two characteristics in the given data i.e. (i) Year and (ii) residence of students.

Thus, the data can be tabulated in the table with the given information as follows:

Year	Students		Total
	Rural	**Urban**	
1990	200	?	500
1991	?	?	?
1992	?	?	?
Total	?	?	?

The data for blank cells are calculated as follows:

In the year 1990, total number of students = 500, rural students = 200.

Thus, urban students = 500 – 200 = 300.

In the year 1991, number of students = 500 + 150 = 650.

Number of urban students = 300 + 75 = 375.

Hence, rural students = 650 – 375 = 275.

In the year 1992, rural students = $275 + \frac{20}{100} \times 275 = 275 + 55 = 330$

Total number of students = $650 + \frac{50}{100} \times 650 = 650 + 325 = 975$.

Hence, urban students = 975 – 330 = 645.

Thus, the completed table is given by

Year	Students		Total
	Rural	**Urban**	
1990	200	300	500
1991	275	375	650
1992	330	645	975
Total	**805**	**1320**	**2125**

Example 2:

In a Factory, the total numbers of employees were 1600. In its Production Dept. there were 550 workers, of which 150 were female. The R&D Dept. had 350 male workers. The total number of workers in the Administrative Dept. was 500. The ratio of male to female workers in the factory is 3:2. Tabulate the above information.

Solution:

From the given information we can see that there are two characteristics i.e. (i) Sex of the workers and (ii) Departments of the Factory.

Thus, the data can be tabulated in the table with the given information as follows:

Workers / Dept.	Male	Female	Total
Production	?	150	550
R & D	350	?	?
Administrative	?	?	500
Total	?	?	**1600**

The data for blank cells are calculated as follows:

The number of male workers in the Production dept. = 550 – 150 = 400.

The total number of workers in R & D Dept. = 1600 – 550 – 500 = 550.

Hence, the number of female workers in this dept. = 550 – 350 = 200.

The ratio of male to female workers is 3:2,

$\therefore$ number of male workers = $\frac{3}{5}$ x 1600 = 960

and number of female workers = $\frac{2}{5}$ x 1600 = 640

Now, the number of female workers in the administrative dept. = 640 – 150 – 200 = 290.

Hence, the number of male workers in this dept. = 500 – 290 = 210.

Thus, the completed table is given by

Dept. \ Workers	Male	Female	Total
Production	400	150	550
R & D	350	200	550
Administrative	210	290	500
Total	**960**	**640**	**1600**

Example 3:

The ratio of male to female workers in ABC Company was 5:3. Out of the total of 160 workers, 25 men were Post Graduates and 20 women were Graduates. The number of men which were HSC was 15 more than that of Post Graduates. Number of Post Graduate women is one-third of HSC women. Tabulate the above data.

Solution:

From the given information we can see that there are two characteristics i.e. (i) Qualification and (ii) Sex of workers.

Thus, the data can be tabulated in the table with the given information as follows:

Dept. \ Workers	Men	Women	Total
Post Graduate	25	?	?
Graduates	?	20	?
HSC	?	?	?
Total	?	?	**160**

Now, the ratio of men to women workers is 5:3 and total number of workers = 160

$\therefore$ number of men = $\frac{5}{8}$ x 160 = 100 and number of women = $\frac{3}{8}$ x 160 = 60

The number of HSC men = 15 + 25 = 40.

Hence, number of Graduate men = 100 – 25 – 40 = 35

$\therefore$ total number of Graduates = 35 + 20 = 55.

Now, let number of women with HSC = x, then number of Post Graduate women = $\frac{x}{3}$

$\therefore x + \frac{x}{3} + 20 = 60$ $\Rightarrow x + \frac{x}{3} = 40$ $\Rightarrow \frac{4x}{3} = 40$

$\therefore x = 30$

Thus, women with HSC = 30 and Post Graduate women = 10.

Total number of Post Graduates = 25 + 10 = 35

Total number of HSC workers = 40 + 30 = 70.

Thus, the completed table is given by

Workers / Dept.	Men	Women	Total
Post Graduate	25	10	35
Graduates	35	20	55
HSC	40	30	70
Total	**100**	**60**	**160**

Example 4:

A survey of the number of students of Almighty Residential School from 2002 to 2005 revealed to following details: In 2002-03 the total number of students was 360, of which 100 were girls. The number of Indian boys outnumbered the NRI boys by 20. The ratio of Indian and NRI girls was 3:2. In 2003-04, the number of Indian boys increased by 60, while the total number of Indians increased by 50%. The number of NRI girls increased by 20 and the total number of NRI's increased by 25%. In 2004-05, in comparison with the previous year, the total number of Indians increased by 200 and the Indian girls increased by 50%. The total number of students were 750, of which the boys were 550. Tabulate the above information.

Solution:

From the given information we can see that there are three characteristics i.e. (i) Gender, (ii) Nationality and (iii) year of enrollment. To tabulate this we require a three-way table.

Thus, the data can be tabulated in the 3-way table with the given information as follows:

Year	Students						Total		
	Indian			NRI					
	Boys *a*	Girls *b*	Total *a+ b*	Boys *c*	Girls *d*	Total *c+d*	Boys *a+c*	Girls *b+d*	Total
2002-03	?	?	?	?	?	?	?	100	360
2003-04	?	?	?	?	?	?	?	?	?
2004-05	?	?	?	?	?	?	550	?	750

For the year 2002-03:

No. of Indian students = 360 – 100 = 260

Since the Indian boys outnumbered the NRI boys by 20, if the no. of NRI boys is x then no. of Indian boys is $x + 20$.

Hence, $(x + 20) + x = 260 \Rightarrow 2x + 20 = 260 \Rightarrow 2x = 240 \Rightarrow x = 120$

Thus, the no. NRI boys = 120 and no. of Indian boys = 140

The ratio of Indian and NRI girls is 3:2

Year	**Students**						**Total**		
	Indian			**NRI**					
	Boys *a*	**Girls *b***	**Total *a+ b***	**Boys *c***	**Girls *d***	**Total *c+d***	**Boys *a+c***	**Girls *b+d***	**Total**
2002-03	140	60	200	120	40	160	260	100	360
2003-04	200	100	300	140	60	200	340	160	500
2004-05	350	150	500	200	50	250	550	200	750

❖❖❖

UNIT IV

5

DIAGRAMS AND GRAPHS

OBJECTIVES

After learning this topic you will be able to know

- What is the importance of diagrams?
- How to interpret the given diagrams i.e. read the given diagram/s and answer the questions asked on the diagram.

5.2 Introduction

5.1.1 Importance of diagram

To understand various trends of the data (which is already classified and tabulated) at a glance and to facilitate the comparison of various situations, the data are presented in the form of diagrams & graphs. Diagrams are useful in many ways, some of the uses are given below:-

i) Diagrams are attractive and impressive.
ii) Diagrams are useful in simplifying the data.
iii) Diagrams save time and labour.
iv) Diagrams are useful in making comparisons.
v) Diagrams have universal applicability.
(i.e. They are used in almost in every field of study like business, economics , administration , social institutions and other areas .)

5.1.2 Limitations of Diagrams

Some of the limitations of diagrams are listed below:

i) Diagrams give only a vague idea of the problem which may be useful for a common man but not for an expert who wishes to have an exact idea of the problem.
ii) The level of precision of values indicated by diagrams is very low.
iii) Diagrams are helpful only when comparisons are desired. They don't lead to any further analysis of data.
iv) They can be a supplement to the tabular presentation but not an alternative to it.

***** Note that we will discuss here only interpretation of diagrams and no construction of diagrams, since construction of diagrams is not in the F.Y.BCom syllabus.

5.2 TYPES OF DIAGRAM

1) **One dimensional diagrams**:
   a) Simple Bar diagrams b) Multiple Bar diagrams (joint Bar Diagrams)
   c) Sub-divided Bar Diagram and d) Percent Bar Diagrams.
2) **Two dimensional diagrams**: Pie or Circular diagram .

Let us solve some examples based on the above diagrams.

a) Simple Bar Diagram:
Example 1
Study the following diagram giving enrollments for C.F.A. and answer the questions given below:

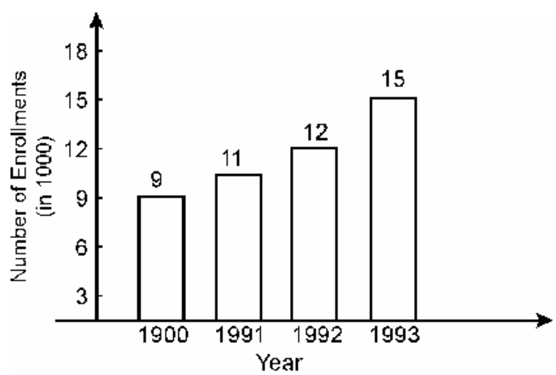


Fig. 5.1

i) Name the above diagram – ***Simple Bar Diagram***.
ii) Which year has least enrollment? – ***The year* 1990**.
iii) What is the percentage increase in enrollments from the year 1991 to 1992? :– $\frac{12-11}{11}$ x100 = **9.09%**
iv) What is the enrollment for the year 1993? – **15,000.**

b) Multiple Bar Diagrams:
Example 2:
The following diagram gives the figures of Indo-US trade during 1987 to 1990. The figures of Indian Exports & Imports are in billion. Answer the following questions from the diagram given below:

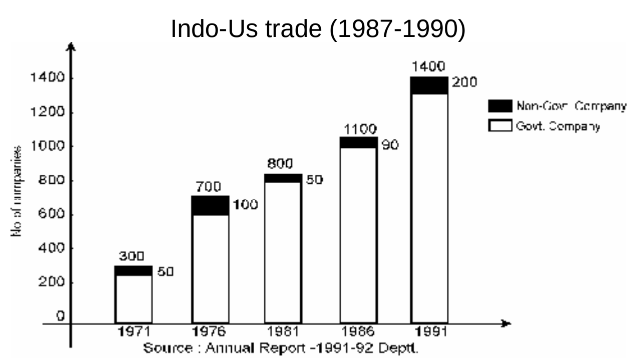


Fig. 5.2

i) Name the above diagram.
ii) Which year shows the maximum exports?
iii) The difference in exports and imports is least in the year ---- and maximum in the year -----
iv) The percentage decrease in exports from year 1989 to year 1990 is -----
v) The import for the year 1987 is ------.

c) Sub-divided Bar Diagrams:
Example 3:
The following diagram shows the number of Govt. and Non-Govt. Companies in various years. Study the diagram and answer the questions given below:-

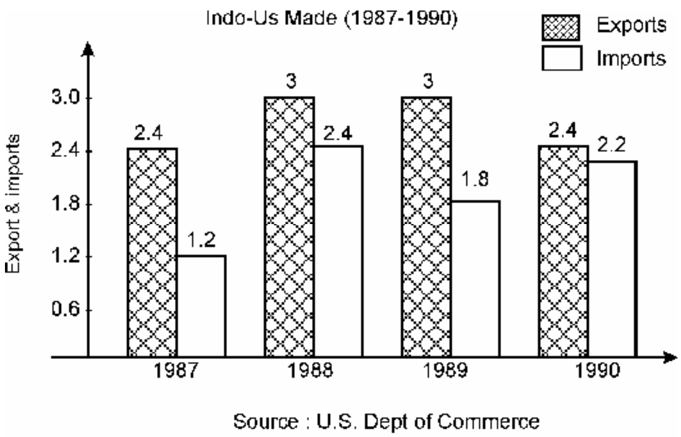


Fig. 5.3

i) Name the above diagram.
ii) The number of Govt . companies in year 1976 is -----.
iii) The increase in Non-Govt. companies from the year 1986 to year 1991 is -----.
iv) In which two years the number of non-Govt. companies is equal ?

d) Percentage Bar Diagrams:
Example 4:
The following diagram gives monthly income and expenditure of two families A and B. Study the diagram and answers the questions given below:

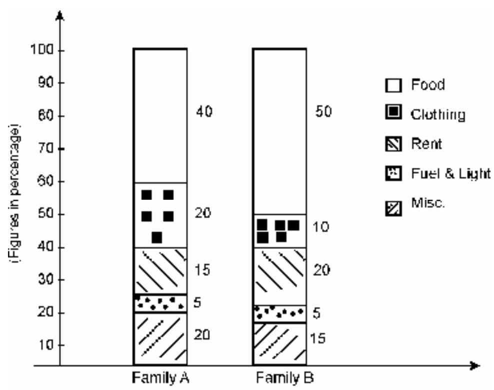


Fig. 5.4

i) Name the above diagram
ii) Which family spends more on clothing ?
iii) If the total income of family A is Rs. 20,000 /- then family A spends Rs. ----- on Rent.
iv) Family A spends equal amount on which two items ?
v) The expenses on item ------ is double than the expenses on item ------ in Family B .
e) pie or circular Diagram

Example 5:
The following diagram shows the expenditure of an average working class family . Study the diagram & answer the questions given below .

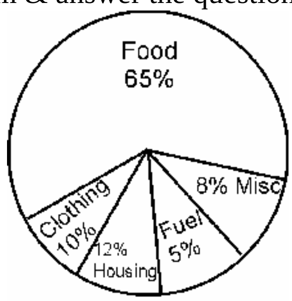


Fig. 5.5

i) Name the above diagram .
ii) Expenses on ------- are maximum and ------- are minimum .
iii) If the total income is Rs.30,000/- then expenses on Housing & fuel together is Rs.----.
(Hint : Housing + fuel = (12+5)% i.e. 17%
Required Expenses = $\frac{17}{100}$ x 30,000 = 5100/-)

Limitations of pie-diagrams:
Pie-diagrams are less effective than bar diagrams for accurate reading and interpretation , particularly when the data is divided into a large number of components is very small .

5.3 GRAPHS

A large variety of graphs are used in practice. Here we will be discussing the graphs of frequency distributions only.
A frequency distribution can be presented graphically in any of the following ways:
1) Histogram
2) Frequency Polygon
3) Frequency Curve
4) 'Ogives' or cumulative frequency curves .

Remark: We will deal with equal class intervals' Histograms only.

1) Histogram: It is a graph of a frequency distribution in which the class intervals are plotted on x-axis and their respective frequencies on y-axis .On each class a rectangle is drawn , the height of each rectangle is taken to be equal to the frequency of the corresponding class . The construction of such a Histogram is shown in the following example.

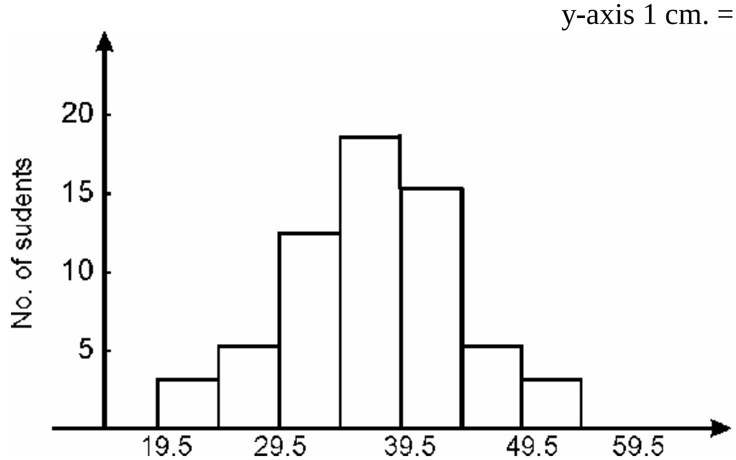
Example 6:
Draw a Histogram for the following distribution giving the marks obtained by 60 students of a class in a college .

Marks :	20-24	25-29	30-34	35-39	40-44	45-49	50-54
No. of students:	3	5	12	18	14	6	2

Solution: Here class intervals given are of inclusive type . State the upper limit of a class is not equal to the lower limit of its following class , the class boundaries will have to be determined . After the adjustment , the distribution will be as below .

Marks :	19.5-24.5	24.5-29.5	29.5-34.5	34.5-39.5	39.5-44.5	44.5-49.5	49.5-54.5
No. of students	3	5	12	18	14	6	2

scale: x-axis 1 cm. = 10 marks
y-axis 1 cm. = 5 students.



5.4 FREQUENCY POLYGON

It is another method of representing a frequency distribution of a graph. Frequency polygons are more suitable than histograms whenever two or more frequency distributions are to be compared.

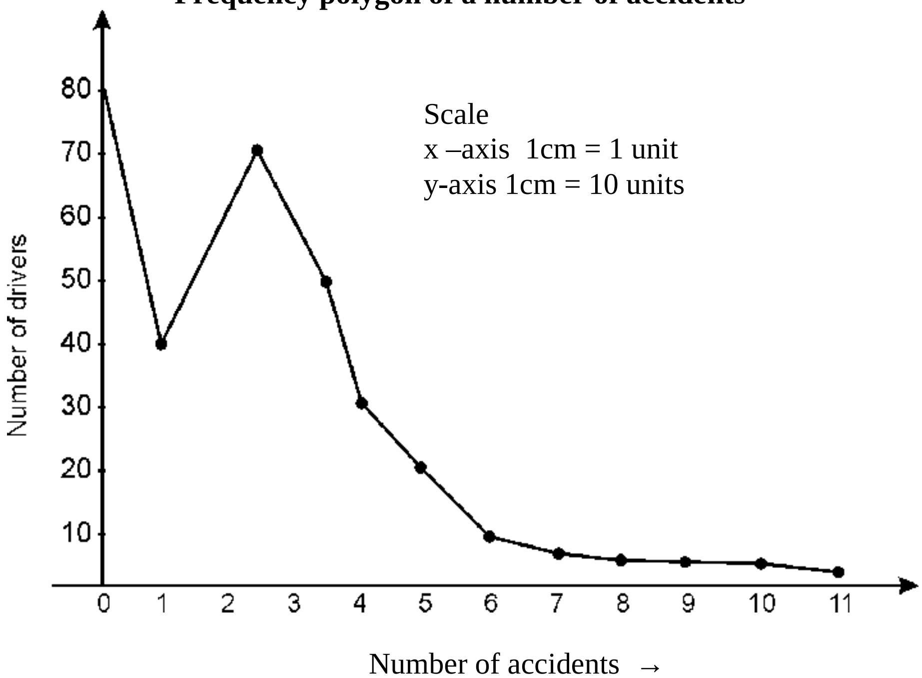
Frequency polygon of a grouped or continuous freq. distribution is a straight line graph. The frequencies of the classes are plotted against the mid-values of the corresponding classes . The points so obtained are joined by straight lines (segments) to obtain the frequency polygon.

Example 7:

The following data show the number of accidents sustained by 313 drivers of a public utility company over a period of 5 years. Draw the frequency polygon

No. of accidents :	0	1	2	3	4	5	6	7	8	9	10	11
No. of drivers :	80	44	68	41	25	20	13	7	5	4	3	2

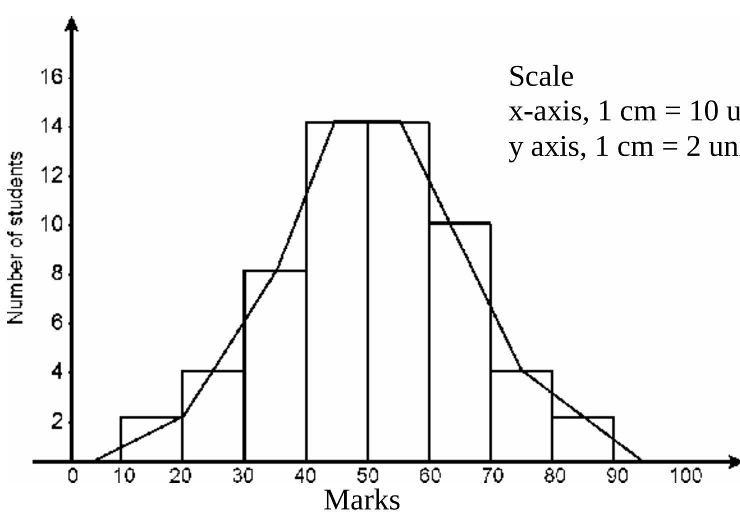
Frequency polygon of a number of accidents



Example 8:

Draw a Histogram & Frequency polygon from the following distribution giving marks of 50 students in statistics .

Marks in Statistics :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students :	0	2	3	7	13	13	9	2	1

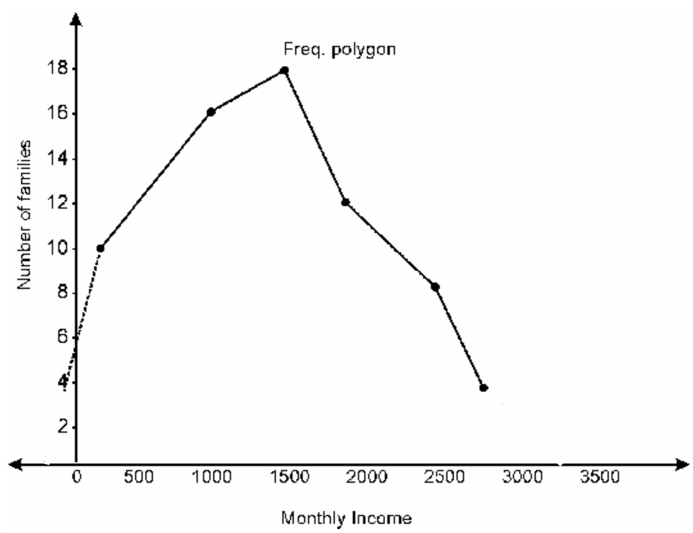


Note that, here we will first draw Histogram and then the mid-points of the top of bars are joined by line segments to get the frequency polygon.

Remark: Note that frequency polygon can be drawn even without converting the given distribution into classes. The frequencies are plotted against the corresponding mid-points (given) and joined by line segments.

Example 9:
Draw a frequency polygon to represent the following distribution.

Monthly Income (in Rs.)	Number of families
0-500	10
500-1000	15
1000-1500	18
1500-2000	12
2000-2500	8
2500-3000	4



5.5 FREQUENCY CURVE

Frequency curve is similar like frequency polygon, only the difference is that the points are joined by a free hand curve instead of line segments as we join in frequency polygon.

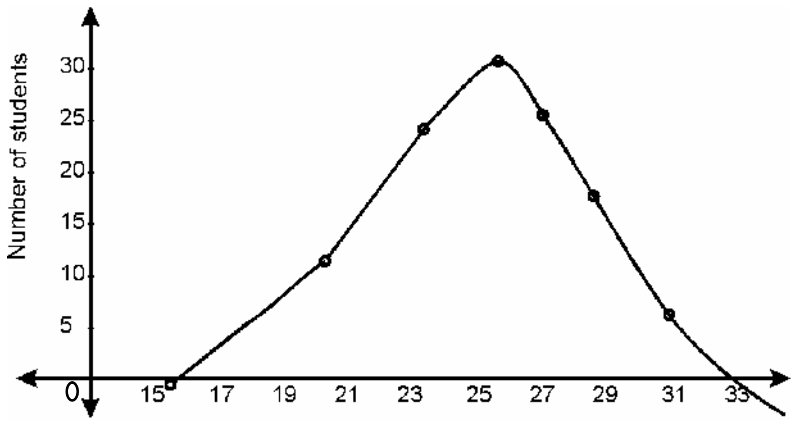
Let us study the following examples to study the concept of frequency curve .

Example 10: Draw a frequency curve for the following data :

Age (yrs) :	17-19	19-21	21-23	23-25	25-27	27-29	29-31
No. of students :	7	13	24	30	22	15	6

Solution: Here we will take ***ages*** on horizontal axis (i.e. x-axis) and ***no. of students*** on vertical axis (i. e. say y-axis)

We will plot the given frequencies (No. of students) against mid-points of the given class interval & then join these points by free hand curve. Extremities (first and the last points plotted) are joined to the mid-points of the neighboring class intervals.

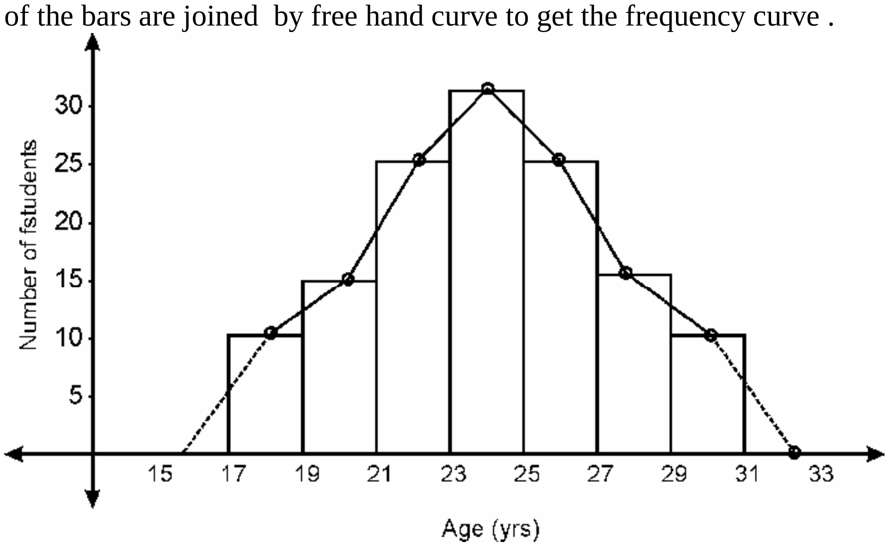


Age yrs

Example 11: Draw a Histogram & frequency curve for the following data:

Age (yrs) :	17-19	19-21	21-23	23-25	25-27	27-29	29-31
No. of students :	7	13	24	30	22	15	6

Solution: First we will draw histogram & then the mid-points of the top of the bars are joined by free hand curve to get the frequency curve .



5.6 Ogive or Cumulative Frequency Curve

Ogive (*pronounced as **ojive***) is a graphic presentation of the cumulative frequency (c.f) distribution of continuous variable. It consists in plotting the c.f. (along the y-axis) against the class boundaries (along x-axis) . Since there are two types of c.f. distributions namely ' less than ' c.f. and 'more than ' c.f. , we have accordingly two types of ogives , namely ,
i) ' less than and equal to ' ogive ,

ii) ' more than and equal to' ogive .

'Less than and equal to ogive : This consists of plotting the 'less than ' cumulative frequencies against the upper class boundaries of the respective classes . The points so obtained are joined by a smooth free hand curve to give 'less than ' ogive . Obviously, 'less than ' ogive is an increasing curve , sloping upwards from left to right .

Note : Since the frequency below the lower limit of the first class (i.e. upper limit of the class preceding the first class) is zero , the ogive curve should start on the left with a cum. freq. zero of the lower boundary of the first class .

'More than and equal to' ogive: This consists of plotting the ' more than ' cum. frequencies against the lower class boundaries of the respective classes . The points so obtained are joined by a smooth free hand curve to give ' equal give . Obviously, 'more than ' ogive is a decreasing curve , slopes downwards from left to right .

Note: We may draw both the 'less than' and 'more than and equal to' ogives on the same graph . If done so, they intersect at a point. The foot of the perpendicular from their point of intersection on the x-axis gives the value of the median .

Remark: Ogives are particularly useful for graphic computation of partition values namely median , quartiles , deciles , percentiles etc . They can also be used to determine graphically the number of proportion of observations below or above a given value of the variable or lying between certain interval of the values of the variable .

We will now study the following examples to understand the concepts discussed above :-

Example 12: The data below give the marks secured by 70 students t a certain examination :

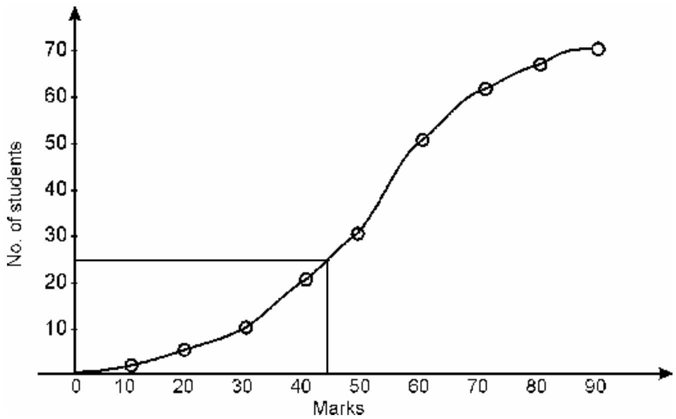
i) Draw the 'less than ' type ogive curve .

ii) Use the ogive curve to estimate the percentage of students getting marks less than 45.

Marks :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students :	2	3	6	11	12	15	10	7	4

Solution: We will take marks on x-axis & No. of students on y-axis . Also we will calculate 'less than ' cumulative frequencies & then we will plot these calculated frequencies against the upper limit of the corresponding class & joining the points so obtained by a smooth free hand curve to get a 'less than' ogive

i) Marks	No.of students		Less than cf
0-10	2	→	2
10-20	3	→	2+3=5
20-30	6	→	5+6=11
30-40	11	→	11+11=22
40-50	12	→	22+12=34
50-60	15	→	34+15=49
60-70	10	→	49+10=59
70-80	7	→	59+ 7=66
80-90	4	→	66+ 4=70



ii) To estimate the number of students getting marks less than 45 , draw perpendicular to the x-axis (representing marks) at x=45 , meeting the 'less than' ogive at point P .
From P draw a perpendicular PM on the y-axis (representing number of students)

Then, from the graph OM = 26.8 (approximate) ≘ 27 is the number of candidates getting score 45 or less .

Hence , the percentage of students getting less than 45 marks is given by
$\frac{27}{70} \times 100$ i.e. 38.57
(Here total students =70) .

Example 13: The following table gives the distribution of monthly income of 600 families in a certain city .

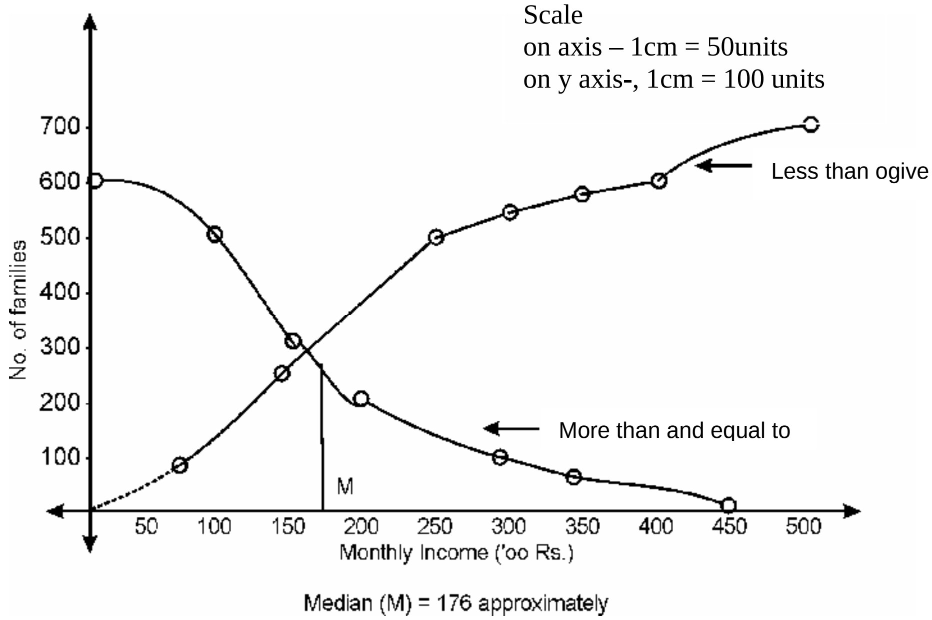
Monthly Income	Below 75	75-150	150-225	225-300	300-375	375-450	Above 450
No. of families	60	170	200	60	50	40	20

Draw a 'less than ' and a 'more than equal to' ogive curve for the above data on the same graph and from there read the median income .

Solution: For drawing the 'less than' and 'more than' equal to ogive we convert the given distribution into 'less than' and 'more than equal to' cumulative frequencies (c.f.) as given in the following table.

Monthly Income ('00 Rs.)	No. of families (*f*)	Less than *c.f.*	More than *c.f*
Below 75	60	60	600
75-150	170	230	540
150-225	200	430	370
225-300	60	490	170
300-375	50	540	110
375-450	40	580	60
450 and above	20	600	20
TOTAL	600		

'Less than' and 'More than & equal to' ogive



Example 14:
Convert the following distribution into ' more than and equal to ' frequency distribution.

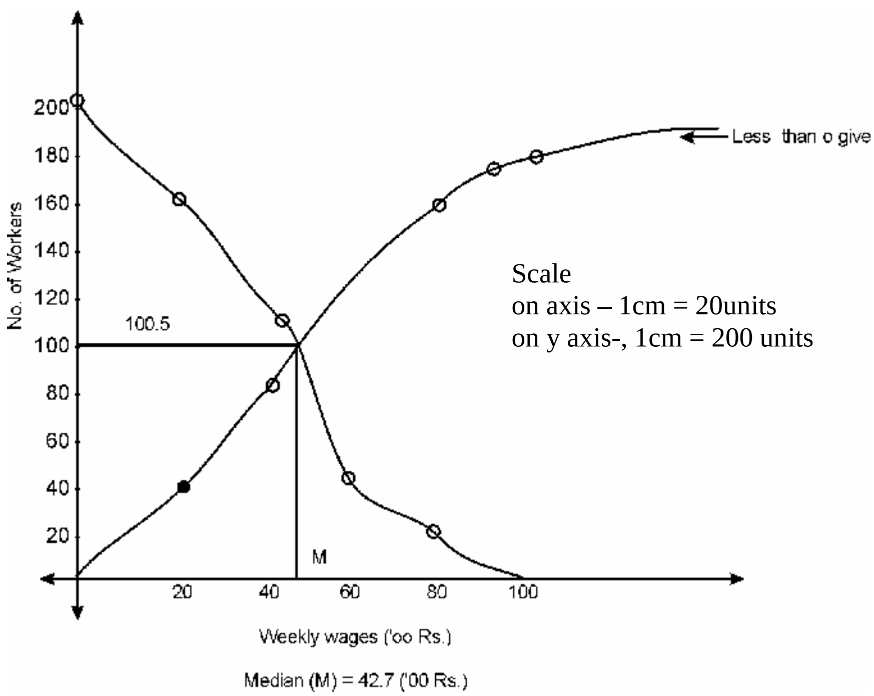
Weekly wages less than ('00 Rs.)	:	20	40	60	80	100
No. of workers	:	41	92	156	194	201

For the data given above , draw ' less than ' and equal to more than ' ogives and hence find the value of median .

Solution:

Weekly wages (in '00 Rs.)	No. of Workers (f)	Cumulative Freq. (c.f) Less than	Cumulative Freq. (c.f) More than
0-20	41	41	160+41=201
20-40	92 -41=51	92	51+109=160
40-60	156-92=64	156	64+45=109
60-80	194-156=38	194	38+7=45
80-100	201-194=7	201	7

'Less than' and 'More than & equal to' ogives are shown in the following figure:



Example 15: Draw ' more than and equal to ' ogive for the following data

Marks :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No.of students :	4	6	10	15	25	22	11	7

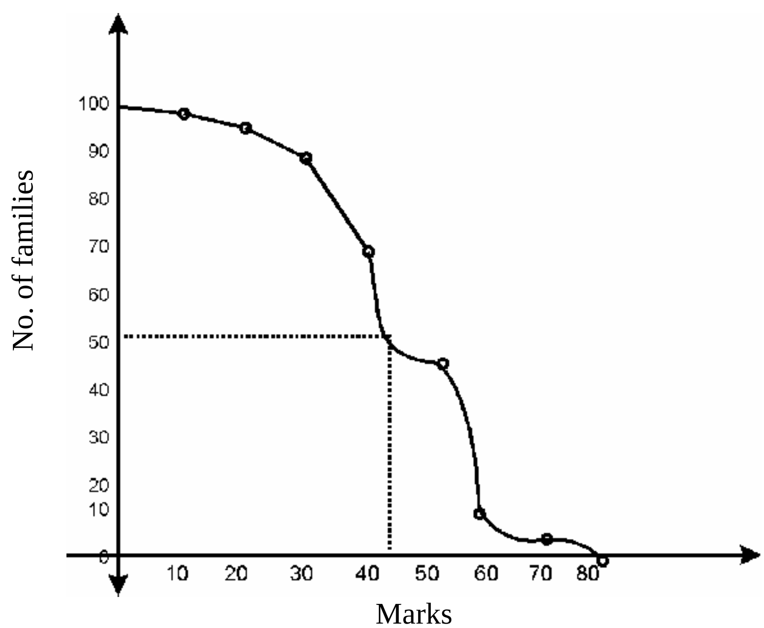
Here find i) median marks ii) Number of students getting 60 or more marks .

Solution:

First we prepare ' more than and equal to ' c.f. distribution table for the given data.

Marks more than (lower limit of class interval)	No. of students	More than c.f.
0	4	100
10	6	96
20	10	90
30	15	80
40	25	65
50	22	40
60	11	18
70	7	7
TOTAL	100	

We draw more than c.f. curve by taking lower limit of class interval on x-axis and more than c.f. on y-axis .



i) To locate median, we find $\frac{N}{2} = \frac{100}{2} = 50$ (N is frequency total)

Now draw a horizontal line parallel to x-axis passing through a point on y-axis (0,50). The line will meet the curve at some point. From that point draw a perpendicular line meeting x-axis. The foot of the perpendicular will be the value of the median.

Therefore, Graphical value of median = 45 (M = median)

ii) To find the number of students getting 60 or more marks, we draw a vertical line passing through (60,0) on x-axis. The line will meet the curve at some point. From that point draw a horizontal line which will meet y-axis at (0,18).

Therefore, the number of students getting 60 or more marks =18.

Note: We can find no. of students getting less than 60 marks by simply calculating :- Total no. of students – 18 = 100 – 18 = 82.

EXERCISE

1) What is the importance of diagrams & graphs in statistical analysis?

2) What is an ogive? Explain its uses.

3) Name different types of diagrams.

4) Draw histogram and locate mode graphically.

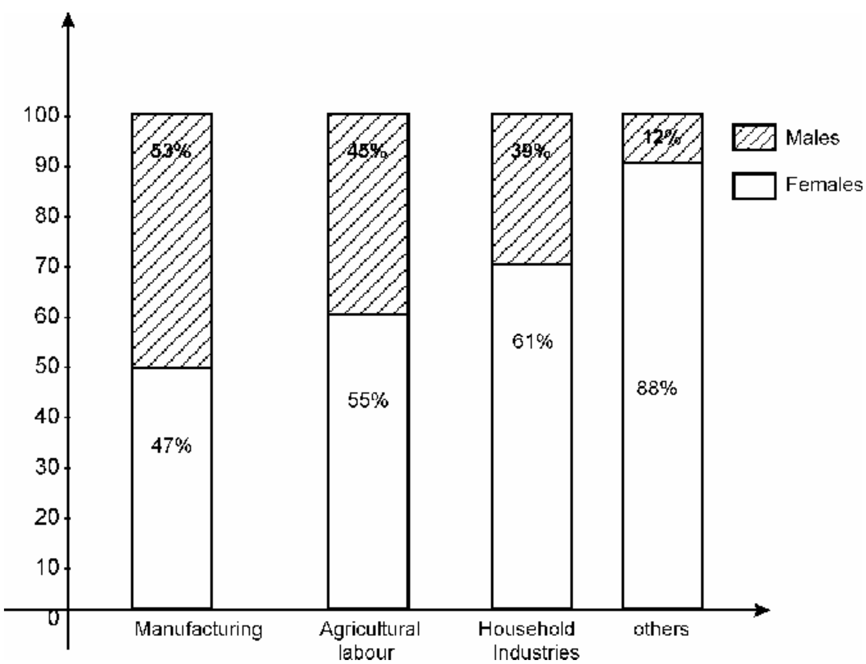
Marks	:	0-10	10-20	20-30	30-40	40-50
No. of students	:	6	11	15	8	3

5) Draw less than ogive curve for the following data .

Wages (in Rs.)	:	70-80	80-90	90-100	100-110	110-120	120-130	130-140
No.of workers	:	85	109	126	134	115	83	68

Hence find the number of workers whose wages are less than Rs. 117.
Also find no. of workers whose wages are Rs. 117 or above.

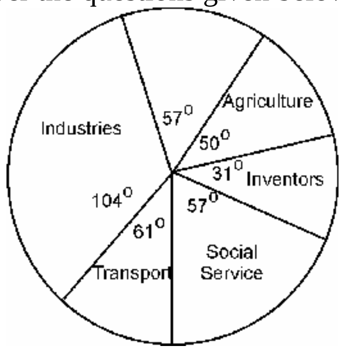
6) Study the following diagram giving the occupation of males and females in India and answer the following questions:



i) Name the above diagram.
ii) In which occupation the proportion of males was the highest in India?

iii) In which industry the absolute difference between the proportion of males and females was the highest?

7) Study the following diagram giving percentage of investment pattern in the fifth plan & answer the questions given below:-



i) Name the above diagram.

ii) What is the highest percentage of investment pattern in the fifth plan ?

iii) Which is the smallest sector in the above diagram ?

iv) What is the contribution of irrigation and power sector ?

8) Draw a Histogram & freq. polygon for the following data :

Marks :	0-10	10-20	20-30	30-40	40-50	50-60
No. of students :	3	8	12	18	10	4

REFERENCES:

1) Business statistics by R.S. Bharadwaj - Excel Books - New Delhi
2) Fundamentals of statistics by S.C. Gupta - Himalaya Publishing House.
3) Statistical Methods by Dr. S. P. Gupta - Sultan Chand & sons . New Delhi

❖❖❖

UNIT IV

6

MEASURES OF CENTRAL TENDENCY

OBJECTIVES

After reading this chapter you will be able to:

1) Define an average.
2) Understand different types of averages.
3) Calculate Arithmetic mean and weighted Arithmetic mean.
4) Calculate Median and Quartiles.
5) Calculate Mode by formula.
6) Graphical location of Median and Mode.

6.1 INTRODUCTION

In the earlier chapter we have discussed the Diagrams and Graphs which are used for presentation of data. Diagrams furnish only approximate information. Graphs are more precise and accurate than diagrams. But they cannot be effectively used for further statistical analysis. To study the characteristics in detail, the data must be further analysed.

There is a tendency in almost every statistical data that most of the values concentrate at the centre which is referred as 'central tendency'. The typical values which measure the central tendency are called measures of central tendency. Measures of central tendency are commonly known as 'Averages.' They are also known as first order measures. Averages always lie between the lowest and the lightest observation.

The purpose for computing an average value for a set of observations is to obtain a single value which is representative of all the items and which the mind can grasp simply and quickly. The single value is the point or location around which the individual items cluster.

The word average is very commonly used in day to day conversation. For example, we often talk of average boy in a class, average height of students, average income, etc. In statistics, the term average has a different meaning. Averages are very much useful for describing the distribution in concise manner. The averages are extremely useful for comparative study of different distributions.

6.2 REQUISITES FOR AN IDEAL AVERAGE

The following are the characteristics which must be satisfied by an ideal average:

(i) It should be rigidly defined.
(ii) It should be easy to calculate and easy to understand.
(iii)It should be based on all observations.
(iv)It should be suitable for further mathematical treatment.
(v) It should be least affected by the fluctuations of sampling.
(vi) It should not be affected by extreme values.

6.3 TYPES OF AVERAGES

The following are the five types of averages which are commonly used in practice.

1. Arithmetic Mean or Mean (A.M)
2. Median
3. Mode
4. Geometric Mean (G.M)
5. Harmonic Mean (H.M)

   Of these, arithmetic mean, geometric mean and harmonic mean are called mathematical averages; median and mode are called positional averages. Here we shall discuss only the first three of them in detail, one by one.

6.4 ARITHMETIC MEAN (A.M)

(i) **For Simple or Ungrouped data**:(where frequencies are not given)
Arithmetic Mean is defined as the sum of all the observations divided by the total number of observations in the data and is denoted by $\overline{x}$, which is read as '*x*-bar'

i.e. $$\overline{x} = \frac{\text{sum of all observations}}{\text{number of observations}}$$

In general, if x_1 x_2……., x_n are the n observations of variable x, then the arithmetic mean is given by

$$\overline{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots x_n}{n}$$

If we denote the sum $x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots\ldots + x_n$ as $\sum x$, then

$$\boxed{\overline{x} = \frac{\Sigma x}{n}}$$

Note: The symbol ∑ is the Greek letter *capital sigma* and is used in Mathematics to denote the sum of the values.

Steps: (i) Add together all the values of the variable x and obtain the total, i.e, $\sum x$.
(ii) Divide this total by the number of observations.

Example 1:
Find the arithmetic mean for the following data representing marks in six subjects at the H.S.C examination of a student.
The marks are 74, 89,93,68,85 and 76.

Solution: $n = 6$

$$\overline{x} = \frac{\Sigma x}{n} \quad \Rightarrow \quad \overline{x} = \frac{74+89+93+68+85+76}{6} = \frac{485}{6}$$

$$\overline{x} = 80.83$$

Example 2: Find arithmetic mean for the following data .
425 , 408 , 441 , 435 , 418

Solution: $n = 5$ and $\overline{x} = \frac{\Sigma x}{n}$

$$\therefore \quad \overline{x} = \frac{425+408+441+435+418}{5} = \frac{2127}{5}$$

$$\therefore \quad \overline{x} = 425.4$$

(ii) For Grouped data (or) discrete data:
(values and frequencies are given)

If x_1, x_2 …… x_n are the values of the variable x with corresponding frequencies $f_1, f_2,$ ………f_n then the arithmetic mean of x is given by

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots f_nx_n}{f_1 + f_2 + \ldots\ldots f_n}$$

Or

$$\boxed{\overline{x} = \frac{\Sigma fx}{\Sigma f}}$$

Where $\sum f = f_1+f_2+\ldots+f_n$ = sum of the frequencies

If we denote $N = \sum f$, then $\overline{x} = \frac{\Sigma fx}{N}$

Steps : (1) Multiply the frequency of each row with the variable x and obtain the total $\sum fx$.
(2) Divide the total $\sum fx$ by the total frequency.

Example 3: Calculate arithmetic mean for the following data .

Age in years:	11	12	13	14	15	16	17
No.ofStudents:	7	10	16	12	8	11	5

Solution:

Age in years (x)	**No.of Students (f)**	fx
11	7	77
12	10	120
13	16	208
14	12	168
15	8	120
16	11	176
17	5	85
Total	$N = 69$	$\sum fx = 954$

$\sum f = N = 69$, $\sum fx = 954$

$$\bar{x} = \frac{\Sigma fx}{N} = 13.83 \text{ years.}$$

Example 4: Calculate the average bonus paid per member from the following data :

Bonus (in Rs.)	40	50	60	70	80	90	100
No. of persons	2	5	7	6	4	8	3

Solution :

Bonus (in Rs) x	**No. of persons (f)**	fx
40	2	80
50	5	250
60	7	420
70	6	420
80	4	320
90	8	720
100	3	300
Total	$N = 35$	$\sum fx = 2510$

Now, $N = 35$ and $\sum fx = 2510$

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{2510}{35} = \text{Rs. } 71.71$$

(iii) **Grouped data** (class intervals and frequencies are given)

Formula : $\boxed{\bar{x} = \frac{\Sigma fx}{N}}$ where N = Σf and x = Mid-point of the class intervals

Steps : (1) Obtain the mid-point of each class interval

(Mid-point = $\frac{\text{lower limit + upper limit}}{2}$)

(2) Multiply these mid-points by the respective frequency of each class interval and obtain the total $\sum fx$.

(3) Divide the total obtained by step (2) by the total frequency N.

Example 5:

Find the arithmetic mean for the following data representing marks of 60 students.

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No.ofStudents	8	15	13	10	7	4	3

Solution:

Marks	No. of students (f)	Mid-point (x)	fx
10-20	8	15	120
20-30	15	25	375
30-40	13	35	455
40-50	10	45	450
50-60	7	55	385
60-70	4	65	260
70-80	3	75	225
Total	N = 60		$\sum fx$ = 2270

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{2270}{60} = 37.83$$

$\therefore$ The average marks are 37.83

Example 6:

Calculate the arithmetic mean of heights of 80 Students for the following data.

Heights in cms	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No. of Students	7	11	15	21	16	6	4

Solution:

The class intervals are of inclusive type. We first make them continuous by finding the class boundaries. 0.5 is added to the upper class limits and 0.5 is subtracted from the lower class limits to obtain the class boundaries as

130-0.5= 129.5 and 134+0.5=134.5

135-0.5= 134.5 and 139+0.5= 139.5 and so on.

Heights (in cms)	**Class boundaries**	**No. of Students(f)**	**Mid-point(x)**	***fx***
130-134	129.5-134.5	7	132	524
135-139	134.5-139.5	11	137	1507
140-144	139.5- 144.5	15	142	2130
145-149	144.5- 149.5	21	147	3087
150-154	149.5-154.5	16	152	2432
155- 159	154.5-159.5	6	157	942
160-164	159.5-164.5	4	162	648
Total		**N = 80**		**$\sum fx$= 1270**

$N = \sum f = 80$ and $\sum fx = 11270$

$$\therefore \bar{x} = \frac{\Sigma fx}{N} = \frac{11270}{80} = 140.88$$

$\therefore$ The average height of students = 140.88 cm.

6.5 WEIGHTED ARITHMETIC MEAN

One of the limitations of the arithmetic mean discussed above is that it gives equal importance to all the items. But these are cases where the relative importance of the different items is not the same. In these cases, weights are assigned to different items according to their importance. The term ‘weight’ stands for the relative importance of the different items.

If x_1, x_2,…., x_n are the n values of the variable x with the corresponding weights w_1, w_2,………., w_n, then the weighted mean is given by

$$x_w = \frac{w_1x_1 + w_2x_2 + ... + w_nx_n}{w_1 + w_2 + ...w_n} = \frac{\Sigma wx}{\Sigma w}$$

Where x_w = Weighted Arithmetic Mean and $\sum$w= Sum of the weights.

Steps:
(1) Multiply the weights w by the variable x and obtain the total $\sum x_w$.
(2) Divide the total $\sum x_w$ by the sum of the weights $\sum$w.

Example 7:
Find the weighted mean for the following data.

x	28	25	20	32	40
W	3	6	4	5	8

Solution:

x	W	*x*w
28	3	84
25	6	150
20	4	80
32	5	160
40	8	320
Total	**26**	**794**

$$\therefore x_w = \frac{\Sigma wx}{\Sigma w} = \frac{794}{26} = 30.54 \text{ units}$$

Example 8:

A candidate obtained the following marks in percentages in an examination. English 64 , Mathematics 93, Economics 72, Accountancy 85 and Statistics 79. The weights of these subjects are 2, 3, 3, 4, 1 respectively. Find the candidate's weighted mean.

Solution:

Subject	Percentage of Marks (*x*)	Weights (w)	w*x*
English	64	2	128
Mathematics	93	3	279
Economics	72	3	216
Accountancy	85	4	340
Statistics	79	1	79
Total		**13**	**1042**

$\sum w = 13$, $\sum wx = 1042$

$$x_w = \frac{\Sigma wx}{\Sigma w} = \frac{1042}{13} = 80.15 \quad \therefore \text{ Weighted Mean Marks} = 80.15$$

6.6 COMBINED MEAN

If there are two groups containing n_1 and n_2 observations with means $\bar{x}_1$ and $\bar{x}_2$ respectively , then the combined arithmetic mean of two groups is given by

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

The above formula can be generalized for more than two groups. If n_1 ,n_2 ,……,n_k are sizes of k groups with means $\bar{x}_1$, $\bar{x}_2$ ……., $\bar{x}_k$ respectively then the mean $\bar{x}$ of the combined group is given by

$$\bar{x} = \frac{n_1 x_1 + n_2 x_2 + ...n_k x_k}{n_1 + n_2 + ...+ n_k}$$

Example 9:

If average salaries of two groups of employees are Rs . 1500 and Rs . 2200 and there are 80 and 70 employees in the two groups. Find the mean of the combined group.

Solution:

Given :	Group I	Group II
	n_1 =80	n_2 =70
	$\bar{x}_1$= 1500	$\bar{x}_2$= 2200

$$\therefore \quad \bar{x} = \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$$

$$= \frac{80\ (1500) + 70\ (2200)}{80 + 70}$$

$$= \frac{120000 + 154000}{150}$$

$$= \frac{274000}{150}$$

$$= 1826.67$$

∴ The average monthly salary of the combined group of 150 employees is Rs . 1826 .67

Example 10:

The mean weight of a group of 50 workers is 58 kgs. The second group consists of 60 workers with average weight 62 kgs. and there are 90 workers in the third group with average weight 56 kgs. Find the average weight of the combined group.

Solution:

Given	Group I	Group II	Group III
No. of workers	n_1 =50	n_2 =60	n_3= 90
Mean weight	$\bar{x}_1$ =58	$\bar{x}_2$ =62	$\bar{x}_3$=56

$$\therefore \quad \bar{x} = \frac{n_1 x_1 + n_2 x_2 + n_3 x_3}{n_1 + n_2 + n_3}$$

$$= \frac{50\ (58) + 60\ (62) + 90\ (56)}{50 + 60 + 90}$$

$$= \frac{11660}{200} = 58.3$$

∴ The average weight of the combined group of 200 workers is 58.3 kgs

Merits of Arithmetic Mean .

(1) It is rigidly defined .
(2) It is easy to understand and easy to calculate .
(3) It is based on each and every observation of the series .
(4) It is capable for further mathematical treatment .
(5) It is least affected by sampling fluctuations.

Demerits of Arithmetic Mean .

(1) It is very much affected by extreme observations.
(2) It can not be used in case of open end classes.
(3) It can not be determined by inspection nor it can be located graphically.
(4) It can not be obtained if a single observation is missing .
(5) It is a value which may not be present in the data .

Exercise 6.1

1. Find the arithmetic mean for the following sets of observations.

(i) 48, 55,83,65,38,74,58
(ii) 154,138,165,172,160,145,157,185
(iii) 2254,2357,2241,2012,2125

Ans: (i) $\bar{x}$= 60.14 (ii) $\bar{x}$=159.5 (iii) $\bar{x}$=2197.8

2. Calculate the mean for the following distribution:

x:	15	17	19	21	23	25
f:	6	11	8	15	5	4

Ans: $\bar{x}$=19.57

3. Find the mean for the following data.

Size of Shoe:	5	6	7	8	9	10
No. of pairs	22	35	28	42	15	12

Ans: $\bar{x}$=7.19

4. Calculate the arithmetic mean for the following data

Daily Wages in Rs:	40-80	80-20	120-60	160-00	200-40	240-80	280-20
No. of workers	8	15	13	19	12	14	10

Ans:$\bar{x}$=181.32

5. Calculate the mean for the following data representing monthly salary of a group of employees.

Salary in Rs:	500-1000	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500
No. of persons:	7	11	13	8	5	4

Ans: $\bar{x}$=1802.08

6. Find the mean for the following data.

Weekly wages in Rs:	50-99	100-149	150-199	200-249	250-299	300-349
No. of workers	12	17	15	20	8	7

Ans: $\bar{x}$=184.63

7. Find the arithmetic mean for the following data

Age in yrs:	10-19	20-29	30-39	40-49	50-59	60-69	70-79
No. of persons:	6	12	19	23	15	8	6

Ans:$\bar{x}$=43.15

8. Find the weighted mean for the following data.

x:	25	22	15	30	18
w:	6	4	3	8	2

Ans: $\bar{x}$w= 24.30

9. A Student Scores 44 in Test I, 32 in Test II, 27 in Test III and 38 in Test IV, These are to be weighted with weights 4,3,3 and 4 respectively. Find the average score.
Ans: xw=36.07

10. The average daily wages for 120 workers in a factory are Rs. 78. The average wage for 80 male workers out of them is Rs.92 Find the average wage for the remaining female workers.
Ans: Rs.50

11. There are three groups in a class of 200 Students. The first groups contains 80 Students with average marks 65, the second group consists of 70 Students with average marks 74. Find the average marks of the Students from the third group if the average for the entire class is 68.
Ans: 64.4

12. Find the combined arithmetic mean for the following data:

	Group I	Group II	Group III
Mean	125	141	131
No.of observations.	150	100	50

Ans: 131.67

6.7 MEDIAN

The median by definition refers to the middle value in a distribution. Median is the value of the variable which divides the distribution into two equal parts. The 50% observations lie below the value of the median and 50% observations lie above it. Median is called a positional average. Median is denoted by M.

(i) For Simple data or ungrouped data:

Median is defined as the value of the middle item of a series when the observations have been arranged in ascending or descending order of magnitude.

Steps: 1) Arrange the data in ascending or descending order of magnitude. (Both arrangements would give the same answer).

2) When n is odd:

Median= value of $\left(\frac{n+1}{2}\right)^{th}$ term.

3) When n is even:

Median= Arithmetic Mean of value of $\left(\frac{n}{2}\right)$ & , $\left(\frac{n}{2}+1\right)^{th}$ terms.

i.e, adding the two middle values and divided by two.
where n = number of observations.

Example 11

Find the median for the following set of observations
65, 38, 79, 85, 54, 47, 72

Solution: Arrange the values in ascending order:
38, 47, 54, 65, 72, 79, 85
n= 7 (odd number)
The middle observation is 65
∴ Median= 65 units.

using formula:

Median = value of $\left(\frac{n+1}{2}\right)^{th}$ term.

= value of $\left(\frac{7+1}{2}\right)$ th term.

= 4^{th} term

Median = 65 units.

Example 12: Find the median for the following data.
25, 98, 67, 18, 45, 83, 76, 35

Solution: Arrange the values in ascending order

18, 25, 35, 45, 67, 76, 83, 98

n = 8 (even number)

The pair 45, 67 can be considered as the middle pair.

∴ Median = A.M of the pair = $\frac{45+67}{2}$

Median = 56

Using formula :

Median = A. M. of values of $\left(\frac{n}{2}\right)^{th}, \left(\frac{n}{2}+1\right)^{th}$ terms

= A. M. of $\left(\frac{8}{2}\right), \left(\frac{8}{2}+1\right)^{th}$ terms

= A. M. of $(4 , 4+1)^{th}$ terms

= A. M. of $(4 , 5)^{th}$ terms

= A. M. of 45 and 67

= $\frac{45 + 67}{2}$

Median = 56

(ii) For ungrouped frequency distribution:

Steps:

1) Arrange the data in ascending or descending order of magnitude with respective frequencies .
2) Find the cumulative frequency (c. f) less than type .
3) Find N/2 , N = total frequency.
4) See the c. f column either equal or greater than N/2 and determine the value of the variable corresponding to it . That gives the value of Median .

Example 13: From the following data find the value of Median.

Income in Rs:	1500	2500	3000	3500	4500
No. of persons	12	8	15	6	5

Solution:

Income (in Rs.) (x)	**No. of persons (f)**	**c.f less than type**
1500	12	12
2500	8	20
3000	15	35
3500	6	41
4500	5	46
	N= 46	

$\frac{N}{2} = \frac{46}{2} = 23$

In c.f column, we get 35 as the first cumulative frequency greater than 23.

The value of x_i corresponding to the c.f. 35 is 3000.

∴ Median = Rs. 3000

Example : 14

Find the median for the following data .

x_i :	25	20	30	40	35	15
f :	19	14	23	12	20	8

Solution : Arrange the values in ascending order .

x_i	f	c.f less than type
15	8	8
20	14	22
25	19	41
30	23	64
35	20	84
40	12	96
	N= 96	

$$\frac{N}{2} = \frac{96}{2} = 48$$

In c.f column, greater than 48 is 64.

∴ The corresponding value of x to the c.f is 30.
Median = 30 units.

(iii) For grouped data.

Steps:

1) Find the c.f less than type
2) Find N/2, N= total frequency.
3) See the c.f column just greater than N/2.
4) The Corresponding class interval is called the Median class.
5) To find Median, use the following formula.

$$M = l_1 + \left[\frac{N/2 - cf}{f}\right](l_2 - l_1)$$

Where

l_1 = lower class-bounding of the median class.
l_2= upper class-bounding of the median class.
f= frequency of the median class.
c.f= cumulative frequency of the class interval preceding the median class.

Example: 15

The following data relate to the number of patients visiting a government hospital daily:

No. of patients :	1000-1200	1200-1400	1400-1600	1600-1800	1800-2000	2000-2200
No. of days :	15	21	24	18	12	10

Solution :

No. of patients	**No. of days**	**c.f. less than type**
1000-1200	15	15
1200-1400	21	36
1400-1600	24	60
1600-1800	18	78
1800-2000	12	90
2000-2200	10	100
	N=100	

N/2 =100/2=50

See the c.f column greater than 50 is 60. The corresponding class interval 1400-1600 is the median class.

$\therefore$ l_1 =1400; l_2=1600; f=24; c.f= 36

$$\therefore M = l_1 + \left[\frac{N/2 - cf}{f}\right](l_2 - l_1) = 1400 + [\frac{50-36}{24}](1600-1400)$$

= 1400+[14] (200) = 1400+116.67

***M* = 1516.67 patients**

Example 16: Find the median for the following distribution:

Weights (in kgs)	30-34	35-39	40-44	45-49	50-54	55-59	60-64
No. of students	4	7	15	21	18	10	5

Solution : Convert the given class intervals into exclusive type

Weights	**No. of students**	**c.f. less than type**
29.5-34.5	4	4
34.5-39.5	7	11
39.5- 44.5	15	26
44.5-49.5	21	47
49.5-54.5	18	65
54.5-59.5	10	75
59.5-64.5	5	80
	N=80	

N/2 =80/2 = 40

Median class is 44.5-49.5

l_1= 44.5 ; l_2 = 49.5 , f =21, c.f. =26

$$M = l_1 + \left[\frac{N/2 - cf}{f} \right] (l_2 - l_1)$$

$$= 44.5 + \left[\frac{40-26}{21} \right] (49.5-44.5)$$

$$= 44.5 + [14/21]\ (5)$$

$$= 44.5 + 3.33$$

M = 47.83 units.

Merits of Median :

1. It is rigidly defined .
2. It is easy to understand and easy to calculate .
3. It is not affected by extreme observations as it is a positional average.
4. It can be calculated , even if the extreme values are not known .
5. It can be located by mere inspection and can also be located graphically.
6. It is the only average to be used while dealing with qualitative characteristics which can not be measured numerically .

Demerits of Median :

1. It is not a good representative in many cases.
2. It is not based on all observations.
3. It is not capable of further mathematical treatment .
4. It is affected by sampling fluctuations.
5. For continuous data case , the formula is obtained on the assumption of uniform distribution of frequencies over the class intervals. This assumption may not be true.

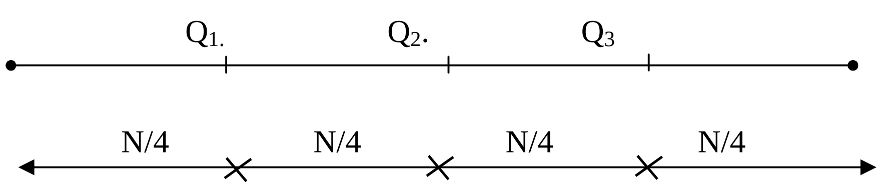
6.8 QUARTILES

When a distribution is divided into four equal parts, each point of division is called as quartile and each part is of 25% (one-fourth) of the total observations. There are three partition values such as Q_1, Q_2, and Q_3.

Q_1 is called first quartile or lower quartile.

Q_3 is called third quartile or upper quartile.

Q_2 is called second quartile which coincides with median. Therefore Q_2 is nothing but Median.



The Steps involved for computing the quartiles is basically the same as that of computing median.

To find Q_1: (First Quartile)

Steps: 1) Find the c.f less than type.

2) Find n/4, n=total frequency.

3) See the c.f. column just greater than n/4.
4) The corresponding class interval is called the quartile class.
5) To find Q_1, use the following formula:

$$Q_1 = l_1 + \left[\frac{N/4 - cf}{f}\right](l_2 - l_1)$$

Where l_1= lower limit of the quartile class
l_2= upper limit of the quartile class.
f= frequency of the quartile class.
c.f= cumulative frequency of the class interval preceding the quartile class.

To find Q_3: (Third Quartile)
Steps: 1) Find the c.f less than type.
2) Find 3N/4, N=total frequency.
3) See the c.f column just greater than 3N/4
4) The corresponding class interval is called the quartile class.
5) To find Q_3, use the following formula.

$$Q_3 = l_1 + \left[\frac{3N/4 - cf}{f}\right](l_2 - l_1)$$

Where l_1 = lower limit of the quartile class.
l_2 =upper limit of the quartile class.
f = frequency of the quartile class.
c.f.= cumulative frequency of the class interval preceding the quartile class.

To find Q_2 (Second Quartile)
Q_2 = Median (Discussed above)

Example 17: Find the three quartiles for the following data.

Commission (in Rs):	100-120	120-140	140-160	160-180	180-200	200-220	220-240
No. of Salesmen:	13	38	45	56	27	17	12

Solution:

Commission (in Rs.)	No. of Salesmen	c.f.less than type
100-120	13	13
120-140	38	51
140-160	45	96
160-180	56	152
180-200	27	179
200-220	17	196
220-240	12	208
	n= 208	

To find Q_1:

$\frac{N}{4} = \frac{208}{4} = 52$

See the c.f. column just greater than 52.

Quartile class: 140-160

$l_1 = 140$, $l_2 = 160$, $f = 45$, $c.f = 51$

$$Q_1 = l_1 + \left[\frac{n/4 - c.f}{f}\right](l_2 - l_1)$$

$$= 140 + \left[\frac{52-51}{45}\right](160-140)$$

$$= 140 + [1/45](20)$$

$$= 140 + 0.44$$

$Q_1 = 140.44$

To find Q_2 = Median :

$N/2 = 208/2 = 104$

See the c.f column just greater than 104.

Median class =160-180

$l_1 = 160$; $l_2 = 180$; $f = 56$; $c.f = 96$

$$M = l_1 + \left[\frac{n/2 - c.f}{f}\right](l_2 - l_1)$$

$$= 160 + \left[\frac{104-96}{56}\right](180-160)$$

$$= 160 + [8/56]\,(20)$$

$$= 160 + 2.86$$

$Q_2 = 162.86$

To find Q_3:

$3N/4 = 3(208/4) = 156$

See the c.f column just greater than 156.

Quartile class=180-200

$l_1 = 180$; $l_2 = 200$; $f = 27$; $c.f = 152$

$$Q_3 = l_1 + \left[\frac{3N/4 - c.f}{f}\right](l_2 - l_1)$$

$$= 180 + \left[\frac{156-152}{27}\right](200-180)$$

$$= 180 + [4/27]\,(20)$$

$$= 180 + 2.96$$

$Q_3 = 182.96$

$\therefore$ Q_1 = 140.44
Q_2 =M= 162.86
Q_3 = 182.96

Graphical location of Quartiles :
The median and the quartiles obtained graphically from the c. f. curve less than type curve as follows:
First draw a cumulative frequency curve less than type.

To locate Q_2 = Median :
Locate n/2 on the Y – axis and from it draw a perpendicular on the c. f. curve. From the point where it meets the c. f. curve draw another perpendicular on the X- axis and the point where it meets the X- axis is called the median .

To locate Q_1 and Q_3 :
Locate n/4 and 3N/4 on the Y- axis and proceed as above to obtain Q_1 and Q_3 respectively on the X- axis.

Note that the values obtained from the graph are approximate figures. They do not represent exact figures.

Example 18:
Draw a 'less than " cumulative frequency curve for the following data and hence locate the three quartiles graphically.

Age in yrs.	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons	15	13	25	22	25	10	5	5

Solution :

Age (in yrs.)	No. of persons.	c.f. less than type
0-10	15	15
10-20	13	28
20-30	25	53
30-40	22	75
40-50	25	100
50-60	10	110
60-70	5	115
70-80	5	120
	N=120	

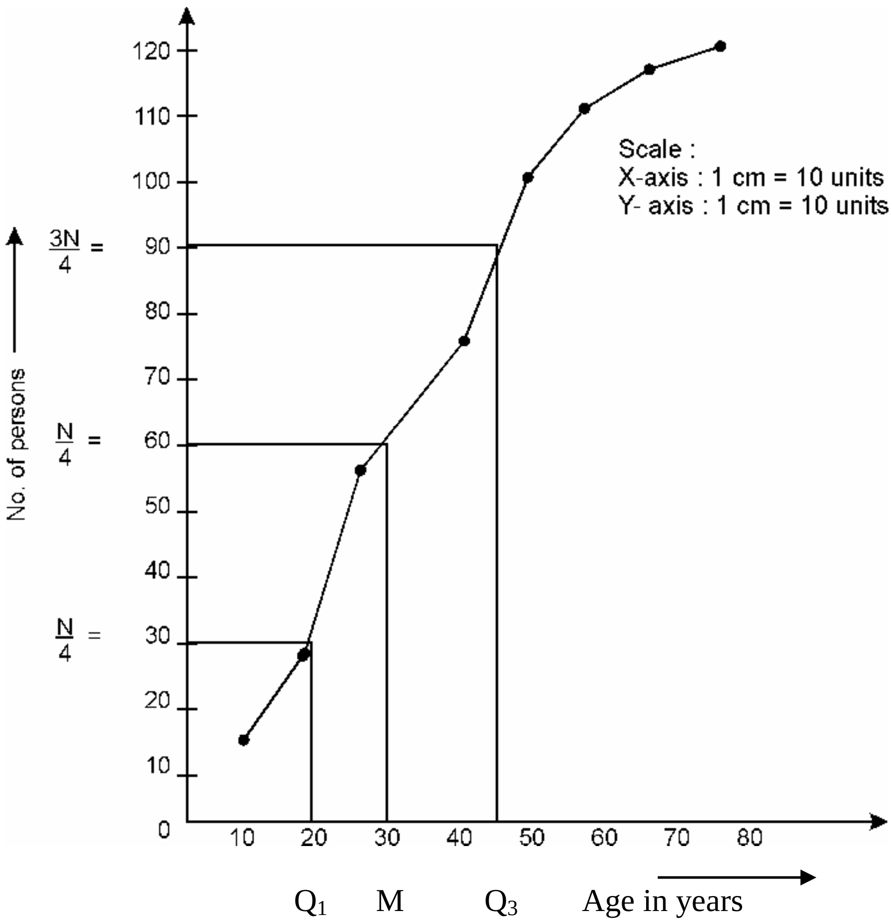
X -axis (upper limit)	Y- axis c.f. $<$ type
10	15
20	28
30	53

40	75
50	100
60	110
70	115
80	120

n/4= 120/4 = 30

n/2 = 120/2 =60

3n/4 = 3 (120/4)
3n/4 =90



From the graph : Q1 = 21 approximately
M = 33 approximately
Q3 = 46 approximately

Exercise 6.2

1. Find the median for the following data
   (i)50 , 28 , 35 , 98 , 75 , 44 , 58
   (ii) 16 , 22 , 10 , 12 , 30 , 37 , 28 , 40 , 15 , 20
   Ans: (i) M=50 (ii) M= 21

2 . Calculate the median for the following distribution :

x:	25	30	35	40	45	50	55
f:	8	14	23	28	10	6	4

Ans: M= 40

3. Find the median for the following data.

Weekly Wages (in Rs)	50-100	100-150	150-200	200-250	250-300	300-350
No. of Workers:	20	18	32	18	12	15

Ans: M= 180.47

4. Find the median height for the following distribution:

Height in cms:	150-154	154-158	158-162	162-166	166-170	170-174
No. of Students:	6	15	23	20	12	10

Ans M=161.83

5. Find the three quartiles for the following distribution:

Income in Rs:	0-500	500-1000	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500
No.of families	3	5	10	18	15	9	6

Ans:Q_1=1425, M=1916.67, Q_3= 2450

6. Find the median and the two quartiles for the following data. Also locate them graphically.

Rainfall (in cms):	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No.of years:	3	7	10	16	12	8	4

Ans:Q_1= 27.5, M=33.13, Q_3=38.75

6. The following is the data representing profits in thousands of Rs. of some companies. Find the quartiles and hence locate them graphically.

Profit:	70-80	80-90	90-100	100-110	110-120	120-130
No.of Companies	11	15	19	23	22	10

Ans:Q_1=89.33, M=102.17,Q_3=113.18

6.9 MODE

(i) For Raw data:
Mode is the value which occurs most frequently ,in a set of observations. It is a value which is repeated maximum number of times and is denoted by Z.

Example 19: Find mode for the following data.
64, 38,35,68,35,94,42,35,52,35
Solution:
As the number 35 is repeated maximum number of times that is 4 times.
∴ Mode=35 units.
(ii) For ungrouped frequency distribution:
Mode is the value of the variable corresponding to the highest frequency.

Example 20: Calculate the mode for the following data.

Size of Shoe:	5	6	7	8	9	10
No.of Pairs:	38	43	48	56	25	22

Solution: Here the highest frequency is 56 against the size 8.

∴ Modal size = 7

(iii) For Grouped data:
In a Continuous distribution first the modal class is determined. The class interval corresponding to the highest frequency is called modal class.

Formula:

$$Z = l_1 + \left[\frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)}\right](l_2 - l_1)$$

Where l_1,= lower boundary of the modal class
l_2= upper boundary of the modal class
f_1= frequency of the modal class
f_0= frequency of the class interval immediately preceding the modal class.
f_2= frequency of the class interval immediately succeeding the modal class.

Example 21
Find the mode for the following data.

Daily wages(in Rs):	20-40	40-60	60-80	80-100	100-120	120-140	140-160
No.of employees:	21	28	35	40	24	18	10

Solution:

Daily wages (in Rs.)	No. of employees
20- 40	21
40-60	28
60-80	35 f_o
80- 100	40 f_1
100-120	24 f_2
120-140	18
140- 160	10

The maximum frequency is 40.

∴ Modal class is 80-100.

∴ l_1= 80; l_2=100; f_o=35; f_1=40, f_2 =24

$$Z = l_1 + \left[\frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)}\right](l_2 - l_1)$$

$$= 80 + \left[\frac{40-35}{(40-35)+(40-24)}\right](100-80)$$

$$= 80 + \left[\frac{5}{5+16}\right] \text{x } 20 = 80 + 4.76$$

Z= 84.76

Modal daily wages = Rs. 84.76

Example: 21

Find the mode for the following data.

Marks:	10-19	20-29	30-39	40-49	50-59	60-69	70-79
No of Students:	8	22	31	44	15	13	10

Solution:

Marks	Class boundaries	No. of Students
10-19	9.5-19.5	8
20-29	19.5-29.5	22
30-39	29.5-39.5	31 f_o
40-49	39.5-49.5	44 f_1
50-59	49.5-59.5	15 f_2
60-69	59.5-69.5	13
70-79	69.5-79.5	10

The maximum frequency is 44.

∴ The modal class is 39.5-49.5

l_1 =39.5 f_0=31

l_2=49.5 f_1=44, f_2=15

$$Z = l_1 + \left[\frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)}\right](l_2 - l_1)$$

$$= 39.5 + \left[\frac{44-31}{(44-31)+(44-15)}\right](49.5-39.5) = 39.5 + \left[\frac{13}{13+29}\right] \text{x } 10$$

= 39.5 + 3.095

= 42.595

Z = 42.60

Modal Marks = 42.60

6.10 GRAPHICAL LOCATION OF MODE

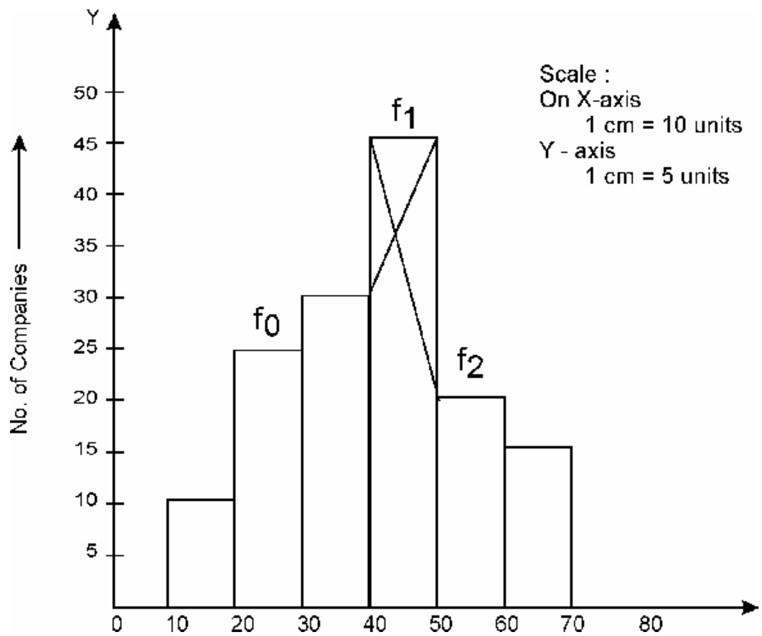
Mode can be obtained from a histogram as follows . The method can be applied for class intervals of equal length having a unique maximum frequency .

In the histogram , the rectangle with the maximum height represents the modal class . The right upper corner of this rectangle is connected with right upper corner of the preceding rectangle by a straight line . Similarly , the left upper corner of the maximum height of the rectangle is connected with left upper corner of the succeeding rectangle by a straight line . These two straight lines are intersecting at a point . From the point of intersection, a perpendicular is drawn on *x*- axis, the foot of which gives the value of Mode.

Example 23:

Draw a histogram for the following data and hence locate mode graphically.

Profit (in lakhs):	10-20	20-30	30-40	40-50	50-60	60-70
No. of : Companies	12	25	31	45	20	15



Mode Profit (in lakhs)

Modal profits = Rs. 44 lakhs.

Merits and Demerits of Mode .

Merits :

1. It is easy to understand and easy to calculate.
2. It is unaffected by the presence of extreme values.
3. It can be obtained graphically from a histogram.
4. It can be calculated from frequency distribution with open-end classes.
5. It is not necessary to know all the items. Only the point of maximum concentration is required.

Demerits :

(1) It is not rigidly defined .
(2) It is not based on all observations.
(3) It is affected by sampling fluctuations.
(4) It is not suitable for further mathematical treatment .

Exercise 6.3

(1) Find the mode for the following data .
(i) 85, 40 , 55 , 35 , 42 , 67 , 75 , 63 , 35 , 10 , 35
(ii) 250 , 300 , 450 , 300 , 290 , 410 . 350 , 300

Ans: (i) Z = 35 (ii) Z= 300

(2) Find the mode for the following data.

x:	15	18	20	22	24
f:	8	6	13	18	10

Ans: Z= 22

(3) Calculate the modal wages for the following distribution.

Wages in Rs:	50-65	65-80	80-95	95-110	110-125	125-140
No. of workers	4	7	15	28	11	5

Ans:Z= 101.5

(4) Find mode for the following data.

Life in hrs:	200-400	400-600	600-800	800-1000	1000-1200	1200-1400	1400-1600
No. of bulbs:	15	22	40	65	38	26	10

Ans: z=896.15

(5) Find the mode for the following data and hence locate mode graphically.

Salary in Rs:	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000
No. of persons:	23	35	50	30	18	12

Ans: z=2214.29

(6) Find the mode for the following data, also locate mode graphically.

Class Interval :	100-150	150-200	200-250	250-300	300-350	350-400	400-450
Frequency:	15	29	31	42	56	35	17

Ans:= z= 320

(7) Find Mean Median and mode for the following data.

Income (in Rs.):	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No.of employees:	15	26	45	32	12	10

Ans: *x*= 3714.29; M=3644.44; Z= 3593.75

(8) Prove that median lies between mean and mode from the following data:

Weekly (wages in Rs.)	100-200	200-300	300-400	400-500	500-600	600-700	700-800
No. of workers.	18	23	32	41	35	20	15

Ans: *x*= 443.48; M=446.34; Z=460

(9) Calculate mean median and mode for the following data.

Height (in.cms)	125-130	130-135	135-140	140-145	145-150	150-155
No. of children	6	13	19	26	15	7

Ans: *x*=140.52; M=140.96; Z=141.94

(10) Find arithmetic mean median and mode for the following data.

Age in yrs:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons:	12	18	23	28	20	16	7

Ans: *x*=43.23; M=43.21; Z= 43.85

❖❖❖

UNIT IV

7

MEASURES OF DISPERSION

OBJECTIVES

After reading this chapter you will be able to:

- Compute different types of deviations like; Range, Quartile Deviation, Standard Deviation and Mean Deviation.
- Compute relative measures of deviation like; coefficient of Range/Quartile Deviation/Mean Deviation/Variation.
- Compute combined standard deviation.

7.1 INTRODUCTION

Why Do We Need to Study the Measure of Dispersion?

The common averages or measures of central tendency indicate the general magnitude of the data and give us a single value which represents the data, but they do not tell us the degree of *spread out* or the extent of variability in individual items in a distribution. This can be known by certain other measures called *measures of dispersion*.

We will discuss the most commonly used statistical measures showing the degree and the characters of variations in data.

Dispersion in particular helps in finding out the variability of the data i.e., the extent of dispersal or scatter of individual items in a given distribution. Such dispersal may be known with reference to the central values or the common averages such as mean, median and mode or a standard value; or with reference to other values in the distribution. The need for such a measure arises because mean or even median and mode may be the same in two or more distribution but the composition of individual items in the series may vary widely. We give an example to illustrate this.

Region	**Rainfall (cms)**				
East	106	106	106	106	106
West	90	98	106	114	122
North	120	130	84	126	70

In the above example note that the average rainfall in all the three regions is the same i.e. 106 cms however it can be easily seen that it would be wrong to conclude that all the three regions have the same climatic pattern. This is so because in East all the values are equal to the average; whereas in the western region they are centered around their mean, and in north they are widely scattered. It may thus be misleading to describe a situation simply with the aid of an average value.

In measuring dispersion, it is necessary to know the ***amount*** of variation and the ***degree*** of variation. The former is designated as absolute measures of dispersion and expressed in the denomination of original variates such as inches, cms, tons, kilograms etc while the latter is designated as related measures of dispersion. We use the absolute measures of dispersion when we compare two sets of data with the same units and the same average type. If the two sets of data do not have the same units then we cannot use the absolute measures and we use the relative measures of dispersion.
Absolute measures can be divided into positional measures based on some items of the series such as (I) Range, (ii)Quartile deviation or semi– inter quartile range and those which are based on all items in series such as (I) Mean deviation, (ii)Standard deviation. The relative measure in each of the above cases are called the coefficients of the respective measures. For purposes of comparison between two or more series with varying size or number of items, varying central values or units of calculation, only relatives measures can be used.

7.2 RANGE

Range is the simplest measure of dispersion.
When the data are arranged in an array the difference between the largest and the smallest values in the group is called the ***Range***.

Symbolically: Absolute Range = L - S, [where L is the largest value and S is the smallest value]

$$\text{Relative Range} = \frac{\textit{Absolute Range}}{\textit{sum of the two extremes}}$$

Amongst all the methods of studying dispersion range is the simplest to calculate and to understand but it is not used generally because of the following reasons:

1) Since it is based on the smallest and the largest values of the distribution, it is unduly influenced by two unusual values at either end. On this account, range is usually not used to describe a sample having one or a few unusual values at one or the other end.
2) It is not affected by the values of various items comprised in the distribution. Thus, it cannot give any information about the general characters of the distribution within the two extreme observations.

For example, let us consider the following three series:
Series: A
6 46 46 46 46 46 46 46
Series: B
6 6 6 6 46 46 46 46

Series: C
6 10 15 25 30 32 40 46

It can be noted that in all three series the range is the same, i.e. 40, however the distributions are not alike: the averages in each case is also quite different. It is because range is not sensitive to the values of individual items included in the distribution. It thus cannot be depended upon to give any guidance for determining the dispersion of the values within a distribution.

7.3 QUARTILE DEVIATION

The dependence of the range on extreme items can be avoided by adopting this measure. Quartiles together with the median are the points that divide the whole series of observations into approximately four equal parts so that quartile measures give a rough idea of the distribution on either side of the average.

Since under most circumstances, the central half of the distribution tends to be fairly typical, quartile range $Q_3 - Q_1$ affords a convenient measure of absolute variation. The lowest quarter of the data (upto Q_1) and the highest quarter (beyond Q3) are ignored. The remainder, the middle half of the data above Q_1 and below Q_3 or $(Q_3 - Q_1)$ are considered. This, when divided by 2, gives the semi-interquartile range or quartile deviation.

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2}$$

In a symmetrical distribution, when Q_3 (75%) plus Q_1 (25%) is halved, the value reached would give Median, i.e., the mid-point of 75% and 25%. Thus, with semi-interquartile range 50% of data is distributed around the median. It gives the expected range between which 50% of all data should lie. The Quartile Deviation is an absolute measure of dispersion . The corresponding relative measure of dispersion is

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

For the following Frequency Distribution we show how to calculate the Quartile Deviation and the coefficient of Quartile Deviation.

x	***f***	**c.*f*.**
10	6	6
15	17	23
20	29	52
25	38	90
30	25	115
35	14	129
40	9	138
90	1	139

It may be noted that:

Corresponding to Q_1, the c.f. = $\frac{n+1}{4}$

Corresponding to Q_2, the c.f. = $\frac{2(n+1)}{4}$

Corresponding to Q_3, the c.f. = $\frac{3(n+1)}{4}$

Thus from the given data :

Q_3 = Value of the variates corresponding to c.f. $\frac{3 \ x \ 140}{4}$ = 105 which corresponds to 30

Q_2 = Value of the variates corresponding to c.f. $\frac{2 \ x \ 140}{4}$ = 70 which corresponds to 25

Q_1 = Value of the variates corresponding to c.f. $\frac{140}{4}$ = 35, which corresponds to 20

Quartile deviation or Q.D. = $\frac{Q_3 - Q_1}{2} = \frac{30-20}{2} = 5$

And Co-efficient of Quartile Deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{30-20}{30+20} = \frac{10}{50} = 0.2$$

Example

Find the Quartile Deviation of the daily wages (in Rs.) of 11 workers given as follows: 125, 75, 80, 50, 60, 40, 50, 100, 85, 90, 45.

Solution: Arranging the data in ascending order we have the wages of the 11 workers as follows:

40, 45, 50, 50, 60, 75, 80, 85, 90, 100, 125

Since the number of observations is odd (11), the 1^{st} Quartile is given by:

$Q_1 = (11 + 1)/4 = 3^{rd}$ observation = 50. … (1)

$Q_3 = 3(11 + 1)/4 = 9^{th}$ observation = 90. … (2)

Q.D. = $\frac{Q_3 - Q_1}{2}$ = (90 – 50)/2 = 20 … from (1) and (2)

Example

The following data gives the weight of 60 students in a class. Find the range of the weights of central 50% students.

Weight in kg	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
No. of students	4	16	12	8	10	5

Solution:

To find range of the weight's of central 50 % students means to find the inter quartile range. For that we require Q_1 and Q_3. The column of less than *c f* is introduced as follows:

Weight in kg	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
No. of students	4	**16**	12	8	**10**	6
cf	**4**	20	32	**40**	50	56

Q_1 class (35 – 40) Q_3 class (50 – 55)

To find Q_1: $N = 56$. Thus $m = N/4 = 14$.

The *cf* just greater than 14 is 20, so 35 – 40 is the 1st quartile class and $l_1 = 35$, $l_2 = 40$, $i = 40 - 35 = 5$, $f = 16$ and $pcf = 4$.

$$\therefore Q_1 = l_1 + \left[\frac{m - pcf}{f} \text{ x } i\right] = 35 + \left[\frac{14-4}{16} \text{ x } 5\right] = 35 + 3.125 = 38.125$$

To find Q_3: $N = 56$. Thus $m = 3N/4 = 42$.

The *cf* just greater than 42 is 50, so 50 – 55 is the 3rd quartile class and $l_1 = 50$, $l_2 = 55$, $i = 55 - 50 = 5$, $f = 10$ and $pcf = 40$.

$$\therefore Q_1 = l_1 + \left[\frac{m - pcf}{f} \text{ x } i\right] = 50 + \left[\frac{42-40}{10} \text{ x } 5\right] = 50 + 1 = 51$$

$\therefore$ inter quartile range = $Q_3 - Q_1 = 51 - 38.125 = 12.875$ kg

Thus, the range of weight for the central 50% students = 12.875kg

Example

Find the semi – inter quartile range and its coefficient for the following data:

Size of shoe	0	1	2	3	4	5	6	7	8	9	10
No. of boys	7	10	15	11	18	10	16	5	12	6	2

Solution: The less than *c f* are computed and the table is completed as follows:

Size of shoe	0	1	**2**	3	4	5	**6**	7	8	9	10
No. of boys	7	10	15	11	18	10	16	5	12	6	2
c f	7	17	32	43	61	71	87	92	104	110	112

Here $N = 112$.

To find Q_1: $m = N/4 = 28$.

The first *c f* just greater than 28 is 32, so the 1st quartile is $Q_1 = 2$

To find Q_3: $m = 3N/4 = 84$.

The first *c f* just greater than 84 is 87, so the 3rd quartile is $Q_3 = 6$

$\therefore$ the semi inter quartile range *i.e.* Q.D. = $\dfrac{Q_3 - Q_1}{2} = \dfrac{6-2}{2} = 2$

Coefficient of Q.D. = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{6-2}{6+2} = 0.5$

7.4 MEAN DEVIATION

Both the range and quartile deviation do not show the scatterness around an average and as such do not give a clear idea of the dispersion of the distribution, these measures also exclude some data and consequently do not give a complete picture based on the entire data.. To study the deviations around the average we now introduce two more measures of dispersion the ***mean deviation*** and the ***standard deviation.*** The mean deviation is also called as the ***average deviation.***

The essence of average deviation lies in the concept of dispersion, which is the average amount of scatter of individual items from either the mean or the median ignoring the algebraic signs.

This measure takes into account the whole data. When it is calculated by averaging the deviations of the individual items from their arithmetic mean, taking all deviation to be positive, the measures is called mean deviation. It may be pointed that we are concerned with the distance of the individual items from their averages and not with their position, which may be either above or below the average.

Suppose that we have sample of six observations 0,2,3,4,4, and 5. The mean of these observations is: $\dfrac{0+2+3+4+4+5}{6} = 3$

Now we obtain the deviation of each observation from the mean of 3. For the first observation, for example, this gives a deviation of 0 – 3 = - 3. Similarly, the other observations are : -1, 0, + 1,+ 1 and + 2. It may seem that a good way to measure dispersion would be to take the mean of six deviations. But this gives 0, since, -3-1+0+1+1+2. For practice in using symbols, we give symbolic definition of the mean deviations. To avoid the cancelling off of the positive deviations with the negative deviations we take only the magnitude (absolute value) of each of these deviations .

So in the above case the M.D. = $\dfrac{\sum |x - x_i|}{n} = \dfrac{3+1+0+1+1+2}{6} = \dfrac{8}{6}$

For a frequency distribution the calculation of M.D. is done as follows

M.D.= $\dfrac{\sum f|d|}{\sum f}$ where f is the frequency of the observation and d= $|x_i - \bar{x}|$

where $\bar{x}$ is the mean.

The coefficient of M.D. = $\frac{M.D.}{Mean}$

We now take an example to calculate the Mean deviation and the coefficient of mean deviation for a frequency distribution.

Example
Find mean deviation from mean for the following distribution:

X	10	15	20	25	30	25	40	90
f	6	17	29	38	25	14	9	1

Solution
Calculation of mean deviation from mean :

x	***f***	**f . x**	**dx** = $\lvert x - 25.43\rvert$	***f* . dx**
10	6	60	15.43	92.58
15	17	255	10.43	177.31
20	29	580	5.43	157.47
25	38	950	0.43	16.34
30	25	750	4.57	114.25
35	14	490	9.57	133.98
40	9	360	14.57	131.13
90	1	90	64.57	64.57
139	3535			**∑*f* dx = 887.63**

In the above example the mean is = $\bar{x} = \frac{\sum f.x}{n} = \frac{3535}{139} = 25.43$ n = Σf

And mean Deviation from Mean = $\frac{\sum (f.\lvert dx\rvert)}{n} = \frac{887.63}{139} = 6.386$ n = Σf

And Coefficient of mean deviation is = $\frac{6.386}{25.43} = 0.25$

The mean deviation tells us that some values were above the mean and some below. On the average, the deviation of all values combined was 6.386. Co-efficient of mean deviation or 0.25.

In case of continuous frequency distribution we take the mid value of each class as the value of x and proceed to do the calculations in the same manner as above.

Example

Find the mean deviation from mean and its coefficient for the following data giving the rainfall in cm in different areas in Maharshtra: 105, 90, 102, 67, 71, 52, 80, 30, 70 and 48.

Solution: Since we have to compute M.D. from mean, we first prepare the table for finding mean and then introduce columns of absolute deviations from the mean.

$$\bar{x} = \frac{105 + 90 + 102 + 67 + 71 + 52 + 80 + 30 + 70 + 48.}{10} = \frac{715}{10} = 71.5$$

x	$d = \lvert x - \bar{x} \rvert$
105	33.5
90	18.5
102	30.5
67	4.5
71	0.5
52	19.5
80	8.5
30	41.5
70	1.5
48	23.5
$\sum x = 715$	$\Sigma d = 182$

Now,

M.D. from mean = $\frac{\Sigma d}{n}$

$$\delta\bar{x} = \frac{182}{10}$$

$$\therefore \delta\bar{x} = 18.2$$

Coefficient of M.D. from mean

$$= \frac{\delta\bar{x}}{\bar{x}} = \frac{18.2}{71.5}$$

Example

The marks obtained by 10 students in a test are given below. Find the M.D. from median and its relative measure.

Marks: 15 10 10 03 06 04 11 17 13 05

Solution: The marks of 10 students are arranged in ascending order and its median is found. The column of absolute deviations from median is introduced and its sum is computed. Using the formula mentioned above, M.D. from median and its coefficient is calculated.

x	$d = \lvert x - M \rvert$
03	5
04	4
05	3
06	2
10	2
10	2
11	3
13	5
15	7
17	9
Total	$\Sigma d = 42$

Since $N = 10$,

Median = A.M. of 5^{th} and 6^{th} observation

$$\therefore M = \frac{6+10}{2} = 8$$

M.D. from median = $\frac{\Sigma d}{n} = \frac{42}{10}$

$\therefore \delta M = 4.2$

Coefficient of M.D. = $\frac{\delta M}{M} = \frac{4.2}{8} = 0.525$

Example

On the Mumbai – Nashik highway the number of accidents per day in 6 months are given below. Find the mean deviation and coefficient of M.D.

No. of accidents	0	1	2	3	4	5	6	7	8	9	10
No. of days	26	32	41	12	22	10	05	01	06	15	10

No. of accidents	No. of days (f)	cf	$d = \|x - M\|$	$f.d$
0	26	26	2	52
1	32	58	1	32
2	41	99	0	0
3	12	111	1	12
4	22	133	2	44
5	10	143	3	30
6	05	148	4	20
7	01	149	5	05
8	06	155	6	36
9	15	170	7	105
10	10	180	8	80
Total	$N = 180$	-	-	$\Sigma fd = 416$

Solution:

Now, $N = 180 \therefore m = N/2 = 180/2 = 90$.

The cf just greater than 90 is 99. The corresponding observation is 2. $\therefore M = 2$

$$\text{M.D. from median} = \delta M = \frac{\Sigma fd}{N} = \frac{416}{180} = 2.31$$

$$\text{Coefficient of M.D.} = \frac{\delta M}{M} = \frac{2.31}{2} = 1.15$$

Example

The following data gives the wages of 200 workers in a factory with minimum wages Rs. 60 and maximum wages as Rs. 200. Find the mean deviation and compute its relative measure.

Wages less than	80	100	120	140	160	180	200
No of workers	30	45	77	98	128	172	200

Solution: The data is given with less than cf, we first convert them to frequencies then find the median and follow the steps to compute M.D. as mentioned above.

Wages in Rs	No. of workers (f)	cf	x	$d = \|x - 141.33\|$	fd
60 – 80	30	30	70	71.33	2139.9
80 – 100	15	45	90	51.33	769.95
100 – 120	32	77	110	31.33	1002.56
120 – 140	21	**98**	130	11.33	237.93
140 – 160	**30**	128	150	8.67	260.1
160 – 180	44	172	170	28.67	1261.48
180 – 200	28	200	190	48.67	1362.76
Total	N = 200	-	-	-	Σfd = 7034.68

Now, $N = 200 \therefore m = N/2 = 200/2 = 100$.

The median class is 140 – 160. $\therefore l_1 = 140$, $l_2 = 160$, $i = 160\text{-}140 = 20$, $f = 30$ and $pcf = 98$

$$\therefore M = l_1 + \left[\frac{m - pcf}{f} \text{ x } i\right] = 140 + \left[\frac{100-98}{30} \text{ x } 20\right] = 140 + 1.33 = 141.33$$

$$\text{M.D. from median} = \delta M = \frac{\Sigma fd}{N} = \frac{7034.68}{200} = 35.1734$$

$$\text{Coefficient of M.D.} = \frac{\delta M}{M} = \frac{35.1734}{141.33} = 0.25$$

7.5 STANDARD DEVIATION

As we have seen range is unstable, quartile deviation excludes half the data arbitrarily and mean deviation neglects algebraic signs of the deviations, a measure of dispersion that does not suffer from any of these defects and is at the same time useful in statistic work is ***standard deviation***. In 1893 Karl Pearson first introduced the concept. It is considered as one of the best measures of dispersion as it satisfies the requisites of a good measure of dispersion. The standard deviation measures the absolute dispersion or variability of a distribution. The greater the amount of variability or dispersion greater is the value of standard deviation. In common language a small value of standard deviation means greater uniformity of the data and homogeneity of the distribution. It is due to this reason that standard deviation is considered as a good indicator of the representativeness of the mean.

It is represented by σ (read as 'sigma'); σ^2 i.e., the square of the standard deviation is called ***variance***. Here, each deviation is squared.

The measure is calculated as the average of deviations from arithmetic mean. To avoid positive and negative signs, the deviations are squared. Further, squaring gives added weight to extreme measures, which is a desirable feature for some types of data. It is a square root of arithmetic mean of the squared deviations of individual items from their arithmetic mean.

The mean of squared deviation, i.e., the square of standard deviation is known as variance. Standard deviation is one of the most important measures of variation used in Statistics. Let us see how to compute the measure in different situation.

s.d = σ= $\sqrt{\frac{\Sigma(x-\overline{x}^2)}{n}}$

For a frequency distribution standard deviation is

$$\sqrt{\frac{\Sigma f\,(x-\overline{x}^2)}{\Sigma f}} = \sqrt{\frac{fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

We will now take an example of a frequency distribution and calculate the standard deviation.

Example
From the following frequency distribution, find the standard deviation using the formula for grouped data:

Class interval	**Frequency**
10 – 20	9
20 – 30	18
30 – 40	31
40 – 50	17
50 – 60	16
60 - 70	9
Total	**100**

Interval	**Mid-point X**	**Frequency *f***	***f*x**	**dx**	**dx^2**	***f* dx^2**
10 – 20	15	9	135	- 24	576	5184
20 – 30	25	18	450	-14	196	3528
30 – 40	35	31	1085	- 4	16	496
40 – 50	45	17	765	6	36	612
50 – 60	55	16	880	16	256	4096
60 - 70	65	9	585	26	676	6084
Total		100	3900			20000

Standard Deviation σ= $\sqrt{\frac{\sum fdx^2}{\sum f}} = \sqrt{\frac{20000}{100}} = \sqrt{200}$ =14.1

The same problem can also be solved with the step deviation method which is useful when the numbers are large.

Taking assumed mean, A = 35, $dx = \frac{x-35}{c}$, where c = 10

Interval	**Mid-point X**	**Frequency *f***	$dx = \frac{x-35}{c}$	**dx²**	**fdx**	***f* dx²**
10 – 20	15	9	-2	4	-18	36
20 – 30	25	18	-1	1	-18	18
30 – 40	35	31	0	0	0	0
40 – 50	45	17	1	1	17	17
50 – 60	55	16	2	4	32	64
60 - 70	65	9	3	9	27	81
Total		100		Total	40	216

S.D. = $\sigma = \sqrt{\frac{\sum f.dx^2}{\sum f} - \left(\frac{\sum f.dx}{\sum f}\right)^2} \, x \, c \quad = \sqrt{\frac{216}{100} - \left(\frac{40}{100}\right)^2} \, x \, 10$

= $\sqrt{2 \, x \, 10}$ = 14.1421

Relative measure of standard deviation or coefficient of variation

$CV = \frac{S.D}{mean}$ x 100 = $\frac{14.14}{39}$ x 100 = 36.26

Example

The marks of internal assessment obtained by FYBMS students in a college are given below. Find the mean marks and standard deviation.

22 30 36 12 15 25 18 10 33 29

Solution: We first sum all the observations and find the mean. Then the differences of the observations from the mean are computed and squared. The positive square root average of sum of square of the differences is the required standard deviation.

x	$d = x - \bar{x}$	d^2
22	-1	1
30	7	49
36	13	169
12	-11	121
15	-8	64
25	2	4
18	-5	25
10	-13	169
33	10	100
29	6	36
$\Sigma x = 230$	-	$\Sigma d^2 =$ 738

I. $\bar{x} = \frac{\Sigma x}{n} = \frac{230}{10} = 23.$

II. $\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{738}{10}}$

$= \sqrt{73.8} = 8.59$

$\therefore \sigma = 8.59$

Example

Find the standard deviation for the following data:

03 12 17 29 10 05 18 14 12 20

Solution: We find the sum of the observations and the sum of its squares. Using formula **2.2**, S.D. is computed as follows:

x	x^2
3	9
12	144
17	289
29	841
10	100
05	25
18	324
14	196
12	144
20	400
$\Sigma x = 140$	$\Sigma x^2 =$ 2472

The formula **2.2** gives the S.D. as below:

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{2472}{10} - \left(\frac{140}{10}\right)^2}$$

$$= \sqrt{247.2 - 196} = \sqrt{51.2}$$

Example

Compute the standard deviation for the following data

x	100	102	104	106	108	110	112
f	5	11	7	9	13	10	12

Solution: *Short-cut method*:

In problems were the value of x is large (consequently its square also will be very large to compute), we use the short-cut method. In this method, a fixed number x_0 (which is usually the central value among x) is subtracted from each observation. This difference is denoted as $u = x - x_0$. Now the columns of fu and fu^2 are computed and the S.D. is calculated by the formula: $\sigma = \sqrt{\frac{\Sigma f.u^2}{N} - \left(\frac{\Sigma fu}{N}\right)^2}$. One can observe that this formula is similar to that mentioned in **2.3**. This formula is called as change of Origin formula.

In this problem we assume x_0 = 106. The table of calculations is as follows:

x	$u = x - 106$	f	fu	fu^2
100	-6	5	-30	180
102	-4	11	-44	176
104	-2	7	-14	28
106	0	9	0	0
108	2	13	26	52
110	4	10	40	160
112	6	12	72	432
Total	-	$N =$ 67	$\Sigma fu =$ 50	$\Sigma f.u^2 =$ 1028

In this table, the column of fu^2 is computed by multiplying the entries of the columns fu and u.

From the table we have: $\Sigma f.u^2 = 1028$, $\Sigma fu = 50$ and $N = 67$.

$$\therefore \sigma = \sqrt{\frac{\Sigma f.u^2}{N} - \left(\frac{\Sigma fu}{N}\right)^2} = \sqrt{\frac{1028}{67} - \left(\frac{50}{67}\right)^2}$$

$$\therefore \sigma = \sqrt{15.34 - 0.56} = \sqrt{14.78}$$

$$\therefore \sigma = 3.84$$

Some important points to be noted regarding Standard deviation:

The standard deviation being an algebraic quantity, it possesses the following important characteristics :

1. It is rigidly defined
2. It is based on all the observations, i.e. the value of the standard deviation will change if any one of the observations is changed.
3. In the case of the value which lies close to the mean, the deviations are small and therefore variance and standard deviation are also small. Variance and standard deviation would thus be zero when all the values are equal.
4. If the same amount is added to or subtracted from all the values, the mean shall increase or decrease by the same amount; also deviations from the mean in the case of each value would remain unchanged and hence variance and standard deviation shall remain unchanged
5. In case a number of samples are drawn from the same population, it may be observed that standard deviation is least affected from sample to sample as compared to other three measures of dispersion.

Limitations: Standard deviation lays down the limits of variability by which the individual observation in a distribution will vary from the mean. In other words, Mean ± 1 standard deviation will indicate the range within which a given percentage of values of the total are likely to fall i.e., nearly 68.27% will lie within mean ± 1 standard deviation, 95.45% within mean ± 2 standard deviation and 99.73% within mean ± 3 standard deviation.

The point may be illustrated by taking an example of distribution of weight of 1000 school students with a mean height of 40 Kgs and standard deviation of 6 Kgs. If the groups of students is a normal one, about two thirds (68.26) of the students would fall within ± 1 standard deviation from the mean. Thus 683 students would weigh between 34 and 46 i.e., 40 ± 6 Kg. Further, when 2 standard deviations are added and subtracted from the mean, the total population covered would be 95.44% and in the case of 3 standard deviation it would cover 99.73%.

❖❖❖

UNIT V

8

ELEMENTARY PROBABILITY THEORY

OBJECTIVES

- To understand the uncertainty (chance) involved in the unpredictable events.
- To find the probability (numeric value of the uncertainty) and various rules of probability to measure the uncertainty.
- To find expected value (Mean) and variance in random experiments.
- Use of normal distribution to find proportions and percentage of the observations referred to certain continuous variables.

8.1 INTRODUCTION

In our day-to-day life conversation we here the statements like Most probably it will rain today. Or a sales manager makes the claim the sales will cross Rs.500 cores.

Both these statements show that the claims are subject to uncertainty and cannot be predicted in advance with 100% guaranty.

Probability measures the certainty in such type unpredicted events. The origin of probability lies in Gambling or the games of choices such as tossing a fair coin, throwing a cubic die or removing a card from a pack of playing cards. Today probability plays an important role in the field of Economics, Finance, and Medicine etc. for making inferences and predictions.

To understand the concept of probability and learn the methods of calculating the probabilities, we should first define understand some basic terms and concepts related to probability.

Random Experiment: Any act or trial in which we are not sure about the result is called as the random experiment.

e.g.. Tossing a fair coin. Throwing a cubic die. Removing a number in the game of Housie.

Outcome: The possible result of the random experiment is called an outcome.

e.g. When we toss a coin, there are two possible outcomes Head(H) and Tail(T).

or when we throw a cubic die the possible outcomes of the no of dots oen the uppermost face are 1,2,3,4,5,or6.
Sample Space: The collection of all the possible outcomes in the Random experiment is called the sample space. It is denoted by S. The outcomes listed in the sample space are called the sample points. The sample space may be finite, countable infinite or infinite in nature. The no of sample points in the sample space is denoted by n(S).

e.g. When we toss a pair of unbiased coins, the sample space is
S ={HH,HT, TH, TT}   n(S) =4

Or when a cubic die is thrown the sample space is
S ={1,2,3,4,5,6}   n(S) =6

Event: An event is a well-defined subset of the sample space. It is denoted by the letters like A,B,C etc. The no of sample points in the event is denoted by n(A).

e.g. In the experiment of throwing a cubic die when the sample space is S ={1,2,3,4,5,6}. We can define the events as follows

Event A: The no of dots appeared is multiple of 3.
A = {3,6}   n(A)=2
B: The no of dots appeared is a prime number.

Random Experiment	Event
• Tossing a pair of coins S ={HH,HT, TH, TT},n(S) =4	A: Both coins show Head. A = {HH} B: At lest one coin show Head. B = {HT,TH,HH} n(B)=3
• Tossing three coines at a time S ={HHH, TTT, HHT, HTH, THH,HTT,THT,TTH}, n(S) =8	A: exactly two Head turns up. A: {HHT, HTH, THH}
• Throwing a pair of cubic dice. S= {(1,1), (1,2), (1,3),(1,4), (1,5), (1,6)… (5,6), (6,6)} n(S)= 36	A: The sum of the dots on the uppermost faces is 6 or 10 A:{(1,5) (5,1), (2,4) (4,2) (3,3) (4,6) (6,4) (5,5)} n(A)= 8 B: The sum of the dots on the uppermost faces is divisible by 4. B:{(1,3),(3,1),(2,2),(2,6),(6,2),(3,5),(5,3) ,(4,4),(6,6)} n(B)=9
• Selecting a two digit number S= {10,11,12,…….99} n(S)=100}	A: The number is a perfect square. A:{ 16,25,36,49,64,81} n(A)=6 B: The number is >99. B:{ } n(B) =0

B = {2,3,5}   n(B) =3

8.2 TYPE OF EVENTS

Simple event- The event containing only one sample point is called a simple event.
e.g Tossing a pair of coins S ={HH,HT, TH, TT}
Now the event defined as
A: Both coins show Head.
Is a simple event.

Null event: It is the event containing no sample point in it is called a null or impossible event. It is the impossible happening and is denoted by 'Φ'.
e.g in the experiment of throwing a cubic die when the sample space is
S ={1,2,3,4,5,6}.We define the event
Event A: The no of dots appeared is a two digit number

A = { } i.e. A= Φ and n(A)=0.

8.3 ALGEBRA OF EVENTS

Union of two events A and B (AUB): union AUB is the event containing all the sample points in A and B together.
AUB = {Elements in either A or B or in both A and B together}

= {x such that $x\in$ A or $x\in$ B} or $\in A^{l}B$ both

e.g. When A={ 1,2,3} and B={2,3,5}
AUB= {1,2,3,5}

Intersection of two events A and B ($A\cap B$): For the events A and B defined on the sample space S associated with the random experiment E, intersection of A and B is the event containing all the sample points common to A and B both.
$A\cap B$ = {Elements in A and B both}

= {x such that $x\in$ A and $x\in$ B}

e.g. When A={ 1,2,3} and B={2,3,5}
$A\cap B$= {2,3}

Mutually exclusive events: The two events A and B are said to be mutually exclusive or disjoint if they have no common element in them. i.e. their intersection is an empty set.
Disjoint events cannot occur simultaneously.

e.g. When A={ 1,2,3} and B={4,5}

Mutually exclusive and exhaustive events: The two events A and B are said to be mutually exclusive and exhaustive if they are disjoint and their union is S.
e.g.when a cubic die is thrown the sample space is
S ={1,2,3,4,5,6}

Now if the events A and B defined on S have the sample points as follows A={1,2,3} and B={4,5,6} then $A \cap B = \Phi$ and AUB=S
Hence A and B are mutually exclusive and exhaustive.

Complement of an event A: Let A be any event defined on the sample space S, then it's complement A' is the event containing all; the sample points in S which are not in A.

A'={elements in S which are not in A}

A= {x s.t. $x \in S$ but $x \notin A$}

e.g. When S ={1,2,3,4,5,6} and A={2,3}
A'={1,4,5,6}.

Probability of an event A: (Classical definition) Suppose S is the sample space associated with the random experiment E, and A is any event defined on the sample space S, then it's probability P(A), is defined as

$$P(A) = \frac{n(A)}{n(S)}$$

In other words probability of A is the proportion of A in S.

Example: A cubic die is thrown up find the probability that, the no of dots appeared is a prime number.

Solution: When a cubic die is thrown the sample space is
S ={1,2,3,4,5,6} n(S) =6
Now we define the event A as,
A: the no of dots appeared is a prime number.
A={2,3,5} n(A) =3
Using the above formula, we get

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

From the above we can note that,
Probability of event A always lies between 0 and 1 i.e. $0 \le P(A) \le 1$.
P(Sample space)=P(S)=1 and $P(\Phi) = 0$.
Also, when events A and B are such that $A \subset B$ then, $P(A) \le P(B)$.

8.4 SOLVED EXAMPLES

Example 1:
An unbiased die is thrown, find the probability that, i) the no of dots is less than 3 ii) the no of dots in divisible by 3.
Solution: when a cubic die is thrown the sample space is
S ={1,2,3,4,5,6} n(S)=6

(i) Let A denote the event that no of dots <3. A={1,2} i.e. n(A)=2

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = 0.33$$

.

(ii) Now let B denote the event of no of dots is divisible by 3.

B: {3,6} and n(B) = 2

$\therefore$ P(B)= $\frac{n(B)}{n(S)} = \frac{2}{6} = 0.33$

Example 2:

Three unbiased coins are tossed at a time. Find the probability that, (a) exactly two Head turns up and (b) at most two Head turns up.

Solution: When three coins are tossed up at a time the sample is

S ={HHH, HHT, HTH, THH,HTT,THT, TTH, TTT}   n(S) =8.

Now to find the required probability, we define the events as follows,

Event A: Exactly two Head turns up.

A= { HHT, THH, HTH}   n(A)= 3

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{3}{8}$

Event B: At most two Head turns up.

B:{HHT, HTH, THH,HTT,THT, TTH, TTT}   n(B) =7.

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{7}{8}$.

Example 3:

A pair of fair dice is rolled. Write down the sample space and find the probability that, a) the sum of dots on the uppermost face is 6 or 10, b) the sum of dots is multiple of 4 and c) the sum of the dots is < 6.

Solution: When a pair of dice is rolled, the sample space is

S={(1,1), (1,2), (1,3),(1,4), (1,5), (1,6)

(2,1)…………………………..(2,6)

……………………….. (5,6), (6,6)}   n(S)= 36.

To find the probability we define the events,

a) event A: the sum of dots on the uppermost face is 6 or 10.

A= {(1,5) (5,1), (2,4) (4,2) (3,3) (4,6) (6,4) (5,5)}   n(A)= 8.

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{8}{36}$

b) event B: The sum of the dots on the uppermost faces is divisible by 4.

B: {(1,3),(3,1),(2,2),(2,6),(6,2),(3,5),(5,3),(4,4),(6,6)} n(B)=9.

$\therefore$ P(B) = $\frac{n(B)}{n(S)} = \frac{9}{36}$ =0.25.

c) event C: the sum of the dots is < 6.

C: { (1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1)}   n(C) = 8.

$\therefore$ P(C)= $\frac{n(C)}{n(S)} = \frac{8}{36}$.

8.5 COUNTING PRINCIPLE AND COMBINATION

In some experiments like selecting cards, balls or players we cannot list out the complete sample space but count the no of sample points in the given sample space. To count the no of points in the sample space in these experiments, we define the concepts of combination. First, we state the **Counting principle** (Fundamental Principle of Mathematics), as follows

Counting principle: If A and B are two different things can be independently performed in m and n different possible ways then, by counting principle

(a) both A and B together can be performed in m.n possible ways.
(b) any one of them i.e. A or B can be performed in m+n possible ways.

Combination: Combination of r different things from n things e.g. selecting 3 balls from 5 balls, or 4 students from the group of 10 students of a class. It is calculation by the formula,

$$^{n}C_{r} = \frac{n!}{r!(n-r)!} \quad \text{where } n! = n(n-1)(n-2)\text{-----}3x2x1.$$

(e.g. 5!= 5x4x3x2x1=120)

Illustrations-

(i) We can select a group of 3 students from 5 students in $^5C_3$ ways and

$$^5C_3 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5x4x3x2x1}{2x13x2x1} = 10.$$

(ii) A student can select 4 different questions from 6 independently in $^6C_4$ ways.

$$^6C_4 = \frac{6!}{2!(6-4)!} = \frac{6!}{2!4!} \frac{6x5x4x3x2x1}{2x1x4x3x2x1} = 15.$$

(iii) A box contains 4 red and 6 green balls then by counting principle,

a) 2 red **and** 3 green together can be drawn in $^4C_2$ **x** $^6C_3$ ways.

b) 3 red **or** 3 green can be drawn in $^4C_3$ **+** $^6C_3$ ways.

Example 4:

Three cards are drawn from the pack of 52 playing cards.Find the probability that (a) all three are spade cards, (b) all three are of same suit, and (c) there are two Kings and one Queen.

Solution: When three cards are drawn from the pack of 52 cards, no of points in the sample is

$$n(S) = {}^{52}C_3 = \frac{52!}{3!(52-3)!} = \frac{51x51x50}{3x2x1} = 22100.$$

Now, we define the event,

a) A: all three are spade cards.

There are 13 spade cards, so 3 of them can be drawn in $^{13}C_3$

$$n(A) = {}^{13}C_3 = \frac{13!}{3!10!} = \frac{13x12x11}{3x2x1} = 286$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{286}{22100} = 0.0129.$$

a) event B: all three are of same suit

There are four suits (Club, Diamond, Heart & Spade) of 13 cards each. Three from them can be drawn in ${}^{13}C_3$ ways each. Now by the counting principle,

$n(B) = {}^{13}C_3 + {}^{13}C_3 + {}^{13}C_3 + {}^{13}C_3 = 4x\ 286 = 1144.$ **<u>(refer to illustrations above)</u>**

Hence $P(B) = \frac{1144}{22100} = 0.051.$

b) event C: there are two Kings and one Queen.

There are 4 kings and 4 Queens in the pack. So 2 kings and 1 Queen can be drawn in ${}^4C_2$ and ${}^4C_1$ respectively.

Therefore by counting principle,

$n(C) = {}^4C_2 \text{ x } {}^4C_1 = 6x4 = 24.$

Where, ${}^4C_2 = \frac{4!}{2!2!} = \frac{4x3x2x1}{2x1x2x1} = 6$ & ${}^4C_1 \frac{4!}{1!3!} = \frac{4x3x2}{3x2x1} = 4.$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{24}{22100}$

Example 5:

A box contains 5 Red and 4 Green balls. Two balls are drawn at random from the box, find the probability that I) Both are of same color ii) only red balls are drawn.

Solution: In the box there 9(4+5) balls in total, so 2 of them can be drawn in ${}^9C_2$ ways.

$n(S) = {}^9C_2 = \frac{9!}{2!7!} = \frac{9x8}{2x1} = 36.$

(i) We define the event A: Both are of same color

$n(A) = {}^4C_2 + {}^5C_2 = \frac{4!}{2!2!} + \frac{5!}{2!3!} = 6+10 = 16.$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{16}{36} = 0.44$

Hence probability of same colour is 0.44

(ii) Let us define event B: only red balls are drawn

Now both the balls will be drawn from 4 red balls only

$n(B) = {}^4C_2 = 6$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = 0.16$

Hence probability of both red balls is 0.16.

Exercise

1. Define the terms, i) Sample space. ii) An event. iii) Mutually exclusive events

2. Define the probability of an event. Also state the properties of the probability of event.

3. An unbiased coin is tossed three times, write down sample points w.r.t. following events: (a) Head occur only two times, (b) Head occur at least 2 times and (c) There are more Heads than Tail.
4. A pair of fair dice is rolled, write down the sample points w.r.t. following events: The sum of the no of dots appearing on the uppermost faces is (i) 7 or 11 (ii) multiple of 3 (iii) a perfect square.
5. A pair of coins is tossed at a time find the probability that,
6. Both the coins show Head. ii) No coin shows Head. iii) Only one Head turns up.
7. A cubic die is thrown, find the probability that the no of dots appeared is (a) A prime number, (b) A number multiple of 2.
8. A box contains 20 tickets numbered 1-20. A ticket is drawn at random from the box, find the probability that, i) the ticket bares a number < 5. ii) the number on the ticket is divisible by 4. iii) it is a cube of a natural number.
9. A card is drawn from the pack of 52 playing cards find the probability that, (a) The card is a king card (b) It is a face card.
10. If two fair dice are thrown , what is the probability the sum of the no of dots on the dice is, a) greater than 8. b) between 5 and 8.
11. Three unbiased coins are tossed at a time find the probability that, (a) exactly one Head turns up. (b) At most 2 Heads turn up. (c) All 3 coins show Heads.
12. Two cards are drawn from the pack of 52 playing cards find the probability that, (a) Both the cards are of same suits. (b) Both are Ace cards. (c) Both are of different colour.
13. A pair of dice is rolled, write down the sample space and find the probability that, the sum of the no of dots appearing on the uppermost faces is,(a) 6 or 10. (b) multiple of 5. (c) a perfect square.
14. Four unbiased coins are tossed at a time find the probability that, (i)exactly 2 Heads turn up. (ii) at most 3 Heads turn up. (iii) at least 3 coins show Heads.
15. Three cards are drawn from the pack of 52 playing cards find the probability that, (i) all 3 cards are of same colour. (ii) Two cards are face cards. 9iii) Only face cards are drawn.
16. Two boxes identical in size and shape respectively contain 3 red, 4 blue and 5 red, 2 blue balls. One ball is drawn at random from each box. What is the probability that both the balls are of same colour.
17. A committee of 4 is to be formed from 3 Professors and 7 students in a college. Find the probability that it includes, a) only 2 Professors. b) there are at least 3 students.
18. A box contains 6 red, 4 green and 3 white balls. Two balls are drawn at random, find the probability that, a) both are of same colour. b) no white ball is drawn. c) the balls are of different colour.
19. Given P(A)=0.6, P(B)=0.5, and $P(A \cap B)= 0.4$. Find P(AUB); P(A/B); P(B/A).
20. For two events A and B; $P(A)=\frac{2}{5}$, $P(B') = \frac{1}{3}$, $P(AUB)=\frac{5}{6}$. Find $P(A \cap B)$; P(A/B); P(only A); P(Only one).
21. For two mutually exclusive events A and B, P(A)=0.7 and P(B)=0.5, find $P(A \cap B)$ and P(A/B)

22. For the independent events A and B, P(A)=$\frac{1}{2}$, P(B) =$\frac{2}{5}$. Find P(A$\cap$B); P(AUB);P(only B).

23. One of the two purses contains 4 Gold coins and 5 Silver coins, another purse contains 3 Gold and 6 Silver coins. A coin is drawn at random from one of the purses, find the probability that it is a silver coin.

24. Two students A and B are solving a problem on Mathematics independently. Their chances of solving the problem are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Find the probability that, i) the problem is solved. ii) it is solved by only one of them.

25. A government contractor applies for 2 tenders of supplying breakfast supply and lunch box supply. His chances of getting the contracts are 0.6 and 0.5 respectively. Find the probability that, (i) he will get either of the contract (ii) he will get only one contract.

8.6 RANDOM VARIABLE AND EXPECTED VALUE

Random variable is a real valued function defined on the sample space.

Suppose S is the sample space associated with the random experiment E, then to every sample point in S we can assign a real number denoted by a variable X called as a random variable on S. e.g. When we toss a coin three times the sample is

S={HHH, HHT, HTH, THH,HTT,THT, TTH, TTT}, now if we define a variable

X: No of tosses showing Heads.

Then X takes values 0,1,2,3 where

{X=0}$\Leftrightarrow$ {TTT}; {X=1}$\Leftrightarrow$ {HTT, THT, TTH};

{X=3}$\Leftrightarrow$ {HHH} {X=2}$\Leftrightarrow$ {HHT, HTH, THH}.

Hence to each sample points in S we have assigned a real number, which uniquely determine the sample point.

The variable X is called as the random variable defined on the sample space S.

We can also find the probabilities of values 0,1,2,3 of the r.v. X as follows

P ({X=0}) = P ({TTT}) = 1/8, P ({X=1}) = P ({HTT, THT, TTH) = 3/8,

P ({X=3}) = P ({HHH}) = 1/8,P ({X=2}) = P ({HHT, HTH, THH}) = 3/8.

Now we can express these probabilities in the form of a table,

X:	0	1	2	3
P (x)	1/8	3/8	3/8	1/8

X	P (x)
0	1/8
1	3/8
2	3/8
3	1/8
Total	1

This is called as the probability distribution of random variable X.

In general probability distribution of X satisfies the following conditions;

(i) all p(x) are positive. i.e. $p(x) \geq 0$
(ii) $\Sigma p(x) = 1$ for all x.

A **random variable X** defined on the sample space S may be finite or infinite, at the same time it may take only countable values (without decimal) such variables are called as discrete random variables. On the other hand some variables like height, weight, income do take the fractional values also and called as the continuous random variables.

Expected value of X ,E(X):
Suppose a random variable X defined on sample space S takes values $x_{1,}$ $x_{2,}x_{3,-----}$ $x_{n.}$ with respective probabilities $p_{1,}$ $p_{2,}$ $p_{3,-----}$ p_n ; $P(x= x_1) = p_{1,}$ it's expected value is defined as,
$E(X) = \Sigma x.p(x)$
Expected value is also called as the mean of X.
Variance of X ; V(X): For the random variable X, variance is defined as,

$$\mathbf{V(X)} = E(X- E(X))^2$$
$$= \mathbf{E(X^2) - [E(X)]^2.}$$
$$= \Sigma x^2.p(x) - (\Sigma x.p(x))^2$$

Root of variance is called as standard deviation S.D.

Solved examples:

Example 6:
A discrete random variable X has the following probability distribution.

x:	-2	-1	0	1	2
p(x)	k	0.2	2k	2k	0.1

Find k. Also find the expected value of random variable.X

Solution: Since X is a random variable with given p(x), it must satisfy the conditions of a probability distribution.
$\Sigma p(x) = 1 \Rightarrow 5k+0.3=1 \Rightarrow k= 0.7/5=0.14$
p(x) values are 0.14;0.2;0.28;0.28;0.1
Now we calculate the expected value by the formula,
$E(X) = \Sigma x.p(x) = 0$ ------ from the table.

x	p(x)	xp(x)
-2	0.14	-2 x 0.14 = -0.28
-1	0.2	-0.2
0	0.28	0
1	0.28	0.28
2	0.1	0.2
Total	1	$0=\Sigma x.p(x)$

Example 7:
A random variable follows the probability distribution given below,

X	0	1	2	3	4
p(x)	0.12	0.23	0.35	0.20	0.10.

Obtain the expected value and variance of X.
Solution: The expected value and variance are given by the formula,

$$E(X) = \Sigma x.p(x) \text{ and } \quad V(X) = \Sigma x^2.p(x) - (\Sigma x.p(x))^2$$

x	p(x)	xp(x)	x^2.p(x)
0	0.12	0x0.12=0	0x0=0
1	0.23	0.23	0.23
2	0.35	0.70	1.40
3	0.20	0.60	1.80
4	0.10	0.40	1.60
Total	1.00	1.93=Σx.p(x)	4.03 = Σx^2.p(x)

Now from the table,

E(X) =Σx.p(x) = 1.93.

V(X) = $\Sigma x^2.p(x) - (\Sigma x.p(x))^2$

$= 4.03 - (1.93)^2$

V(X) = **0.35.**

Hence, Mean E(X) =1.93 units and V(X) = 0.35 units.

Example 8:

Find mean and variance of the random variable X whose probability distribution is given by

X:	-2	-1	0	1	2
P(x)	1/16	1/8	5/8	1/8	1/16.

Solution: For the random variable X, we have

From the table we get,

E(X) = Σx.p(x) = 0.

And V(X) = $\Sigma x^2.p(x) - (\Sigma x.p(x))^2$

$$= \frac{12}{16} - 0 = \frac{12}{16}$$

x	**p(x)**	**xp(x)**	**x^2.p(x)**
-2	1/16	-2/16	-2x(-2/16) = 4/16
-1	1/8	-1/8	1/8
0	5/8	0	0
1	1/8	1/8	1/8
2	1/16	2/16	4/16
Total	**1.00**	**0=Σx.p(x)**	**12/16 = Σx^2.p(x)**

Example 9:

A uniform die is thrown find the expcted value of the random variable X denoting the no on the uppermost face.

Solution: When a uniform die is thrown the random variable

X: the no on the uppermost face, takes the possible values 1,2,3,4,5 or 6.

With the same probability of occurrence.

Therefore we can find the mean or expected value of X by using the formula,

$E(X) = \Sigma x.p(x)$

$= \frac{21}{6} = 3.5$ ------ from the table.

x	p(x)	xp(x)
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6
Total	6/6 =1	21/6 =Σx.p(x)

Hence the mean of X is 3.5.

EXERCISE II

1. A random variable X has the following probability distribution:

X:	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

Find the value of k. Find the expected value and variance of x.

2. A random variable X has the following probability distribution:

X:	0	1	2	3	4	5
P(x)	0.1	0.1	0.2	0.3	0.2	0.1

Find the expected value and variance of x.

3. An unbiased coin is tossed four times. Find the expected value and variance of the random variable defined as number of Heads.

8.7 NORMAL DISTRIBUTION

Normal distribution deals with the calculation of probabilities for a continuous random variable like Height of players, Marks of students, or Wages of workers. We define the normal distribution as follows.

A continuous random variable X is said follow a normal distribution with parameters μ and 6, written as X~ N(μ , 6^2) if it's probability function is given by

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{where } x,\ \mu \in R,\ \sigma > 0$$

$$= 0 \qquad \text{otherwise.}$$

Here the constants are μ = Mean(X); σ = S.D.(X)

π = 3.142 and e= 2.718 (approx).

Before we learn to calculate the probabilities on normal distribution, we state the characteristics of the normal distribution stated below.

Characteristics of the normal distribution

i) The graph of normal distribution is a bell shaped curve.

ii) The area under the curve reads the probabilities of normal distribution hence total area
is 1 (one).

iii) The curve is symmetric about it's mean μ. Hence,

Area on l.h.s. of μ = Area on r.h.s. of μ = 0.5. Since area reads probability,

$$P(X<\mu) = P(X>\mu) = 0.5 = 50\%.$$

iv) Hence mean μ divides the curve into two equal parts so it is also the median.

The curve has it's maximum height at x = μ, therefore it the mode of the distribution.

v) Hence for normal distribution **Mean = Median = Mode =** μ

For the probability calculations, we define the variable

$$Z = \frac{x - Mean}{S.D.} = \frac{x-\mu}{\sigma}$$

Mean (Z) = 0 and S.D.(Z)= 1. Z is called a standard normal variable (s.n.v.)

Also P (X) = P (Z) = Area (Z). The area (probability) values of z are tabulated.

vi) The lower (Q_1) and upper quartiles (Q_3) are equidistant from the mean

μ. i.e. $\mu - Q_1 = Q_3 - \mu \Rightarrow \mu = \frac{Q_1+Q_3}{2}$

vii) The mean deviation (M.D.) of normal distribution is $\frac{4}{5}\sigma$

viii) The quartile deviation (Q.D.) of normal distribution is 0.67σ.

Area under the normal curve between

(i) $\mu \pm \sigma$ is 68.27% (ii) $\mu \pm 2\sigma$ is 95.45%

(iii) $\mu \pm 3\sigma$ is 99.73%. (iv) $\mu \pm 1.645\sigma$ is 90%

(v) $\mu \pm 1.96\sigma$ is 95% (vi) $\mu \pm 0.67\sigma$ is 50%.

Solved examples:

Example 10:

A continuous random variable X follows a normal distribution with mean 50 and S.D. of 10. Find the following probabilities for X,

a) P (X≥55) b) P(45≤X≤60) c) P(X≤45).

Solution: For the normal variable X , we have

Mean (X)= μ = 50 and standard deviation = 6= 5.

∴ X (N (μ, 6^2) = N (50, 10^2).

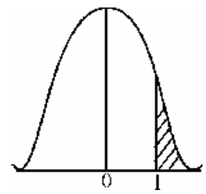
We define the variable $Z = \frac{x-\mu}{\sigma} = \frac{x-50}{5}$

Mean (Z) = 0 and S.D.(Z)= 1. Z is called a standard normal variable (s.n.v.)

Also P (X) = P (Z) = Area (Z).

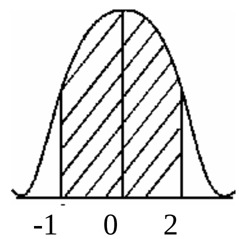
a) P (X≥55)= P($\frac{x-50}{5} \geq \frac{55-50}{5}$)

= P(Z≥1)
= Area r.h.s. of +1
= 0.5- Area from 0 to 1
= 0.5-0.3413 = 0.1587.



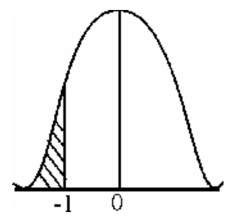
b) P(45≤X≤60) = P($\frac{45-50}{5} \leq \frac{x-50}{5} \leq \frac{60-50}{5}$)

= P(-1≤Z≤2)
= Area between -1 & +2
= Area from -1 to 0+ Area from 0 to 2.
= 0.3413 + 0.4772 = 0.8185.



c) P(X≤45)=P($\frac{x-50}{5} \leq \frac{45-50}{5}$)

= P(Z≤-1)
= Area on l.h.s. of -1.
= 0.5 - Area from -1 to 0.
= 0.5 - 0.3413 = 0.1587.



Example 11:

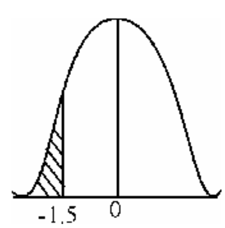
The marks of 150 students in the class is said to follow a normal with mean 60 and S.D. of 10. Find, the expected no of students scoring marks below 45.Percentage of students scoring marks between 55 and 70.

Solution: Let X: Marks of students;
Mean(X) = 60 and S.D.(X) = 6 = 10.
X has normal distribution with μ =60 and 6 = 10.
i.e. X(N (μ , 6^2) = N (60, 10^2)

We define Z= $\frac{x-\mu}{\sigma} = \frac{x-60}{10}$.

∴ Mean (Z) = 0 and S.D.(Z)= 1. Z is called a standard normal variable (s.n.v.)
Also P (X) = P (Z) = Area (Z).
Now, to find the expected no of students scoring marks below 45, we find P(marks less than 45)

= P(X ≤ 45) =.P($\frac{x-60}{10} \leq \frac{45-60}{10}$)
= P(Z ≤ -1.5)
= Area on l.h.s. of -1.5.
= 0.5 - Area from -1.5 to 0.
= 0.5 - 0.4332 = 0.0668 = 6.68%.



Expected no of students = 6.68%(150) =10.
Similarly, to find percentage of students scoring marks between 55 and 70.
Consider, P(marks between 55 and 70)

= P(55≤X≤70)

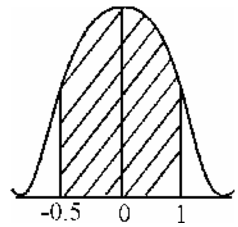
=P($\frac{55-60}{10} \le \frac{x-60}{10} \le \frac{70-60}{10}$)

= P(-0.5 ≤ Z ≤ 1)

= Area between -0.5 & +1

= Area from -0.5 to 0+ Area from 0 to 1.

= 0.1915 + 0.3413= 0.5328 = 53.28%.



∴ 53.28% students have scored marks between 55 and 70.

Example 12:

The height of 250 soldiers in a military camp confirms a normal distribution with mean height of 155cms.and S.D. of 20cms. Find the proportion of soldiers with height above 170 cms. Also find the height of the shortest soldier in the group of tallest 20% soldiers.

Solution: Let r.v. X denotes the height of a soldier.

Mean(X) = 155 and S.D.(X) = σ = 20.

∴ X has normal distribution with μ =155 and σ = 20.

i.e. X N (μ , σ^2) = N (155, 20^2)

We define Z= $\frac{x-\mu}{\sigma} = \frac{x-155}{20}$.

∴ Mean (Z) = 0 and S.D.(Z)= 1. Z is called a standard normal variable (s.n.v.)

Also P (X) = P (Z) = Area (Z).

Now, to find the proportion of soldiers with height above 170 cms, we find,

P(soldier's height is above 170 cms)

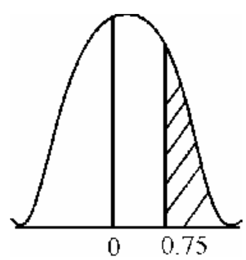
P (X ≥ 170) = P$\left(\frac{x-155}{20} \ge \frac{170-155}{20}\right)$

= P(Z ≥ 0.75)

= Area r.h.s. of 0.75

= 0.5- Area from 0 to 0.75.

= 0.5-0.2734 = 0.2766.



∴ Proportion of soldiers with height above 170 cms is (0.2766x250):250 i.e. 69 : 250.

Now, let the height of the shortest soldier in the group of tallest 20% soldiers be *h*.

P(height less than *h*) =20%=0.2 i.e. P(X ≤ *h*) =.0.2

Consider, P(X ≤ *h*) = P($\frac{x-155}{20} \le \frac{h-155}{20}$ = t say) i.e. P(Z ≤ t) = 0.2.

Area on l.h.s. of t =0.2(t is less than 0 i.e. negative.(since area on l.h.s.< 0.5)

area from t to 0 = 0.3.

Now from the normal area table, area from 0 to 0.84 is 0.3.

Hence, t = -0.84 (t= $\frac{h-155}{20}$ = -0.84 i.e. h = 155 + 20(-0.84) =155-16.8= 138.2.

Therefore, the height of the shortest soldier in the group of tallest 20% soldiers is 138 cms.

Example 13:

The daily wages of 300 workers in a factory are normally distributed with the average wages of Rs.2500 and S.D. of wages equals to Rs.500. Find the percentage of workers earning wages between Rs.3000 and Rs.4000. Also find the wages of the lowest paid worker in the group of highest paid 30% workers.

Solution: Let r.v. X denotes the wages of a worker.

Mean(X) = 2500 and S.D.(X) = σ = 500.

$\therefore$ X has normal distribution with μ =2500 and σ = 500.

i.e. X(N (μ , σ^2) = N (2500, 500^2)

We define Z= $\frac{x-\mu}{\sigma} = \frac{x-2500}{500}$

$\therefore$ Mean (Z) = 0 and S.D.(Z)= 1. Z is called a standard normal variable (s.n.v.)

Also P (X) = P (Z) = Area (Z).

Now, to find the percentage of workers earning wages between Rs 3000 and Rs 4000.

Consider, P(wages between Rs.3000 and Rs 4000)

= P($3000 \le X \le 4000$)

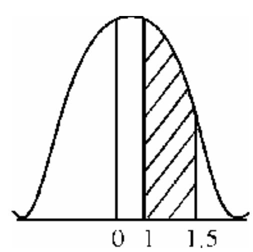
=P($\frac{3000-2500}{500} \le \frac{x-2500}{500} \le \frac{4000-2500}{500}$)

= P($1 \le Z \le 1.5$)

= Area between +1 & +1.5

= Area from 0 to 1.5 - Area from 0 to 1.

= 0.4332 - 0.3413= 0.0919 = 9.19%.



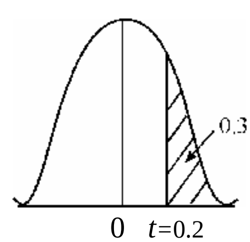
$\therefore$ 9.91% workers are earning wages between Rs 3000 and Rs 4000.

Now, let the wages of the lowest paid worker in the group of highest paid 30% workers be h.

P(wages greater than h) =30%=0.3 i.e. P(X $\ge$ h) = 0.3.

Consider, P(X $\ge$ h) = P($\frac{x-2500}{500} \ge \frac{h-2500}{500}$ = t say)

i.e. P(Z $\ge$ t) = 0.3.



$\therefore$ Area on r.h.s. of t =0.3 (t is greater than 0 i.e. positive.(since area on r.h.s.< 0.5)

area from 0 to t = 0.2.

Now from the normal area table, area from 0 to 0.52 is 0.2.

Hence, t = 0.52 $\Rightarrow \frac{h-2500}{500}$ = 0.52 $\Rightarrow$ h = 2500 + 500(0.52) = 2760.

Therefore, the wages of the lowest paid worker in the group of highest paid 30% workers are Rs. 2760.

EXERCISE III

1. Define a normal variable. State the properties of normal distribution.

2. What is mean by a standard normal variable. What are the mean and standard of a a standard normal variable.

3. A continuous random variable X follows a normal distribution with mean 50 and S.D. of 10. Find the following probabilities for X,

a) $P(X \geq 55)$ b) $P(45 \leq X \leq 60)$ c) $P(X \leq 45)$.

Given, Area under the normal curve,
From 0 to 1 is 0.3413.
From 0 to 2 is 0.4772.

4. The marks of 150 students in the class is said to follow a normal with mean 60 and S.D. of 10. Find, the expected no of students scoring marks below 45.Percentage of students scoring marks between 55 and 70.

Given, Area under the normal curve,
From 0 to 0.5 is 0.1915.
From 0 to 1 is 0.3413.
From 0 to 1.5 is 0.4332.

5. The height of 250 soldiers in a military camp confirms a normal distribution with mean height of 155cms.and S.D. of 20cms. Find the proportion of soldiers with height above 170 cms. Also find the height of the shortest soldier in the group of tallest 20% soldiers.

Given, Area under the normal curve,
From 0 to 1.5 is 0.4332.
From 0 to 0.84 is 0.3.

6. The daily wages of 300 workers in a factory are normally distributed with the average wages of Rs.2500 and S.D. of wages equals to Rs.500. Find the percentage of workers earning wages between Rs.3000 and Rs.4000. Also find the wages of the highest paid worker in the group of lowest paid 30% workers.

Given, Area under the normal curve,
From 0 to 1 is 0.3413.
From 0 to 1.5 is 0.4332.
From 0 to 0.52 is 0.2.

7. A normal distribution has mean $\mu = 15$ and 6= 5. Find the following probabilities.

$P(X \geq 20)$ $P(10 \leq X \leq 17.5)$ $P(X \leq 12)$.

Given, Area under the normal curve,
From 0 to 0.4 is 0.1554.
From 0 to 0.5 is 0.1915.
From 0 to 1 is 0.3413.

8. The weights of 450 students in a school are normally distributed with the average weight of 50 kg. and S.D.5 kg. Find the percentage of students with weight:

i) less than 45 kg. ii) Between 40 and 47 kg.

Given, Area under the normal curve,
From 0 to 0.4 is 0.1554. From 0 to 0.5 is 0.1915.From 0 to 1 is 0.3413.

❖❖❖