M.Sc. Part-I
Sample Questions For Online exam

Q. 1 (1 point) Let $\mathcal{B} = \{(1, 0, -1), (1, 1, 1), (2, 2, 0)\}$ be a basis for $\mathbb{C}^3$. The dual basis $\mathcal{B}^*$ of $\mathcal{B}$ for $\mathbb{C}^{3^*}$ is given by

1. $f_1(x_1, x_2, x_3) = x_1, f_2(x_1, x_2, x_3) = x_2, f_3(x_1, x_2, x_3) = x_3$
2. $f_1(x_1, x_2, x_3) = -x_1, f_2(x_1, x_2, x_3) = -x_2, f_3(x_1, x_2, x_3) = x_3$
3. $f_1(x_1, x_2, x_3) = x_1 - x_2, f_2(x_1, x_2, x_3) = x_1 - x_2 + x_3,$
   $f_3(x_1, x_2, x_3) = -\frac{1}{2}x_1 + x_2 - \frac{1}{2}x_3$
4. $f_1(x_1, x_2, x_3) = 1, f_2(x_1, x_2, x_3) = 0, f_3(x_1, x_2, x_3) = -1$

Q. 2 (1 point) Let V be the vector space of all polynomial functions over the field of real numbers. Let a and b be fixed real numbers and let f be a linear functional on V defined by $f(p) = \int_a^b p(x)dx$. If D is the differentiation operator on V, then $D^t f$, D^t is transpose of D, is given by

1. 1
2. 0
3. $p(b) - p(a)$
4. $p(a) - p(b)$

Q. 3 (1 point) Let n be a positive integer and let V be the vector space of all polynomial functions over the field of real numbers which have degree at most n.If D is the differentiation operator on V, then dimension of the null space of D^t, D^t is transpose of D, is given by

1. 0
2. $n - 1$
3. 1
4. $n + 1$

Q. 4 (1 point) Let D_1, D_2 be the functions on set of 3×3 matrices over the field of real numbers defined by, for $A = [A_{ij}]_{3\times3}$, $D_1(A) = A_{11} + A_{22} + A_{33}$ and $D_2(A) = -A_{11}^2 + 3A_{11}A_{22}$. Then

1. D_1 is a 2-linear function
2. D_2 is a 2-linear function
3. both D_1 and D_2 are 2-linear functions
4. both D_1 and D_2 are not 2-linear functions

Q. 5 (1 point) The determinant of the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is

1. 0
2. $(a-b)(c-a)(c-b)$
3. -1
4. $(b-a)(c-a)(b-c)$

Q. 6 (1 point) A is a 2×2 unitary matrix. Then eigen value of A are

1. $1, -1$.
2. $1, -i$.
3. $i, -i$.
4. $-1, i$.

Q. 7 (1 point) A is 5×5 matrix, all of whose entries are 1, then

1. A is not diagonalizable.
2. A is idempotent.
3. A is nilpotent.
4. The minimal polynomial and the characteristics polynomial of A are not equal.

Q. 8 (1 point) A is a 5×5 matrix over $\mathbb{R}$, then $(t^2+1)(t^2+2)$

1. is a minimal polynomial.
2. is a characteristics polynomial.
3. is minimal as well as characteristics polynomial.
4. is not minimal as well as characteristics polynomial.

Q. 9 (1 point) M is a 2-square matrix of rank 1, then M is

1. diagonalizable and non singular.
2. diagonalizable and nilpotent .
3. neither diagonalizable nor nilpotent.
4. either diagonalizable or nilpotent.

Q. 10 (1 point) Let $P(\mathbb{R})$ be vector space of all polynomials over $\mathbb{R}$. $T_i : P(\mathbb{R}) \to P(\mathbb{R})$ such that $T_1(f(x)) = \int_0^x f(t)dt$ and $T_2(f(x)) = f'(x)$. Then

1. T_1 is 1-1, T_2 is not.
2. T_2 is 1-1, T_1 is not.
3. T_1 is onto and T_2 is 1-1.
4. T_1 and T_2 both are 1-1. .

Q. 11 (1 point) What is the order of the subgroup generated by $20(mod 30)$ in the cyclic group $\mathbb{Z}_{30}$?

1. 20
2. 10
3. 6
4. 3

Q. 12 (1 point) Let A be a real 34 matrix of rank 2. Then the rank of $A^t A$, where denotes A^t the transpose A, is:

1. exactly 2
2. exactly 3
3. exactly 4
4. at most 2 but not exactly 2

Q. 13 (2 points) Which one of the following is not true for Cantor set?

1. Closed
2. Bounded
3. Compact
4. connected

Q. 14 (2 points) Let f and g are the continuous functions from $\mathbb{R} \to \mathbb{R}$. Then the set $\{x \colon f(x) = g(x) + 1\}$ is always

1. Closed
2. Bounded
3. Open
4. connected

Q. 15 (2 points) Suppose $K \subset Y \subset X$. Then K is compact relative to X if and only if ..... (Complete true statement by choosing correct option from following)

1. K is compact relative to Y
2. $K = \{0\}$.
3. K is closed relative to Y.
4. K is open relative to Y.

Q. 16 (2 points) Let E^0 denotes the set of all interior points of a set E. Then which of following is true.

1. If $E \subset E^0$ then is closed.
2. $\overline{E^0} = E$
3. E^0 is neither open nor closed.
4. E is open if and only if $E^0 = E$.

Q. 17 (2 points) Suppose E is an open set in $\mathbb{R}^n$, f maps E into $\mathbb{R}^m$, and $x \in E$. If there exists a linear transformation A of $\mathbb{R}^n$ into $\mathbb{R}^m$ such that

$$\lim_{h \to 0} \frac{|f(x+h) - f(x) - Ah|}{|h|} = 0,$$

then which of following is correct.

1. f is differentiable at x and $f'(x) = A^T$.
2. f is not differentiable at x.
3. f is differentiable at x and $f'(x) = A$
4. f is differentiable at x and $f'(x) \neq A$.

Q. 18 (2 points) A mapping f of set E into $\mathbb{R}^k$ is said to be bounded ....

1. if there exists real number M such that $|f(x)| \geq M$ for all $x \in E$.
2. if there exists a closed ball $B(0, r) \subset E$ and real number $M > 0$ such that $|f(x)| \geq M$ for all $x \in B(0, r)$.
3. if there exists real number M such that $|f(x)| \leq M$ for all $x \in E$.
4. if there exists a ball $B(x_0, r) \subset E$, such that $|f(x)| \geq \dfrac{1}{r}$ for all $x \in B(x_0, r)$.

Q. 19 (2 points) If $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x, y) = xy$, then $D_f(a, b)(x, y) =?$

1. $abxy$
2. $bx + ay$
3. $ax + by$
4. xy

Q. 20 (2 points) Which of following is a statement of Heine Borel theorem.

1. Let F be an open covering of a closed and bounded set A in $\mathbb{R}^n$. Then a finite sub cover of F also covers A.
2. Let F be an open covering of a closed and bounded set A in $\mathbb{R}^n$. Then any finite sub cover of F does not cover A.
3. Let F be an open covering of a closed set A in $\mathbb{R}^n$. Then a finite sub cover of F also covers A.
4. Let F be an open covering of set A in $\mathbb{R}^n$. Then a finite sub cover of F also covers A.

Q. 21 (2 points) A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is said to be differentiable at c if ....(choose correct option from following to complete a true statement)

1. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c + v) = f(c) + T_c(v) + \|v\| E_c(v)$, where $E_c(v) \to 0$ as $v \to 0$.

2. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c+v) = f(c) + T_c(v) + \|v\| E_c(v)$, where $E_c(v) \to \infty$ as $v \to 0$.
3. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c+v) = f(c) + T_c(v) + E_c(v)$, where $E_c(v) \to 0$ as $v \to 0$.
4. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c+v) = f(c) + T_c(v)$, where $T_c(v) \to 0$ as $v \to 0$.

Q. 22 (2 points) Consider the function $f : (0,1) \to \mathbb{R}$ defined by $f(x) = \dfrac{1}{x}$. Then .....

1. f is not continuous but bounded in $(0,1)$.
2. f is neither continuous nor bounded in $(0,1)$.
3. f is not continuous.
4. f is continuous but not bounded in $(0,1)$.

Q. 23 (3 points) Let $z, w \in \mathbb{C}$. Then $|| \, z \, | - | \, w \, || \leq$ ....

1. $| \, z \, | - | \, w \, |$.
2. $| \, z - w \, |$.
3. $z - w$.
4. $z + w$.

Q. 24 (3 points) The radius of convergence of the power series $\sum\limits_{n=0}^{\infty} (8+6i)^n z^n$ is ....

1. 10.
2. $\dfrac{1}{10}$.
3. 100.
4. 20.

Q. 25 (3 points) The length of the curve $\gamma(t) = 3e^{it}$, $t \in [0, 2\pi]$, is ....

1. 2π
2. π
3. $\dfrac{\pi}{2}$
4. 6π

Q. 26 (3 points) $\displaystyle\int_\gamma \frac{z^2+z+2}{z} \, dz =$ .... ,where γ is the circle $|z| = 5$

1. $i\pi$
2. $4i\pi$
3. $10i\pi$

4. zero

Q. 27 (3 points) Let f be analytic in open disk $B(a;R) \subset \mathbb{C}$ and suppose $|f(z)| \leq M$ for all z in $B(a;R)$. Then $|f^{(2)}(a)| \leq$ ....

1. $\dfrac{Mn!}{R^n}$
2. $\dfrac{M}{R^n}$
3. $\dfrac{6M}{R^3}$
4. $\dfrac{2M}{R^2}$

Q. 28 (3 points) $\displaystyle\int_\gamma \frac{\sin z + z^2}{z-4}\,dz =$ .... , where γ is the circle $|z| = \dfrac{3}{2}$.

1. $2\pi i$
2. $4\pi i$
3. zero
4. 16π

Q. 29 (3 points) Let G be a domain in $\mathbb{C}$ and suppose that f is a non constant analytic function on G. Then for any open set U in G , ...

1. $f(U)$ is closed.
2. $f(U)$ is open.
3. $f(U)$ is neither open nor closed.
4. $f(U)$ is both open and closed.

Q. 30 (3 points) Let $z = z_0$ be an isolated singularity of f and let $f(z) = \displaystyle\sum_{n=-\infty}^{\infty} c_n (z-z_0)^n$ be its Laurent series expansion in $ann(z_0; 0, R)$. If $c_n = 0$ for $n \leq -1$ then $z = z_0$ ....

1. is a pole of order n.
2. is a removable singularity.
3. is a non isolated singularity.
4. is an essential singularity.

Q. 31 (3 points) If f is analytic in $0 < |z-\alpha| < R$ and f has a pole of order 8 at $z = \alpha$ so that $f(z) = \dfrac{g(z)}{(z-\alpha)^8}$, where g is analytic in $|z-\alpha| < R$ then the residue of f at $z = \alpha$ is ....

1. $\dfrac{g^{(8)}(\alpha)}{7!}$

2. $\dfrac{g^{(5)}(\alpha)}{4!}$
3. $\dfrac{g^{(8)}(\alpha)}{8!}$
4. $\dfrac{g^{(7)}(\alpha)}{7!}$

Q. 32 (3 points) Let $f(z) = \dfrac{z^2}{(z-3)(z+2)^2}$. Then the residue of f at $z = 3$ is ....

1. $\dfrac{4}{16}$
2. $\dfrac{9}{24}$
3. $\dfrac{9}{16}$
4. $\dfrac{9}{25}$

Q. 33 (3 points) Let $f(z) = z^6 - 5z^4 + z^3 - 2z$ then $f(z)$ has .... zeros, counting multiplicities , inside the circle $|z| = 1$.

1. 3
2. 6
3. 4
4. 2

Q. 34 (3 points) Let $z = x + iy$ and $w = u + iv$. The image of straight line $x = \dfrac{1}{4}$ in the complex plane under the transformation $w = \frac{1}{z}$ is ....,

1. $u^2 + v^2 - 4u = 0$
2. $u^2 + v^2 - 4u = 4$
3. $u^2 + v^2 - 4v = 0$
4. none of these

Q. 35 (3 points) Let $S(z) = \dfrac{4z+5}{6z+7}$ be a mobius transformation. Then $S^{-1}(z) =$ ....

1. $s(z) = \dfrac{7z-5}{-6z+4}$
2. $s(z) = \dfrac{7z-5}{6z-4}$
3. $s(z) = \dfrac{7z+5}{-6z+4}$
4. $s(z) = \dfrac{6z-5}{4z-7}$

Q. 36 (4 points) If $56x + 72y = 40$, where x and y are integers. Then

1. There finitely many possibilities for x and y.
2. $x = -15 + 7t, y = 20 - 9t$, where $t \in \mathbb{Z}$.
3. $x = 20 + 9t, y = -15 - 7t$, where $t \in \mathbb{Z}$.
4. $x = 20 + 7t, y = -15 - 9t$, where $t \in \mathbb{Z}$.

Q. 37 (4 points) The equation $x^3 + 6x^2 + 11x + 6 = 0$ has

1. Three non-real roots.
2. Three real roots.
3. Two non-real roots and one real root.
4. Two real roots and one non-real root.

Q. 38 (4 points) Let f be a function from a set with $k + 1$ or more elements to a set with k elements. Then

1. f is not injective.
2. f is injective.
3. f is bijective.
4. f is injective but can not be surjective.

Q. 39 (4 points) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are selected?

1. 6.
2. 7.
3. 9.
4. 8.

Q. 40 (4 points) At a party, seven gentlemen check their hats. In how many ways can their hats be returned so that at least two of the gentlemen receive their own hats?

1. $7! - D_7$.
2. $7! - D_7 - 7 \times D_6$..
3. $7! - D_7 - D_6$.
4. $D_7 - D_6$.

Q. 41 (4 points) If $r(m, n)$ denote the Ramsey number then which of the following is true.

1. $r(3, 5) < 14$ and $r(4, 4) < 18$.
2. $r(3, 5) = 14$ and $r(4, 4) < 18$.
3. $r(3, 5) = 14$ and $r(4, 4) = 18$.

4. $r(3,5) < 14$ and $r(4,4) = 18$.

Q. 42 (4 points) Find the exponential generating function for the sequence $\{a_n\}$, where $a_n = n+1, n = 0, 1, 2, \dots$.

1. $(x+1)e^x$
2. xe^x
3. e^x
4. $2xe^x$

Q. 43 (4 points) Find the coefficient of x^{10} in the power series of $1/(1-x)^3$

1. 64
2. 65
3. 67
4. 66

Q. 44 (4 points) Find the sequence with $f(x) = e^{3x} - 3e^{2x}$ as its exponential generating function.

1. $a_n = 3^{n+1} - 3 \times 2^{n+1}$
2. $a_n = 3^{n-1} - 3 \times 2^{n-1}$
3. $a_n = 3^n - 3 \times 2^n$
4. $a_n = 3^{n+1} + 3 \times 2^{n+1}$

Q. 45 (4 points) Suppose a necklace can be made from beads of three colors—black, white, and red. How many different necklaces with 3 beads are there?

1. 8
2. 9
3. 10
4. 11

Q. 46 (4 points) Consider $y'' + P(x)y' + Q(x)y = 0$. If both $P(x)$ and $Q(x)$ are analytic at p then p is called

1. singular point
2. ordinary point
3. regular singular point
4. both regular and ordinary point

Q. 47 (5 points) The number of distinct equivalence classes of the relation of congruence modulo m is.

1. m

2. $m+1$
3. $m-1$
4. zero

Q. 48 (5 points) If every two elements of a poset are comparable then the poset is called

1. sub ordered poset
2. totally ordered poset
3. sub lattice
4. semigroup

Q. 49 (5 points) The composition of function is associative but not ——

1. commutative
2. associative
3. distributive
4. idempotent

Q. 50 (5 points) The set of all rational numbers in the interval (0, 1).

1. countable
2. uncountable
3. finite
4. none of these

Q. 51 (5 points) The set of all real numbers in the interval $(0, 1)$ is ——.

1. countable
2. uncountable
3. finite
4. infinite

Q. 52 (5 points) The set of rational numbers $\mathbb{Q}$ is ———

1. countable
2. uncountable
3. finite
4. infinite

Q. 53 (5 points) There exists no —— from a set to its power set

1. relation
2. injection

3. surjection
4. bijection

Q. 54 (5 points) Self-complemented, distributive lattice is called ——

1. Boolean algebra
2. Modular lattice
3. Complete lattice
4. Self dual lattice

Q. 55 (5 points) If S is uncountable and T is countable then $S \setminus T$ is ———.

1. countable
2. uncountable
3. finite
4. infinite

Q. 56 (5 points) Let S be a partially ordered set. If every totally ordered subset of S has an upper bound, then S contains a —— element.

1. minimal
2. maximal
3. lub
4. glb

Q. 57 (5 points) Let $X = \{a, b, c\}$. Then $|\mathcal{P}(X)| =$

1. 3
2. 5
3. 8
4. 24

Q. 58 (5 points) A mixture of candies contains 6 mints, 4 toffees, and 3 chocolates. If a person makes a random selection of one of these candies, find the probability of getting a toffee or a chocolate.

1. $\frac{9}{13}$
2. $\frac{10}{13}$
3. $\frac{7}{13}$
4. $\frac{13}{7}$