Answer Key forTheory of Computer Science
Q.P Code 78070 (Nov 2019)

Ans 1c) L (R) = { w | w Ɛ {0, 1} *} with at the most three zeros

$$RE = 1^* (0 + 01^*0 + 01^*01^*0) 1^*$$

Ans 1 d) $E \rightarrow E+E \mid E*E \mid (E) \mid id$ is ambiguous grammar.
Let us derive a string id + id * id

LMD 1:

E ⟶ E + E
⟶ id + E
⟶ id + E * E
⟶ id + id * E
⟶ id + id * id

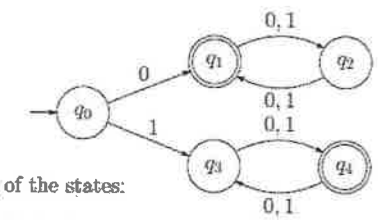
LMD 2 :

E ⟶ E * E
⟶ E + E * E
⟶ id + E * E
⟶ id + id * E
⟶ id + id * id

More than one LMD's for string id + id * id

Ans 2 a) L (R) = { w | w starts with 0 and has odd length or starts with 1 and has even length }

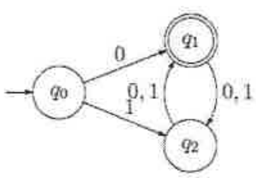
Solution:



The meaning of the states:

- q_0 - initial state
- q_1 - the string the DFA read so far starts with 0 and has an odd length
- q_2 - the string the DFA read so far starts with 0 and has an even length
- q_3 - the string the DFA read so far starts with 1 and has an odd length
- q_4 - the string the DFA read so far starts with 1 and has an even length

As it was correctly pointed out, this DFA can be simplified by combining the states q_1 and q_4, as well as q_2 and q_3:

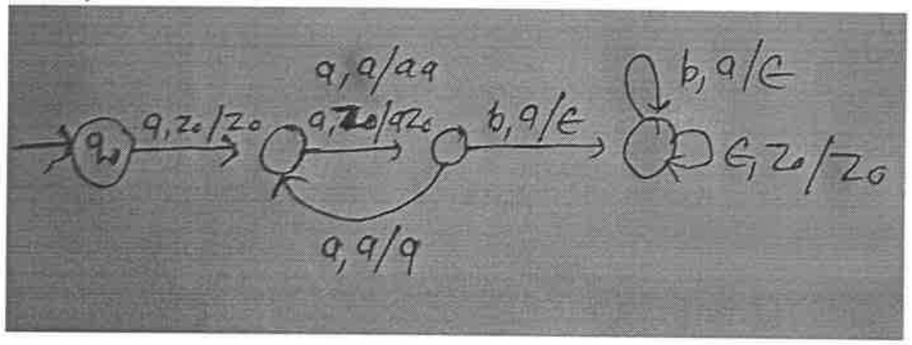


Ans 2b) Prove that $L = \{a^i : i \text{ is a prime number}\}$ is not regular.

1. We don't know m, but assume there is one.
2. Choose a string $w = a^i$ where i is a prime number and $|xyz| = i > m+1$. (This can always be done because there is no largest prime number.) Any prefix of w consists entirely of a's.
3. We don't know the decomposition of w into xyz, but since $|xy| \leqslant m$, it follows that $|z| > 1$. As usual, $|y| > 0$,
4. Since $|z| > 1$, $|xz| > 1$. Choose $j = |xz|$. Then $|xy^jz| = |xz| + |y||xz| = (1 + |y|)|xz|$. Since $(1 + |y|)$ and $|xz|$ are each greater than 1, the product must be a composite number. Thus $|xy^jz|$ is a composite number.

Hence $L = \{a^i : i \text{ is a prime number}\}$ is not regular.

Ans 3 a)



Ans 3 b)

CFG $S \rightarrow iCtS \mid iCtSeS \mid a$

$C \rightarrow b$

For the string "ibtaeibta

LMD :

$S \Rightarrow iCtSeS$

$S \Rightarrow ibtSeS$ $\quad C \rightarrow b$

$S \Rightarrow ibtaeS$ $\quad S \rightarrow a$

$S \Rightarrow ibtaeiCtS$ $\quad S \rightarrow iCtS$

$S \Rightarrow ibtaeibtS$ $\quad C \rightarrow b$

$S \Rightarrow ibtaeibta$ $\quad S \rightarrow a$

RMD :

$S \Rightarrow iCtSeS$

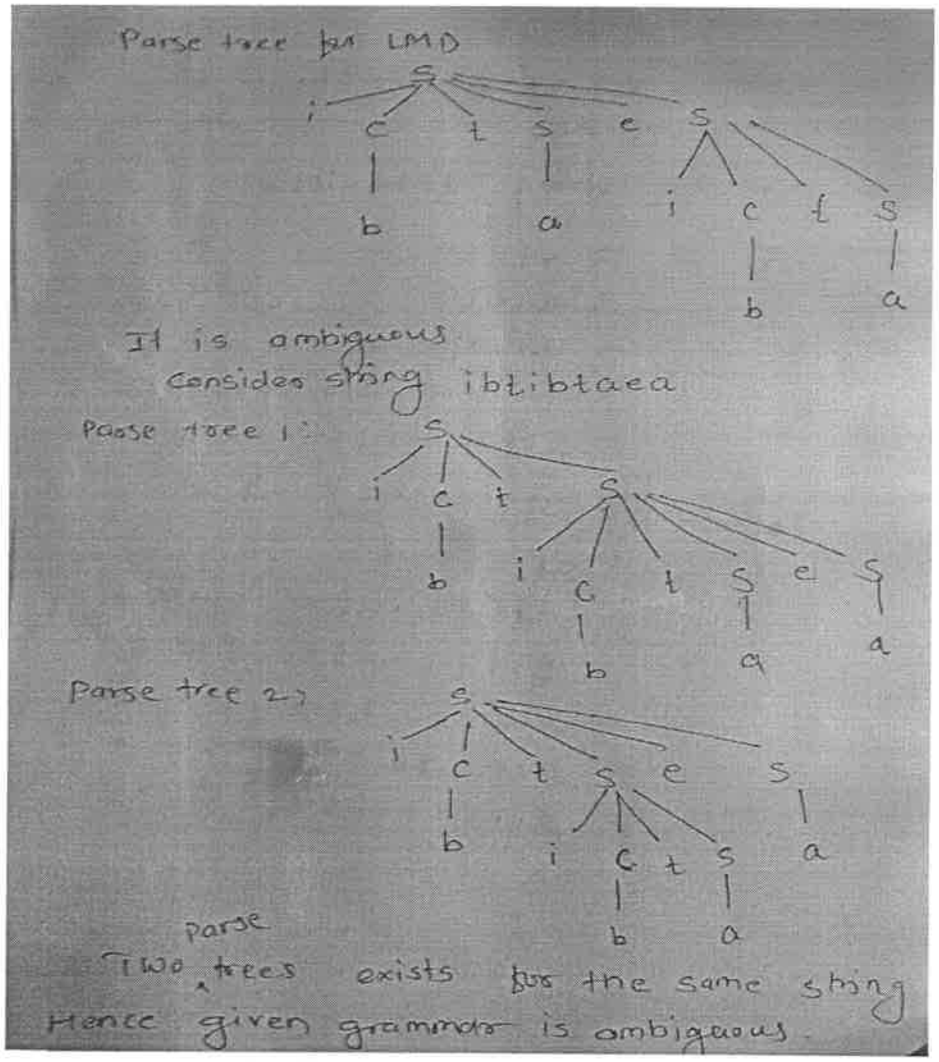
$S \Rightarrow iCtSeiCtS$ $\quad S \rightarrow iCtS$

$S \Rightarrow iCtSeiCta$ $\quad S \rightarrow a$

$S \Rightarrow iCtSeibta$ $\quad C \rightarrow b$

$S \Rightarrow iCtaeibta$ $\quad S \rightarrow a$

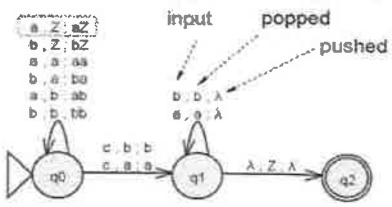
$S \Rightarrow ibtaeibta$ $\quad C \rightarrow b$



Ans 4 a) PDA accepting the language L={wcw^R | n≥0 }

$\delta(q_0, a, Z) = (q_0, AZ)$
$\delta(q_0, a, A) = (q_0, AA)$
$\delta(q_0, a, B) = (q_0, AB)$
$\delta(q_0, b, Z) = (q_0, BZ)$
$\delta(q_0, b, A) = (q_0, BA)$
$\delta(q_0, b, B) = (q_0, BB)$

$\delta(q_0, c, Z) = (q_1, Z)$
$\delta(q_0, c, A) = (q_1, A)$
$\delta(q_0, c, B) = (q_1, B)$
$\delta(q_1, a, A) = (q_1, \varepsilon)$
$\delta(q_1, b, B) = (q_1, \varepsilon)$
$\delta(q_1, \varepsilon, Z) = (q_2, Z)$



Ans 4 b)

(a) Eliminate ϵ-productions.

Since C can produce ϵ, A can also produce ϵ, so B can produce ϵ, so S can produc
therefore, we need to change the productions for every symbol.

$$
\begin{aligned}
S &\to 00 \mid 0A0 \mid 11 \mid 1B1 \mid B \mid BB \\
A &\to C \\
B &\to S \mid A \\
C &\to S
\end{aligned}
$$

(b) Eliminate any unit productions in the resulting grammar.

We note that A, B, and C will all just produce S again. So we remove them.

$$
\begin{aligned}
S &\to 00 \mid 0A0 \mid 1B1 \mid BB \\
A &\to 00 \mid 0A0 \mid 1B1 \mid BB \\
B &\to 00 \mid 0A0 \mid 1B1 \mid BB \\
C &\to 00 \mid 0A0 \mid 1B1 \mid BB
\end{aligned}
$$

(c) Eliminate any useless symbols in the resulting grammar.

The variable C has now become unreachable. We also remove A and B, because they are exactly equal to S. Our grammar becomes:

$$S \to 00 \mid 0S0 \mid 1S1 \mid SS$$

(d) Put the resulting grammar into Chomsky Normal Form.

To make this a CNF grammar, we first create variables $A \to 0$ and $B \to 1$. We then divide the two productions of length 3 using variables C and D.

Our final CNF grammar is:

$$
\begin{aligned}
S &\to AA \mid AC \mid BD \mid SS \\
A &\to 0 \\
B &\to 1 \\
C &\to SA \\
D &\to SB
\end{aligned}
$$

Ans 5b) Question was misprinted, the string given to construct mealy and Moore machine was to convert each occurence of 101 by 111 which was not possible, If students could identify it and wrote the answer as follows then it should be considered as correct.

As Mealy and Moore machines are Finite Automata with Output, they can remember through states only. So Mealy and Moore machine can be constructed to change the last bit(MSB) only. Hence not possible to construct the given machine.