Paper Code: 77376.
CBSGS.

Mechanical Vibrations
TE (Mech/Auto).

1 (b). $K = 1066.67$ N/m
$M = 0.1876$ kg

(c) $\omega_n = 80.87$ rad/s
$c = 80870.27$ Ns/m.

(d). $K = 974.77$ N/m.
New $t_p = 0.159$ s.

2 (a). $m_{eq} = m_1 + \frac{I_p}{r_2^2} + m_2\left(\frac{r_1}{r_2}\right)^2$
$k_{eq} = k_1 + k_2\left(\frac{r_1}{r_2}\right)^2$
$c_{eq} = c$.
$\boxed{c = 1816.20 \text{ Ns/m}}$

(b) $m\ddot{x} + \left[\frac{Tl}{a(l-a)}\right]x = 0$.
$\therefore \omega_n = \sqrt{\frac{Tl}{ma(l-a)}}$ rad/s.

3 (a). $K = 3EI/l^3 = 2343750$ N/m.
$\omega_n = 228.218$ rad/s.
$\frac{X}{F_0/k} = MF \quad (\xi = 0)$.
$r = 0.856$ or $r = 1.145$
$\omega = 195.35$ or 256.75
$N < 1865.45$ or > 2451.73 rpm

(b) $f_n = 3.33$ Hz; $\omega_n = 20.94$ rad/s
$K_t = 82.73$ Nm/rad.
$\omega_d = 18.85$ rad/s
$\omega_d = \sqrt{1-\xi^2}\,\omega_n \Rightarrow \boxed{\xi = 0.435}$.
$\boxed{\delta = 3.0964}$.

4 (a) $Error_{(max)}$ when $\frac{dE}{dr} = 0$.
i.e. when $r = r^* = 2.202$.
$\therefore \% Error_{max} = -0.022\%$.

(b). $\omega_{n1} = 0$.
$\omega_{n2}; \omega_{n3} \neq 0$.

5 (a) Coulomb damping.
$\mu = 0.025$.
$F = 5$ N
$\omega_d = \omega_n = 14.14$ rad/s

6 (a). $\alpha_{11} = 0.0392/c$
$(\alpha_{22} = 0.08/c)$
$\alpha_{33} = 0.08/c$
$\alpha_{44} = 0.0392/c$.
Dunkerley's method.
$C = 161.78$.
New $\omega_n = 15.80$ rad/s

Mechanical Vibrations.

Subject Code: 37503; Paper Code: 77576.

Paper Solution:

Sem VI / CBSGS / Mech-Auto / December 10, 2019.

(1). (b). $f_n = 12\ Hz = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

Also; $6\ Hz = \frac{1}{2\pi}\sqrt{\frac{k-800}{m}}$.

$\therefore \frac{6}{12} = \sqrt{\frac{k-800}{\cancel{m}}} \cdot \sqrt{\frac{\cancel{m}}{k}}$.

$\therefore \frac{6}{12} = \sqrt{\frac{k-800}{k}}$.

$\therefore 0.25k = k - 800$.

$\therefore \underline{k = 1066.67\ N/m}$.

$\therefore \underline{m = 0.1876\ kg}$.

(c). $m = 500\ kg$; $\delta_{st} = 1.5\ mm$, $\xi = 1$.

$c = ?$

$\omega_n = \sqrt{g/\delta_{st}} = \underline{80.87\ rad/s}$.

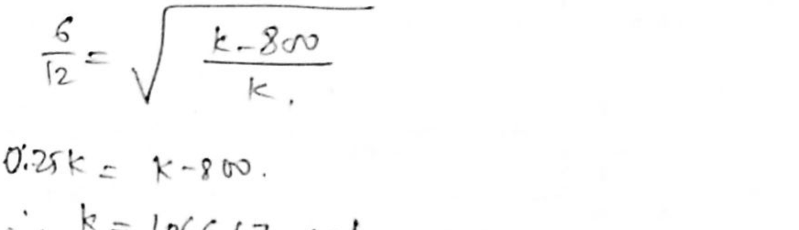
$\frac{c}{m} = 2\xi\omega_n \Rightarrow c = 2m\xi\omega_n = \underline{80,870.27\ Ns/m}$.

(d). $m = 5\ kg$; $t_{p_1} = 0.45\ sec$.

$\therefore 0.45 = 2\pi\sqrt{\frac{5}{k}}$.

$\therefore k = 974.77\ N/m$.

$t_{p_2} = ?$



$\therefore t_{p_2} = 2\pi\sqrt{\frac{2.5}{3899.10}} = \underline{0.159\ s}$.

2(a). $c = ?$

$\xi = 1.25$

$r_1 = 0.1\,m$

$r_2 = 0.3\,m$

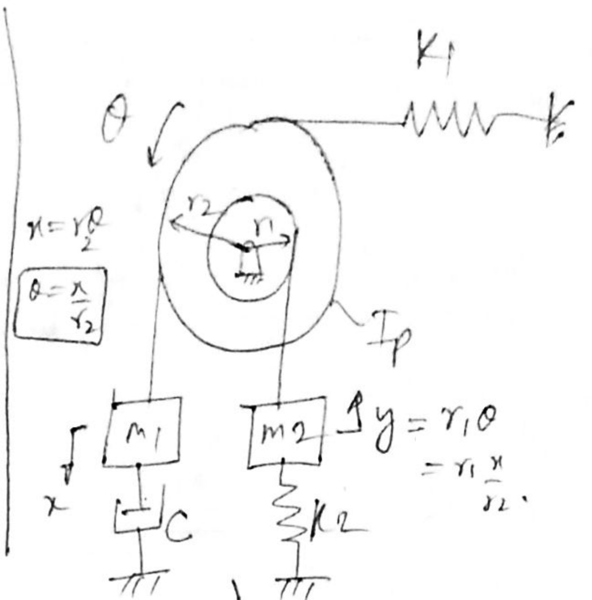
$I_p = 1.1\ kgm^2$

$m_1 = 10\ kg.$

$m_2 = 25\ kg.$

$k_1 = 1\times10^4\ N/m$

$k_2 = 1\times10^5\ N/m.$



$x = r_2\theta$

$\boxed{\theta = \frac{x}{r_2}}$

$y = r_1\theta = r_1\frac{x}{r_2}.$

$$KE = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}I_p\dot{\theta}^2 + \frac{1}{2}m_2\dot{y}^2$$

$$= \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}\left(\frac{I_p}{r_2^2}\right)\dot{x}^2 + \frac{1}{2}m_2\left(\frac{r_1}{r_2}\right)^2\dot{x}^2$$

$$= \frac{1}{2}\underbrace{\left[m_1 + \frac{I_p}{r_2^2} + m_2\left(\frac{r_1}{r_2}\right)^2\right]}_{m_{eq}\ kg}\dot{x}^2$$

$$PE = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2\left(\frac{r_1}{r_2}\right)^2x^2$$

$$= \frac{1}{2}\underbrace{\left[k_1 + k_2\left(\frac{r_1}{r_2}\right)^2\right]}_{k_{eq}\ N/m.}x^2$$

$$WD = -\int_{x_1}^{x_2} c\dot{x}\,dx.$$

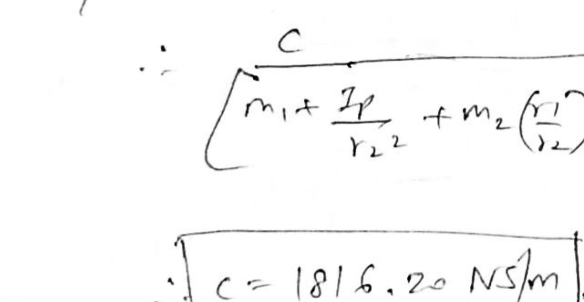
$\therefore\ c_{eq} = c.\ Ns/m.$

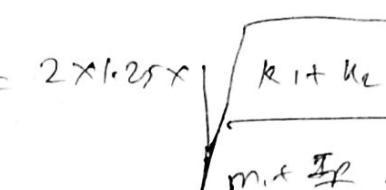
$$\frac{c_{eq}}{m_{eq}} = 2\xi\omega_n$$

$$\therefore \frac{c}{\left[m_1 + \frac{I_p}{r_2^2} + m_2\left(\frac{r_1}{r_2}\right)^2\right]} = 2\times1.25\times\sqrt{\frac{k_1 + k_2\left(\frac{r_1}{r_2}\right)^2}{m_1 + \frac{I_p}{r_2^2} + m_2\left(\frac{r_1}{r_2}\right)^2}}.$$

$\therefore\ \boxed{c = 1816.20\ Ns/m}.$

(b).





from Newton's 2nd law,

$$m\ddot{x} + T(\sin\alpha + \sin\beta) = 0.$$

$$\therefore m\ddot{x} + T\left[\frac{x}{a} + \frac{x}{(l-a)}\right] = 0. \Rightarrow m\ddot{x} + Tx\left[\frac{l-\cancel{a}+\cancel{a}}{a(l-a)}\right] = 0$$

$$\therefore m\ddot{x} + \left[\frac{Tl}{a(l-a)}\right]x = 0.$$

$$\therefore \omega_n = \sqrt{\frac{Tl}{a(l-a)m}}.$$

3 (a). $m = 45$ kg.
$l = 1.6$ m.
$E = 2\times10^{11}$ N/m².
$I = 1.6\times10^{-5}$ m⁴.
$f_0 = 125$ N.
$X \leq 0.2\times10^{-3}$ m.

$\omega = ?$ or $N = ?$

$K = \frac{3EI}{l^3} = 2343750$ N/m.
$\omega_n = \sqrt{K/m} = 228.218$ rad/s.

$$\frac{X}{(f_0/K)} = \frac{1}{\sqrt{(1-r^2)^2 + \cancel{(2\xi r)^2}}}$$

$$\therefore \frac{0.2\times10^{-3}}{(125/2343750)} = \frac{1}{\sqrt{(1-r^2)^2 +}} = \frac{1}{|1-r^2|}$$

$\therefore |1-r^2| = 0.2667$

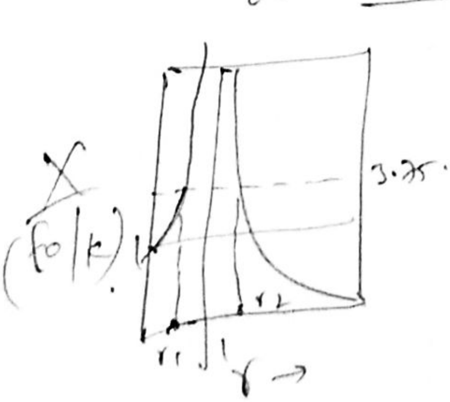
ie $1-r^2 = 0.2667$ or $r^2 - 1 = 0.2667$.

ie; $r = 0.856$ or $r = 1.125$

ie $\omega = 0.856\,\omega_n$ or $\omega = 1.125\,\omega_n$

ie; $\omega = 195.35$ rad/s or $\omega = 256.745$ rad/s

or $N = 1865.45$ rpm or $N = 2451.73$ rpm.



$\therefore N \leq 1865.45$ rpm
or
$N \geq 2451.73$ rpm.

3 (b). Torsional Pendulum.

$$f_n = \frac{200}{60} = 3.33 \text{ Hz}.$$

$$\therefore \omega_n = \underline{20.94 \text{ rad/s}}.$$

$$I = 0.2 \text{ kgm}^2.$$

$$\omega_n = \sqrt{\frac{K_t}{I}} \Rightarrow \underline{K_t = 87.73} \; \underline{\text{Nm/rad}}.$$

$$f_d = \frac{180}{60} = 3 \text{ Hz}.$$

$$\therefore \omega_d = 18.8495 \text{ rad/s}.$$

Now; $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$.

$$\therefore 18.8495 = \sqrt{1-\zeta^2}\,.(20.94)$$

$$\therefore \boxed{\zeta = 0.435}.$$

Now:

$$\theta_0 = 2^\circ = \left(\frac{2\times\pi}{180}\right)^c = \underline{0.0349^c}.$$

$$\dot{\theta}_0 = 0. \text{ rad/s}.$$

$$\delta = \ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}. = \ln\left(\frac{\theta_0}{\theta_1}\right).$$

$$\therefore \ln\left(\frac{2^\circ}{\theta_1^\circ}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow 3.0354.$$

$$\therefore \boxed{\theta_1 = 0.096^\circ}.$$

4(a). Max. % error of vibrometer = ?

$4 \leq r < \infty$, $\xi = 0.63$.

% Error in vibrometer is given by:- (E).

$$E = \frac{\cancel{Y} - \cancel{Z}}{Y} = 1 - \frac{Z}{Y}$$

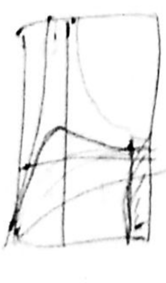
$$\text{Error factor} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

Max error occurs when $dE/dr = 0$.

ie; $r = r^* = \frac{1}{\sqrt{1-2\xi^2}} = \frac{1}{\sqrt{1-2(0.63)^2}}$

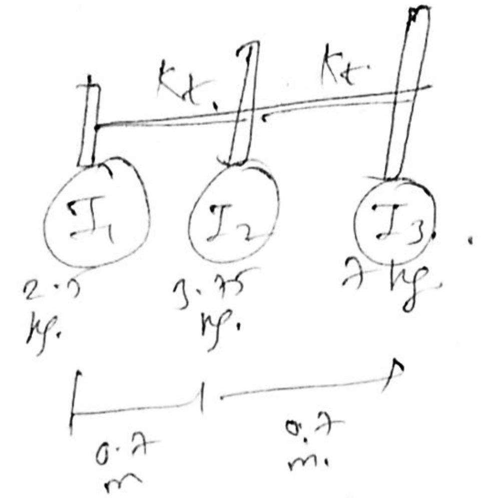
$= 2.202$.

$$\therefore \% \text{ Error} = 1 - \frac{2.202^2}{\sqrt{(1-2.202^2)^2 + (2 \times 0.63 \times 2.202)^2}}$$



$= -0.022\%$.

4(b).



$d = 10$ cm.

ω_n's = ?

$$K_t = \frac{T}{\theta} = \frac{9\times10^8 \times \pi (0.1)^4}{0.7\times 32}$$

$$= \underline{12622.47} \ \frac{Nm}{rad.}$$

$k_1 = 0.2$ m, $k_2 = 0.3$ m, $k_3 = 0.4$ m.

$G = 9\times10^8$ N/m².

$$I_1\ddot{\theta}_1 + K_t(\theta_1 - \theta_2) = 0.$$

$$I_2\ddot{\theta}_2 - K_t(\theta_1 - \theta_2) + K_t(\theta_2 - \theta_3) = 0.$$

$$I_3\ddot{\theta}_3 - K_t(\theta_2 - \theta_3) = 0.$$

$$\therefore \ I_1\ddot{\theta}_1 + K_t\theta_1 - K_t\theta_2 = 0.$$

$$I_2\ddot{\theta}_2 - K_t\theta_1 + 2K_t\theta_2 - K_t\theta_3 = 0$$

$$I_3\ddot{\theta}_3 - K_t\theta_2 + K_t\theta_3 = 0.$$

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} + \begin{bmatrix} K_t & -K_t & 0 \\ -K_t & 2K_t & -K_t \\ 0 & -K_t & K_t \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 3.75 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} + 12622.47\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}.$$

$$[D] = \underbrace{\begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.267 & 0 \\ 0 & 0 & 0.1429 \end{bmatrix}}_{(M)^{-1}}\underbrace{\begin{bmatrix} 12622.47 & -12622.47 & 0 \\ -12622.47 & 2(12622.47) & -12622.47 \\ 0 & -12622.47 & 12622.47 \end{bmatrix}}_{(K)}$$

$$\therefore [D] = \begin{bmatrix} 5049 & -5049 & 0 \\ -3370.2 & 6740.4 & -3370.2 \\ 0 & -1803.75 & 1803.75 \end{bmatrix}.$$

$$\left| D - \lambda [I] \right| = 0$$

$$\therefore \begin{vmatrix} (5049-\lambda) & (-5049) & 0 \\ (-3370.2) & (6740.4-\lambda) & (-3370.2) \\ 0 & (-1803.75) & (1803.75-\lambda) \end{vmatrix} = 0.$$

$\therefore$ ~~$\lambda = 0 \Rightarrow \omega_n = 0$.~~

$$\left[(5049-\lambda)\left\{(6740.4-\lambda)(1803.75-\lambda)\right\} - (3370.2)(1803.75)\right]$$

$$+(5049)(-3370.2)(1803.75-\lambda) = 0.$$

$\therefore \lambda_1 = 0. \Rightarrow \underline{\omega_{n1} = 0}.$

5(a).

$M = 20\ kg.$
$k = 4000\ N/m.$

Coulomb damping.

$\frac{4f}{k} = \Delta = 5\ mm.$

$F = \mu N = \mu mg.$

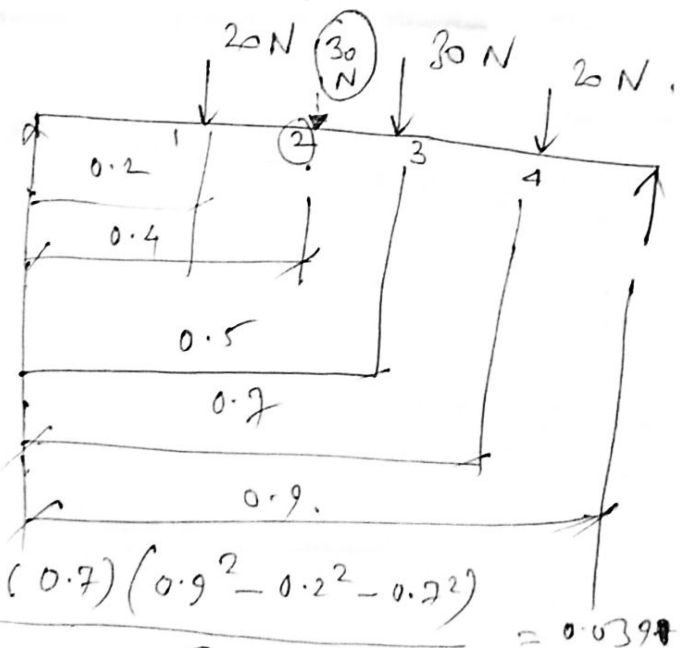
$\therefore \frac{4 \times \mu (20)(9.81)}{4000} = 5 \times 10^{-3}.$

$\therefore \boxed{\mu = 0.025}.$

$\therefore \boxed{F = 5\ N}.$

$\omega_d = \omega_n = \sqrt{\frac{k}{m}} = \boxed{14.14\ rad/s}$

6(a).



$\alpha_{11} = \frac{(0.2)(0.7)(0.9^2 - 0.2^2 - 0.7^2)}{C} = 0.0392/C$

$\alpha_{33} = \frac{(0.5)(0.4)(0.9^2 - 0.5^2 - 0.4^2)}{C} = 0.08/C$

$\alpha_{44} = \frac{(0.7)(0.2)(0.9^2 - 0.7^2 - 0.2^2)}{C} = 0.0392/C.$

$\alpha_{22} = \frac{(0.4)(0.5)(0.9^2 - 0.4^2 - 0.5^2)}{C} = \alpha_{33} = 0.08/C.$

Using Dunkerley's method;

$$y_1 = \alpha_{11} W_1 = \frac{20 \times 0.0392}{c} = 0.784/c.$$

$$y_3 = \alpha_{33} W_3 = \frac{0.08}{c} \times 30 = 2.4/c.$$

$$y_4 = \alpha_{44} W_4 = \frac{0.0392}{c} \times 20 = 0.784/c.$$

$$\therefore \omega_n = \sqrt{\frac{g}{\sum_{i=1,3,4} y_i}} \Rightarrow 20 = \sqrt{\frac{g}{\sum y_i's}}$$

$$\therefore \underline{c = 161.79}.$$

$$\therefore y_1 = \underline{4.845 \times 10^{-3}} \text{ m.} = y_4.$$

$$\therefore \underline{y_3 = 0.0148 \text{ m.}}$$

Now; $y_2 = \alpha_{22} W_2 = \frac{0.08}{c} \times 30 = \frac{2.4}{c} = \underline{0.0148 \text{ m.}}$

$$\therefore \text{New } \omega_n = \sqrt{\frac{g}{\sum_{i=1}^{4} y_i's}} = \boxed{15.80 \text{ rad/s.}}$$

Using Dunkerley's method,

$$y_1 = \alpha_{11} W_1 = 20 \times \frac{0.0392}{c} = 0.784/c.$$

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$$y_4 = \alpha_{44} W_4 = \frac{0.0392}{c} \times 20 = 0.784/c.$$

$$\therefore \omega_n = \sqrt{\frac{g}{\sum_{i=1,3,4} y_i}} \Rightarrow 20 = \sqrt{\frac{g}{\sum y_i}}$$

$$\therefore c = 161.79.$$

$$\therefore y_1 = 4.845 \times 10^{-3} \text{ m} = y_4.$$

$$y_3 = 0.0148 \text{ m}$$

Now, $y_2 = \alpha_{22} W_2 = \frac{0.08}{c} \times 30 = \frac{2.4}{c} = 0.0148$ m.

$$\therefore \text{New } \omega_n = \sqrt{\frac{g}{\sum_{i=1}^{4} y_i}} = \boxed{15.80 \text{ rad/s.}}$$