paper code: 88964 / signals & systems 78088

Q.1 a) $x(n) \leftrightarrow X(z)$, then $Z\{x(n-k)\} \leftrightarrow z^{-k} X(z)$

→ $$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n-k) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n-k) \cdot z^{-(n-k)} \cdot z^{-k}$$

$$= z^{-k} \sum_{n=-\infty}^{\infty} x(n-k) \cdot z^{-(n-k)}$$

Put $n-k = m$

$$= z^{-k} \sum_{m=-\infty}^{\infty} x(m) \cdot z^{-m}$$

$$= z^{-k} X(z)$$

b)

Power signal	Energy signal.
i) signal having finite non-zero power are called as power signals.	i) signals having finite non-zero energy are called energy signals.
ii) All Periodic signals are power signals.	ii) All non-periodic signals are energy signals.
iii) Power signals can exist over an infinite time. Aren't time limited.	iii) energy signals exist over a short period of time i.e they are time limited.
iv) Energy of power signal is infinite.	iv) Power of energy signal is zero.

$x(t) = \cos \omega t$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} \cos^2 \omega t \cdot dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \frac{1+\cos 2\omega t}{2} \cdot dt$$

$$= \frac{1}{2T}\left\{\int_{-T/2}^{T/2} 1 \cdot dt + \int_{-T/2}^{T/2} \cos 2\omega t \cdot dt\right\}$$

$$= \frac{1}{2T}\left\{\left[t\right]_{-T/2}^{T/2} + \frac{1}{2}\left[\sin 2\omega t\right]_{-T/2}^{T/2}\right\}$$

$$= \frac{1}{2T}\left\{\left[\frac{T}{2}-\left(-\frac{T}{2}\right)\right] + \frac{1}{2}\left[\sin 2\omega \frac{T}{2} - \sin 2\omega \frac{T}{2}\right]\right\}$$

$$= \frac{1}{2T}\left\{\left[T\right] + \frac{1}{2}\left[\sin \omega T - \sin \omega T\right]\right\}$$

$$= \frac{1}{2} \qquad \therefore \text{power signal.}$$

Q:1 c) $y(t) = e^{t \cdot x(t)}$

i) Memoryless → As past samples aren't present.

ii) Linearity :- Apply zero i/p then o/p is $y(t) = e^{0} = 1$

$x_1(t) \xrightarrow{T} y_1(t) = e^{t \cdot x_1(t)}$

$x_2(t) \xrightarrow{T} y_2(t) = e^{t \cdot x_2(t)}$

$y'(t) = y_1(t) + y_2(t) = e^{t \cdot x_1(t)} + e^{t \cdot x_2(t)} \longrightarrow (a)$

$[x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = e^{t[x_1(t) + x_2(t)]}$

$\therefore y''(t) = e^{t \cdot x_1(t)} \cdot e^{t \cdot x_2(t)} \longrightarrow (b)$

Since $y'(t) \neq y''(t)$ sym is non-linear.

iii) Time invariant :-

Delay i/p by k units & denote o/p by $y(t, k)$

$y(t, k) = e^{t \cdot x(t-k)} \longrightarrow (a)$

Replace 't' by $(t-k)$ throughout the eq^n.

$\therefore \quad y(t-k) = e^{(t-k) \cdot x(t-k)} \longrightarrow (b)$

Since $y(t, k) \neq y(t-k)$ sym is time variant.

iv) Causal — sym doesn't contain any future term so it is causal system.

v) Stable :- As i/p is bounded, the o/p of system is always bounded. Thus, it is stable system.

Q.1 d) $x(t) = \sin^2(200\pi t)$

$$= \left[\frac{1 - \cos 2(200\pi t)}{2}\right]$$

$$= \frac{1}{2} - \frac{1}{2}\cos(400\pi t)$$

comparing with, $x(t) = A_0 - A_1 \cos \omega_1 t$

$\therefore$ $\omega_1 = 400\pi$

$2\pi f_1 = 400\pi$

$f_1 = 200$ (max^m freq)

$\therefore$ Nyquist rate $= 2 f_{max}$

$= 2 \times 200 = 400 Hz$

Q.2 a) i) $x(t) = 2\cos\left(\frac{2\pi}{3}\right)t + 3\cos\left(\frac{2\pi}{7}\right)t$

comparing with,

$x(t) = A\cos\omega_1 t + A\cos\omega_2 t$

$\therefore$ $\omega_1 = 2\pi f_1 = \frac{2\pi}{3}$ & $\omega_2 = 2\pi f_2 = \frac{2\pi}{7}$

$\therefore$ $f_1 = \frac{1}{3}$ $\therefore$ $f_2 = \frac{1}{7}$

$\therefore$ $T_1 = 3$ $T_2 = 1/f_2 = 7$

$\therefore$ since $\frac{T_1}{T_2}$ is rational number. It is a periodic signal.

$\therefore$ $\frac{T_1}{T_2} = \frac{3}{7}$ $\Rightarrow$ fundamental period is 21.

$\therefore$ $T = 21$

iii) $x(n) = \cos^2\left(\frac{\pi}{4}n\right)$ As $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\therefore$ $x(n) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{2}n\right)$

First term is const. Comparing 2^{nd} term with,

$$\therefore \omega n = \frac{\pi}{2}n$$

$$2\pi f n = \frac{\pi}{2}n$$

$$\therefore f = \frac{1}{4} = \frac{k}{N}$$

Ratio is of two integers. so it is a periodic signal with period $N = 4$ samples.

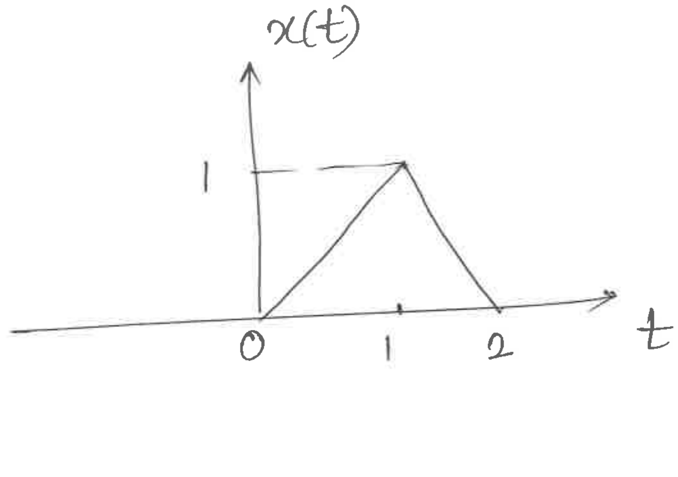
Q.2 b) $x(t) = \begin{cases} t & , 0 \le t \le 1 \\ 2-t & , 1 \le t \le 2 \end{cases}$

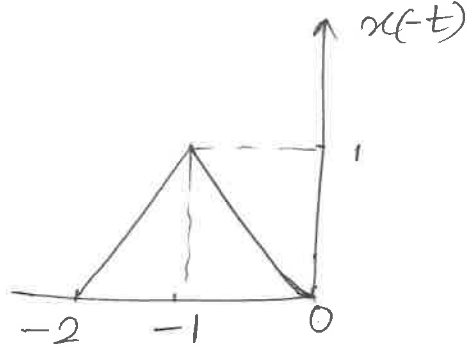
→ 0 to 1, it is ramp wave.

for $2-t$, $1 \le t \le 2$

$\therefore$ $t=1 \Rightarrow 2-1 = 1$
$t=1.5 \Rightarrow 2-1.5 = 0.5$
$t=2 \Rightarrow 2-2 = 0$

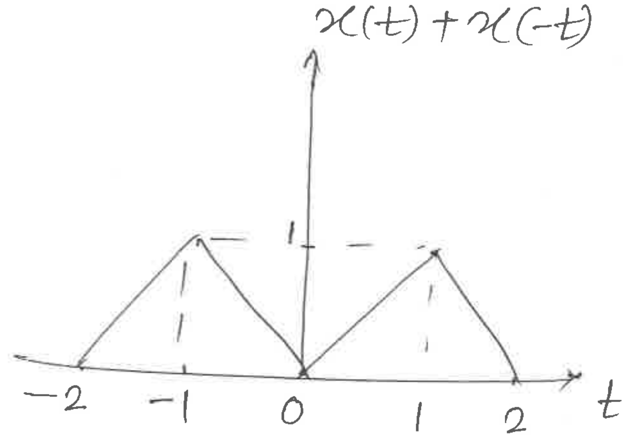
signal, $x(t)$ is

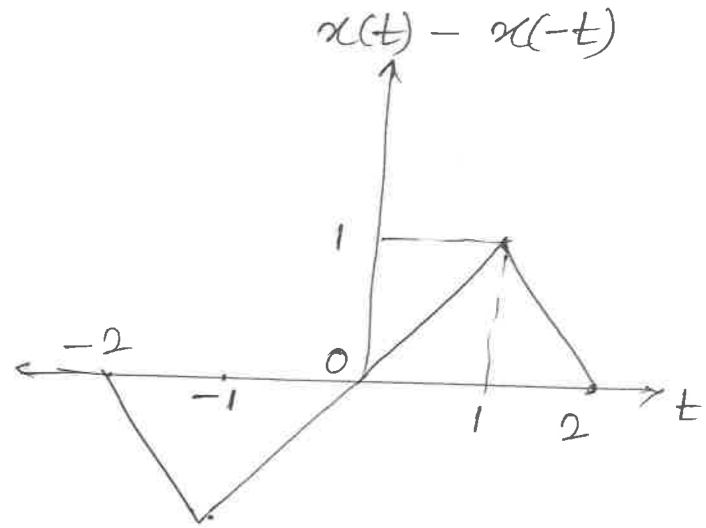


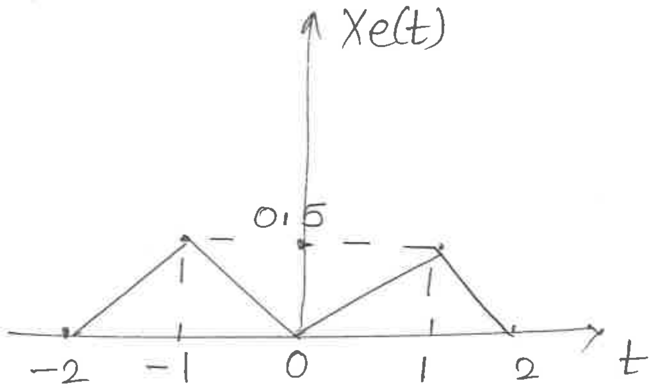


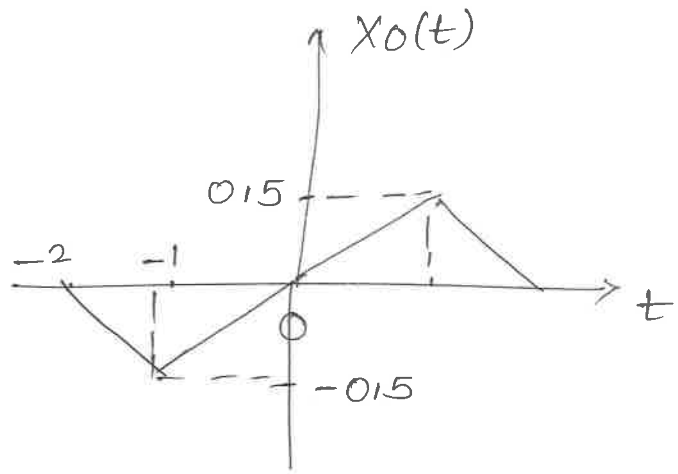
$$X_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$X_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$









Q.2 c) $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-1)$

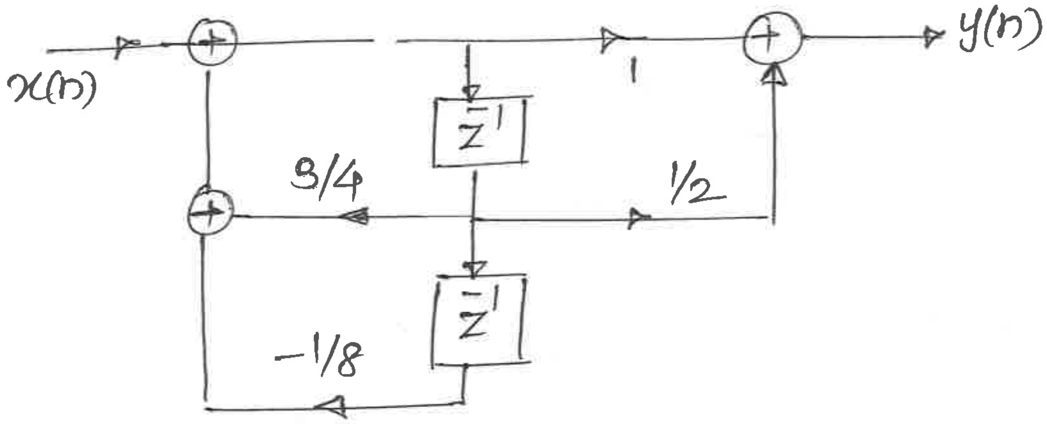
Taking Z.T on both sides,

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}\cdot Y(z) = X(z) + \frac{1}{2}z^{-1}\cdot X(z)$$

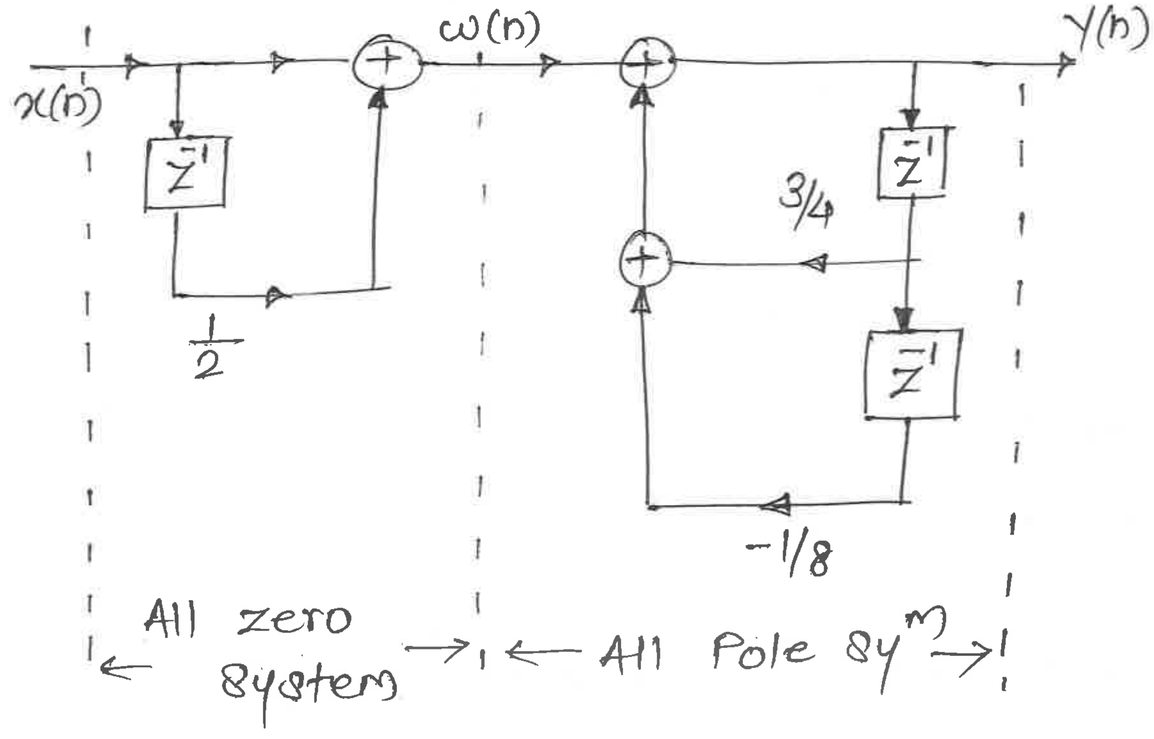
$$\therefore Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X(z)\left[1 + \frac{1}{2}z^{-1}\right]$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Direct form - II Realization :-



Direct form - I Realization :-



Q.3 a) $X(z) = \dfrac{1 - 2z^{-1}}{1 - \frac{7}{12}z^{-1} + \frac{1}{12}z^{-2}}$

$\longrightarrow \quad X(z) = \dfrac{z^2 - 2z}{z^2 - \frac{7}{12}z + \frac{1}{12}}$

$\therefore \dfrac{X(z)}{z} = \dfrac{z-2}{z^2 - \frac{7}{12}z + \frac{1}{12}}$

$= \dfrac{z-2}{\left(z-\frac{1}{3}\right)\left(z-\frac{1}{4}\right)}$

$\therefore \dfrac{X(z)}{z} = \dfrac{A}{\left(z-\frac{1}{3}\right)} + \dfrac{B}{\left(z-\frac{1}{4}\right)}$

$\therefore$ Using P.F.E, we get $A = -20$ &
$B = 21$

$\therefore \dfrac{X(z)}{z} = \dfrac{-20}{z-\frac{1}{3}} + \dfrac{21}{z-\frac{1}{4}}$

$\therefore X(z) = \dfrac{-20\,z}{z-\frac{1}{3}} + \dfrac{21\,z}{z-\frac{1}{4}}$

Differnt ROCs :-

1) ROC: $|z| > \frac{1}{3}$

$x(n) = -20\left(\frac{1}{3}\right)^n u(n) + 21\left(\frac{1}{4}\right)^n u(n)$

ii) ROC: $|z| > \frac{1}{4}$

$$\therefore x(n) = 20\left(\frac{1}{3}\right)^n \cdot u(-n-1) + 21\left(\frac{1}{4}\right)^n \cdot u(-n-1)$$

iii) ROC: $\frac{1}{4} < |z| < \frac{1}{3}$

$$\therefore x(n) = 20\left(\frac{1}{3}\right)^n \cdot u(-n-1) + 21\left(\frac{1}{4}\right)^n \cdot u(n)$$

Q.3 b) $x_1(t) = u(t)$, $x_2(t) = e^{-t} \cdot u(t)$

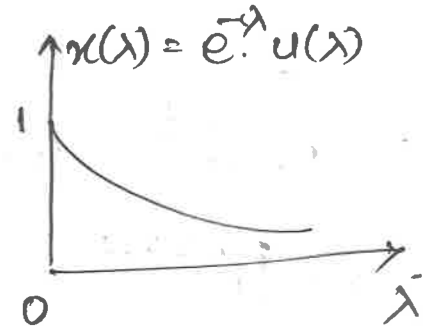
$\rightarrow$ Let $x_1(t) = u(t)$, $h(t) = u(t)$

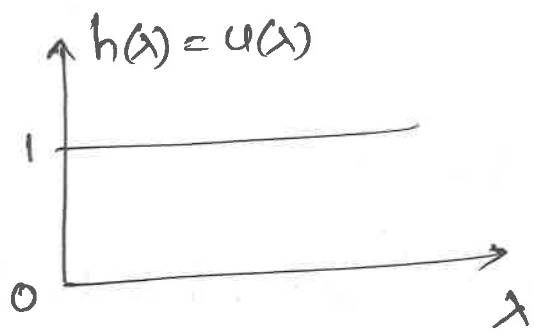
& $x_2(t) = e^{-t} \cdot u(t)$, $x(t) = e^{-t} \cdot u(t)$

As eq^n of convolution, $y(t) = x(t) * h(t)$

$$\therefore y(t) = \int_{-\infty}^{\infty} x(\lambda) \cdot h(t-\lambda) \cdot d\lambda \longrightarrow ①$$

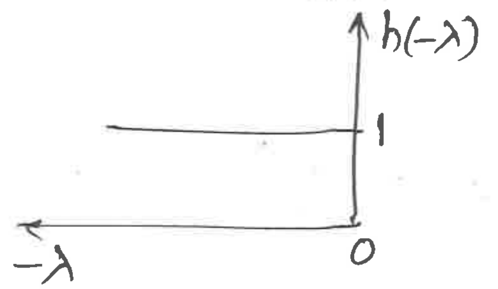
So, $x(\lambda) = e^{-\lambda} \cdot u(\lambda)$ & $h(\lambda) = u(\lambda)$.





i) first Put $t=0$ in eq^n ①,

$$\therefore y(0) = \int_{-\infty}^{\infty} x(\lambda) \cdot h(-\lambda) \cdot d\lambda$$



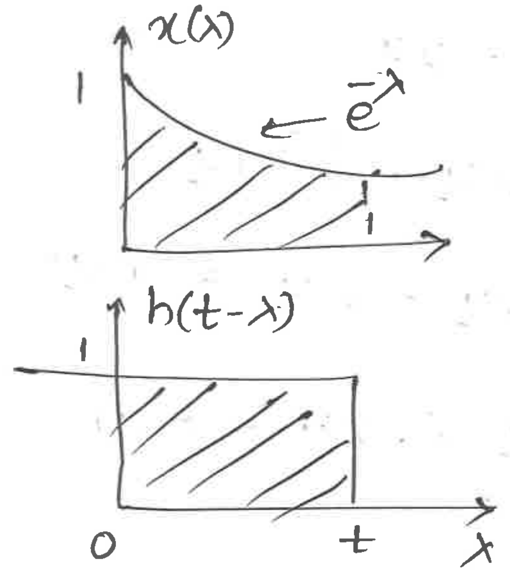
The result of integration of mul^n of $x(\lambda)$ and $h(-\lambda)$, as there is no overlap.

$\therefore \; y(0) = 0$

ii) As $t > 0$ eq^n ① becomes,

$$\therefore \quad y(t) = \int_{-\infty}^{\infty} x(\lambda) \cdot h(t-\lambda) \cdot d\lambda$$

there is a overlapping.



$$\therefore \; y(t) = \int_0^t x(\lambda) \cdot 1 \cdot d\lambda$$

$$= \int_0^t e^{-\lambda} \cdot d\lambda$$

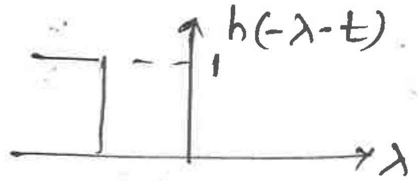
$$= \left[-e^{-\lambda}\right]_0^t$$

$$= -\left[e^{-t} - e^{0}\right]$$

$$= 1 - e^{-t} \quad \text{for } t > 0$$

iii) for $t < 0$.

$$\therefore \; y(t) = \int_{-\infty}^{\infty} x(\lambda) \cdot h(-t-\lambda) \cdot d\lambda$$



there is no overlapping. The Result of convolution is zero. $\therefore \; y(t) = 0$ for $t < 0$

output of convolution is, $y(t) = \begin{cases} 1-e^{-t} & \text{for } t>0 \\ 0 & \text{for } t \le 0 \end{cases}$

Q.4 a) $y''(t) + 3y'(t) + 2y(t) = 2x(t)$

i) Taking L.T on both sides,

$$s^2 \cdot Y(s) + 3s\,Y(s) + 2Y(s) = 2X(s)$$

$$\therefore \quad Y(s)\left[s^2 + 3s + 2\right] = 2X(s) \longrightarrow (1)$$

$$\therefore \quad \boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{2}{(s+1)(s+2)}}$$

ii) Using P.F.E,

$$H(s) = \frac{2}{s+1} + \frac{-2}{s+2}$$

I.L.T

$$\boxed{h(t) = 2e^{-t}\cdot u(t) - 2e^{-2t}\cdot u(t)}$$

iii) $y(t) = ?$ if $x(t) = 4e^{-3t}\cdot u(t)$

$$\therefore \quad X(s) = \frac{4}{s+3}$$

Putting '$X(s)$' value in eq^n (1)

$$\therefore \quad Y(s)\left[s^2+3s+2\right] = 2\cdot\frac{4}{s+3}$$

$$\therefore \quad Y(s) = \frac{8}{(s+1)(s+2)(s+3)}$$

Using P.F.E.,

$$\therefore Y(s) = \frac{-4}{s+1} - \frac{8}{s+2} + \frac{4}{s+3}$$

$\because$ I.L.T,

$$\therefore y(t) = \left[-4e^{-t} - 8.e^{-2t} + 4e^{-3t}\right] \cdot u(t)$$

Q.4 b) $h(n) = \{1, 2, 1, -1\}$ $x(n) = \{1, 2, 3, 1\}$

$\rightarrow$ $y(n) = x(n) * h(n)$

$$Y(z) = X(z) \cdot H(z)$$

$$X(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n} = \sum_{n=0}^{3} x(n) \cdot z^{-n}$$

$$= 1 + 2z^{-1} + 3z^{-2} + z^{-3}$$

$$H(z) = \sum_{n=0}^{3} h(n) \cdot z^{-n} = 1 + 2z^{-1} + z^{-2} - z^{-3}$$

$$\therefore Y(z) = X(z) \cdot H(z)$$

$$= (1 + 2z^{-1} + 3z^{-2} + z^{-3}) \cdot (1 + 2z^{-1} + z^{-2} - z^{-3})$$

$$= 1 + 4z^{-1} + 8z^{-2} + 8z^{-3} + 3z^{-4} - 2z^{-5} - z^{-6}$$

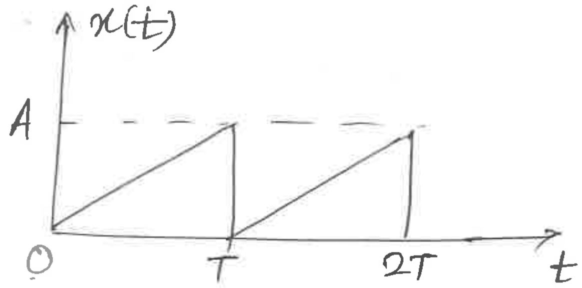
By defn of one-sided Z.T,

$$\therefore Y(z) = \sum_{n=0}^{\infty} y(n) \cdot z^{-n}$$

$$= y(0) \cdot z^{0} + y(1) \cdot z^{-1} + y(2) \cdot z^{-2} + y(3) \cdot z^{-3} + \dots$$

Res. $y(n) = \{\underset{\uparrow}{1}, 4, 8, 8, 3, -2, -1\}$

Q. 5 a)



Given signal is neither even nor odd signal.

$$x(t) = \frac{At}{T} \quad \text{for } t = 0 \text{ to } T.$$

$$a_0 = \frac{2}{T}\int_0^t x(t) \cdot dt = \frac{2}{T}\int_0^T \frac{At}{T} \cdot dt$$

$$= \frac{2A}{T^2}\left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{T^2}{2}\right] = A$$

$$a_n = \frac{2}{T}\int_0^T x(t) \cdot \cos n\Omega_0 t \cdot dt$$

$$= \frac{2}{T}\int_0^t \frac{At}{T} \cos n \cdot \Omega_0 t \cdot dt$$

$$= \frac{2A}{T^2}\int_0^t t \cdot \cos n\Omega_0 t \cdot dt$$

$$= \frac{2A}{T^2}\left[t \cdot \left(\frac{\sin n\Omega_0 t}{n\Omega_0}\right) - \int 1 \times \left(\frac{\sin n\Omega_0 t}{n\Omega_0}\right) dt\right]_0^T$$

$$a_n = \frac{2A}{T^2}\left[t \cdot \frac{\sin n\Omega_0 t}{n\Omega_0} - \left(\frac{-\cos n\Omega_0 t}{n^2\Omega_0^2} \right) \right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{t \cdot \sin n \cdot \frac{2\pi}{T} t}{n \cdot \frac{2\pi}{T}} + \frac{\cos n \cdot \frac{2\pi}{T} \cdot t}{n^2 \frac{4\pi^2}{T^2}} \right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{T \cdot \sin n \cdot \frac{2\pi}{T} \cdot T}{n \cdot \frac{2\pi}{T}} + \frac{\cos n \cdot \frac{2\pi}{T} \cdot T}{n^2 \cdot \frac{4\pi^2}{T^2}} \right]$$

$$= \frac{2A}{T^2}\left[\frac{T^2}{2n\pi} \cdot \sin n \cdot 2\pi + \frac{T^2}{4n^2\pi^2} \cos n 2\pi - 0 - \frac{T^2}{4n^2\pi^2} \right]$$

$$= \frac{2A}{T^2}\left[0 + \frac{T^2}{4n^2\pi^2} - \frac{T^2}{4n^2\pi^2} \right] = 0$$

$$b_n = \frac{2}{T}\int_0^T x(t) \sin \cdot n\Omega_0 t \cdot dt$$

$$= \frac{2}{T}\int_0^T \frac{At}{T} \sin n\Omega_0 t \cdot dt = \frac{2A}{T^2}\int_0^t t \sin n\Omega_0 t \cdot dt$$

$$= \frac{2A}{T^2}\left[t\left(\frac{-\cos n\Omega_0 t}{n\Omega_0} \right) - \int 1 \times \left(\frac{-\cos n\Omega_0 t}{n\Omega_0} \right) dt \right]_0^T$$

$$= \frac{2A}{T^2}\left[-t \frac{\cos n\Omega_0 t}{n\Omega_0} + \frac{\sin n\Omega_0 t}{n^2\Omega_0^2} \right]_0^T$$

$$= \frac{2A}{T^2}\left[-t\cdot\frac{\cos n\frac{2\pi}{T}t}{n\cdot\frac{2\pi}{T}} + \frac{\sin n\cdot\frac{2\pi}{T}t}{n^2\frac{4\pi^2}{T^2}}\right]_0^T$$

$$= \frac{2A}{T^2}\left[-\frac{T\cdot\cos n\frac{2\pi}{T}\cdot T}{n\cdot\frac{2\pi}{T}} + \frac{\sin n\cdot\frac{2\pi}{T}\cdot T}{n^2\cdot\frac{4\pi^2}{T^2}} + 0\right]$$

$$= \frac{2A}{T^2}\left[-\frac{T^2}{2n\pi}\cos n2\pi + \frac{T^2}{4n^2\pi^2}\sin n\cdot 2\pi\right] = \frac{-A}{n\pi}$$

$$\therefore b_1 = \frac{-A}{\pi},\quad b_2 = \frac{-A}{2\pi},\quad b_3 = \frac{-A}{3\pi},\quad b_4 = \frac{-A}{4\pi},\ \ldots$$

The Trignometric F.S of $x(t)$ is given,

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cdot\cos n\Omega_0 t + \sum_{n=1}^{\infty} b_n\cdot\sin n\Omega_0 t$$

As $a_n = 0$

$$\therefore x(t) = \frac{a_0}{2} + b_1\sin\Omega_0 t + b_2\sin 2\Omega_0 t + b_3\sin 3\Omega_0 t + \ldots$$

$$\therefore x(t) = \frac{A}{2} - \frac{A}{\pi}\sin\Omega_0 t - \frac{A}{2\pi}\sin 2\Omega_0 t - \frac{A}{3\pi}\sin 3\Omega_0 t - \ldots$$

$$= \frac{A}{2} - \frac{A}{\pi}\left[\frac{\sin\Omega_0 t}{1} + \frac{\sin 2\Omega_0 t}{2} + \frac{\sin 3\Omega_0 t}{3} + \ldots\right]$$

Q. 5 b) signal is expressed as,

$$x(t) = A \cdot u(t) + A \cdot u\left(t - \frac{T}{4}\right) - A \cdot u\left(t - \frac{3T}{4}\right) - A \cdot u(t-T)$$

As $u(t) \xleftrightarrow{FT} \frac{1}{j2\pi f}$ As T.S. property,

$A \cdot u(t) \longleftrightarrow \frac{A}{j2\pi f}$ $x(t-t_0) \longleftrightarrow e^{-j2\pi f t_0} \cdot X(f)$

$$\therefore A \cdot u\left(t - \frac{T}{4}\right) \longleftrightarrow e^{-j2\pi f\left(\frac{T}{4}\right)} \cdot \frac{A}{j2\pi f}$$

$$= e^{j\frac{\pi f T}{2}} \cdot \frac{A}{j2\pi f}$$

$$\therefore A \cdot u\left(t - \frac{3T}{4}\right) \longleftrightarrow e^{-j2\pi f\left(\frac{3T}{4}\right)} \cdot \frac{A}{j2\pi f}$$

$$= e^{-j\frac{3\pi f T}{2}} \cdot \frac{A}{j2\pi f}$$

$$\because A \cdot u(t-T) \longleftrightarrow e^{-j2\pi f T} \cdot \frac{A}{j2\pi f}$$

$$\therefore X(f) = \frac{A}{j2\pi f} + e^{-j\frac{\pi f T}{2}} \cdot \frac{A}{j2\pi f} + e^{-j\frac{3\pi f T}{2}} \cdot \frac{A}{j2\pi f} + e^{-j2\pi f T} \cdot \frac{A}{j2\pi f}$$

$$= \frac{A}{j2\pi f}\left[1 + e^{-j\frac{\pi f T}{2}} + e^{-j\frac{3\pi f T}{2}} + e^{-j2\pi f T}\right]$$

Q.6 a) Parseval's Th^m of F.S :- $P = \sum_{n=-\infty}^{\infty} |C_n|^2$

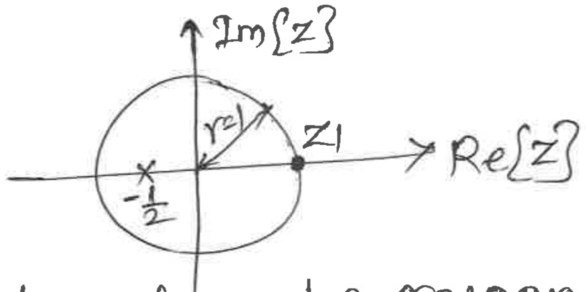
b) $sgn(t) \xleftrightarrow{F.T} \frac{2}{j\omega}$

c) $y(n) + \frac{1}{2} y(n-1) = x(n) - x(n-1)$

$Y(z) + \frac{1}{2} z^{-1} \cdot Y(z) = X(z) - z^{-1} \cdot X(z)$

$\therefore Y(z)\left[1 + \frac{1}{2} z^{-1}\right] = X(z)\left[1 - z^{-1}\right]$

$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{z-1}{z+\frac{1}{2}}$ $\quad \therefore z_1 = 1 \;\&\; P_1 = -\frac{1}{2}$



d) Stability:-

LTI system is stable if it's impulse response is absolutely summable.

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad , \quad \text{BIBO}$$

Causality:-

- LTI sym is causal if o/p of system depends only on present & past inputs o/p, and not on the future inputs and outputs.
- LTI sym is causal if it's impulse response $h(n) = 0$ for $n < 0$