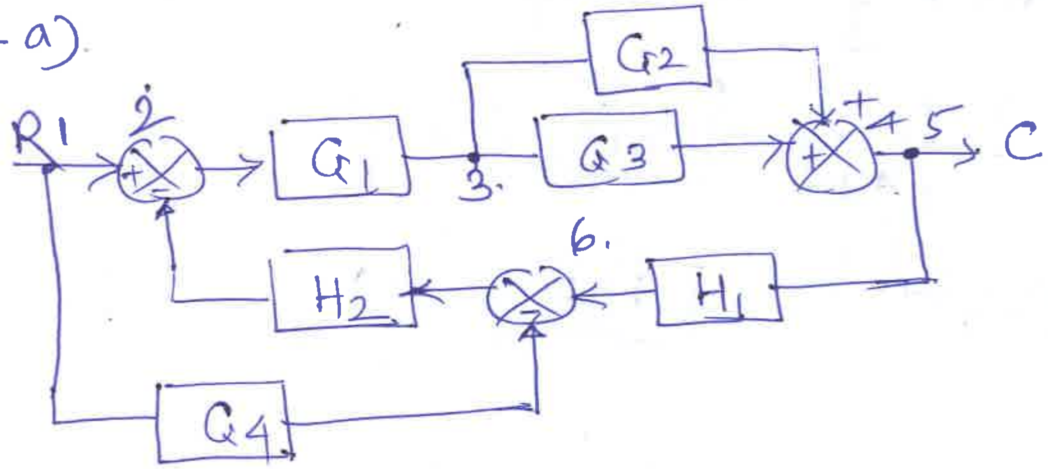
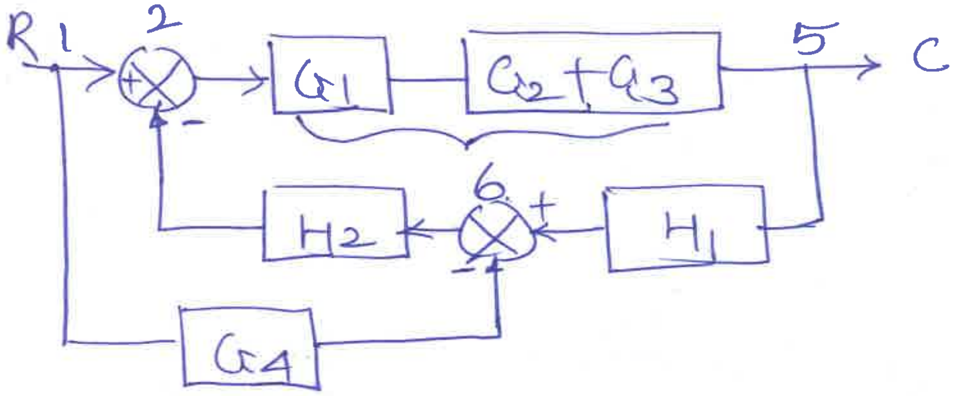
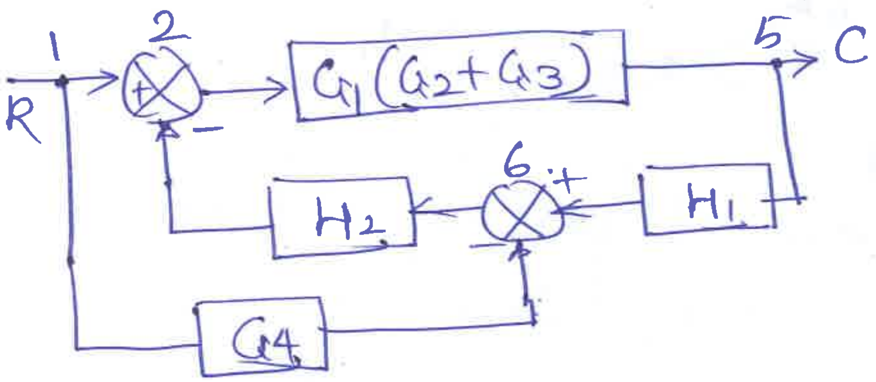
Q. 2 a)



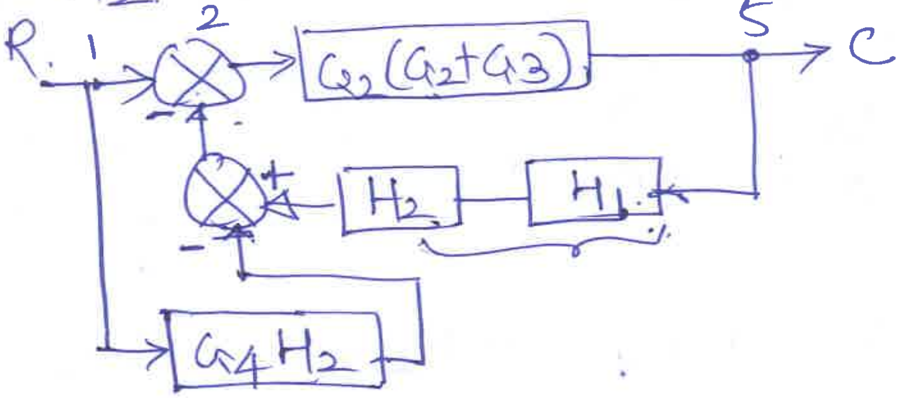
Sol^n: 1 G_2 & G_3 blocks in ~~series~~ parallel



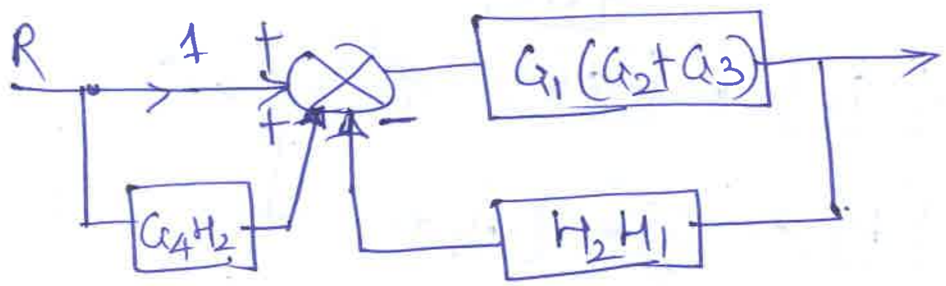
2 G_1 & G_2+G_3 are blocks in series

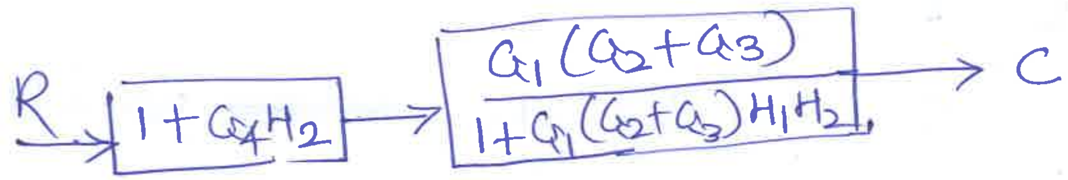


3 shifting summing block ahead of H_2

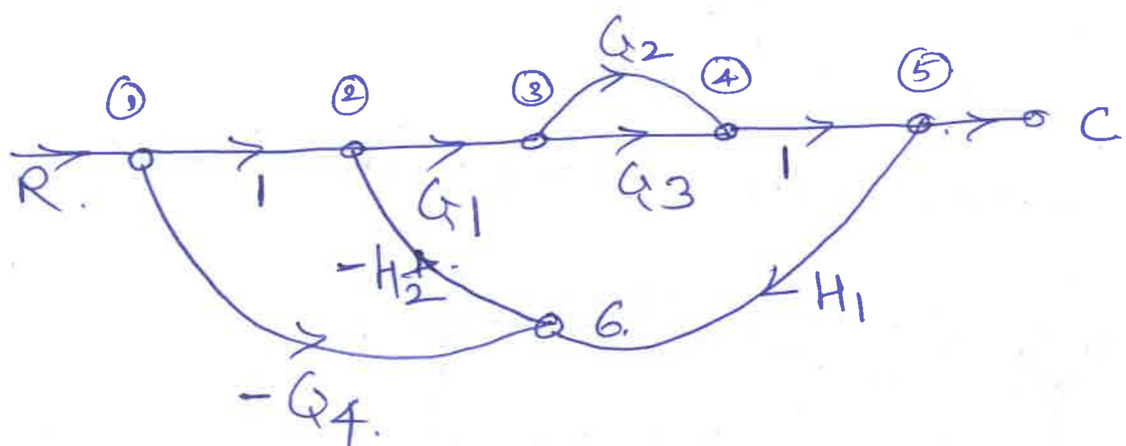


Combining summing blocks H_1H_2 in series ②.





2b) SFG :



No. of forward paths → 4.

$P_1 = G_1G_3$ $\quad P_2 = G_1G_2$

$P_3 = G_4H_2G_1G_3$ $\quad P_4 = G_4H_2G_1G_2$.

No. of loops → 2

$L_1 = -G_1G_3H_1H_2$ $\quad L_2 = -G_1G_2H_1H_2$.

Both the loops are touching the forward paths.

$$\Delta = 1-(L_1+L_2) = 1+G_1G_3H_1H_2+G_1G_2H_1H_2$$
$$= 1+G_1(G_2+G_3)H_1H_2.$$

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1.$$

$$\therefore \frac{C}{R} = \frac{G_1G_2 + G_1G_3 + G_1G_2G_4H_2 + G_1G_3G_4H_2}{\Delta}.$$

$$= \frac{G_1(G_2+G_3) + G_1G_4H_2(G_2+G_3)}{\Delta}$$

$$\frac{C}{R} = \frac{(G_2+G_3)G_1[1+G_4H_2)}{\Delta}$$

3a)
$$\dot{x} = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{B.} u. \quad y = \underbrace{[0 \ \ 1]}_{C} x$$

④

For controllability check
Controllability Matrix $U_c = [B : AB]$

$$AB = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$U_c = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} \qquad |U_c| = -1-3 = -4 \neq 0.$$

$\therefore$ Rank of $U_c = 2$ and hence system is completely controllable.

Observability Matrix $U_0 = \begin{bmatrix} C \\ CA \end{bmatrix}$

$$CA = [0 \ \ 1] \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} = [1 \ \ -2]$$

$$U_0 = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \qquad |U_0| = 0 - 1 = -1 \neq 0.$$

$\therefore$ Rank of U_0 is 2 and hence the system is completely observable.

3b)
$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A.} x + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u, \quad y = \underbrace{[1 \ \ 0]}_{C} x$$

Transfer f^n T.F. $= C[sI - A]^{-1} B$.

$$[sI-A]^{-1} \longrightarrow \quad s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = sI - A$$

$$[sI-A] = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

(3) $|SI-A| = s(s+3)+2 = s^2+3s+2.$

$$[SI-A]^{-1} = \frac{1}{|SI-A|}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \qquad B=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C\,[SI-A]^{-1} = \frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & 1 \end{bmatrix}$$

$$\text{T.F.} = C[SI-A]^{-1}B = \frac{s+3}{s^2+3s+2}$$ is the T.F.

4 a)Sketch Bode plot for a system whose open loop transfer function is

$$G(s) = \frac{0.354(s+1)(1+0.05s)}{s(1+0.025s)}$$

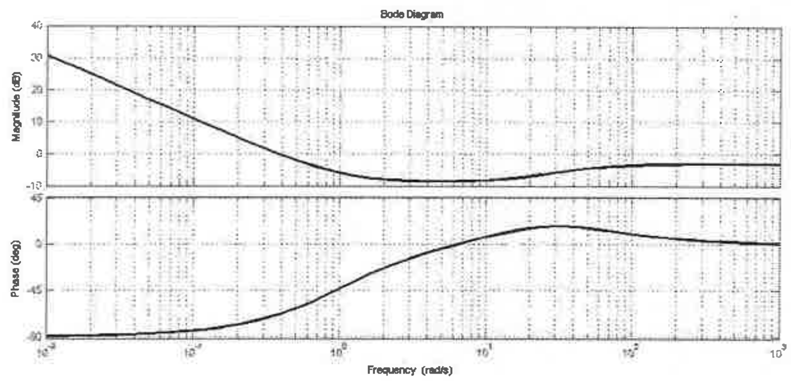
Ans:

Replace s by jw

Bode Magnitude Plot:

There are 4 factors, $\frac{0.354}{jw}$, $\frac{1}{(1+0.025\text{w})}$, $(1+jw)$, $(1+0.05jw)$ of G(jw)
Corner frequencies are 1, 20 and 40
First factor Bode plot is straight line with a slope of -20dB/decade passing through -9dB at w=1
At w=1, there is a zero, so slope changes by 20db/decade, net slope=0;
At w=20, zero, slope changes by 20 dB/decade, net slope=20dB/decade;
At w=40, pole, slope changes by -20 dB/decade, net slope=0;

Bode Phase Plot: $\angle G(jw)H(jw) = -90 + \tan^{-1}(w) + \tan^{-1}(0.05w) + \tan^{-1}(0.025w)$
Plots are as shown:



OR

4 b) Sketch root locus for a unity feedback system, whose open loop transfer function is given as $G(s) = \frac{K}{s(s+2)(s+4)}$.

Ans:

1. There are 3 poles (n=3) and no zeroes (m=0), so three branches of root locus originate at open loop poles and terminate on infinity. Branches shown in different colour.

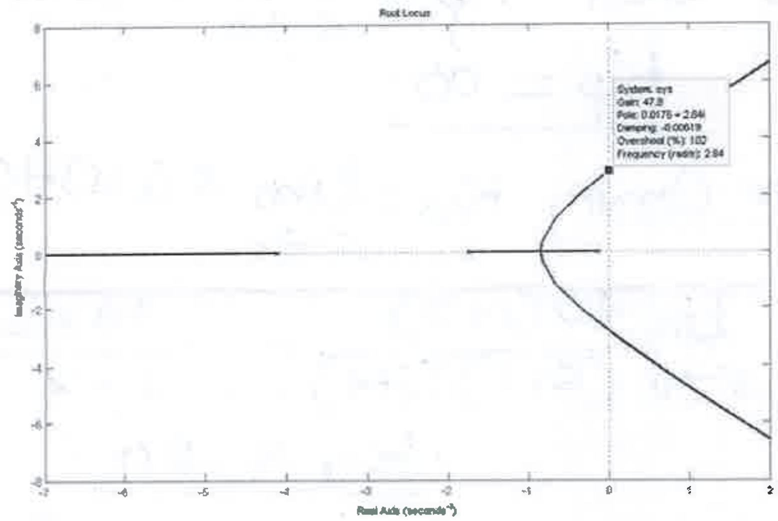
2. Angles of asymptotes $\phi_A = \pm\frac{(2q+1)180^0}{n-m}$ $q = 0,1...(n-m-1)$. These angles are 60, 180 and 300 degrees respectively

3. Centroid, the point of intersection of these asymptotes

$$\sigma_A = \frac{\sum \text{Real part of poles} - \sum \text{Real part of zeros}}{\text{n-m}}$$

$$= \frac{-2-4}{3} = -2$$

4. Breakaway point obtained by solving $\frac{dK}{ds} = 0$



5a) $G(s) = \frac{K}{s(s^2+4s+13)}$ → OLTF

Char. eqn is $1 + G(s)H(s) = 0$.

$$1 + \frac{K}{s(s^2+4s+13)} = 0. \quad (\because H(s) = 1)$$

$s^3 + 4s^2 + 13s + K = 0$. which can be arranged in an array.

Routh Array:

s^3	1	13.
s^2	4	K
s^1	$\frac{-K+52}{4}$	0.
s^0	K	

For stability $K > 0$.

$-K + 52 > 0$.

$52 > K$.

$\therefore$ Range of 'K' for stability is

$\underline{0 < K < 52}$

⑨

6a) $G(s) = \dfrac{40(s+2)}{s(s+1)(s+4)}$ $H(s) = 1.$

a) Type of system → Type 1

b) Static Error Coefficients.

Position Error Coeff $K_p = \lim_{s \to 0} G(s)H(s)$

$$\underline{K_p = \infty}$$

Velocity Error Const. $K_v = \lim_{s \to 0} sG(s)H(s)$

$$\lim_{s \to 0} \frac{40(s+2)}{(s+1)(s+4)} = \frac{40 \times 2}{1 \cdot 4}$$

$$\underline{K_v = 20.}$$

Acceleration error const. $K_a = \lim_{s \to 0} s^2 G(s)H(s)$

$$\lim_{s \to 0} \frac{40 s^{\cancel{2}}(s+2)}{\cancel{s}(s+1)(s+4)} = 0. \quad , \underline{K_a = 0.}$$

c) Steady state error for ramp input with magnitude $\underline{4}$.

$$R(s) = \frac{4}{s^2}$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$ → Laplace of $e(t)$ i.e. error.

Steady state error $e_{ss} = \lim_{s \to 0} sE(s).$

$$= \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)} = \lim_{s \to 0} \frac{s \cdot 4/s^2}{1+G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{4}{s + sG(s)H(s)} = 4/K_v.$$

$$e_{ss_{ramp}} = 4/20.$$