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SS new
Sol Set 2

SE (Electronics and Telecommunication Engg.) Sem IV (CBCGS/T1434) (R2016)

T10020/Signals and Systems

Solution

3 Hours

Total marks: 80

- Question no. 1 is compulsory
- Attempt any Three questions from remaining

Q1. Answer any 4 questions from the given questions:

a. If system matrix A= [-3, 1 ; -2, 0] find the sate transition matrix.

$$\dot{q} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} q + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x(t)$$

$$(sI - A) = \begin{bmatrix} (s+3) & 1 \\ 2 & s \end{bmatrix}$$

The STM is

$$\phi(s) = [sI - A]^{-1} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$Q(s) = [sI - A]^{-1} q(0) + [sI - A]^{-1} B X(s)$$

$$(sI - A)^{-1} q(0) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} (s-1) \\ -(s+5) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(s-1)}{(s+1)(s+2)} \\ \frac{(s+5)}{(s+1)(s+2)} \end{bmatrix}$$

b. Find the fundamental frequency of the following continuous signal:

$$x(t) = cos(\frac{10\pi}{3}t) + sin(\frac{5\pi}{4}t)$$

The frequencies and periods of the two terms are, respectively,

$$\omega_1 = \frac{10\pi}{3},\ f_1 = \frac{5}{3},\ T_1 = \frac{3}{5}, \qquad \omega_2 = \frac{5\pi}{4},\ f_2 = \frac{5}{8},\ T_2 = \frac{8}{5}$$

The fundamental frequency f_0 is the GCD of $f_1 = 5/3$ and $f_2 = 5/8$:

$$f_0 = GCD(\frac{5}{3}, \frac{5}{8}) = GCD(\frac{40}{24}, \frac{15}{24}) = \frac{5}{24}$$

Alternatively, the period of the fundamental T_0 is the LCM of $T_1 = 3/5$ and $T_2 = 8/5$:

$$T_0 = LCM(\frac{3}{5}, \frac{8}{5}) = \frac{24}{5}$$

Now we get $\omega_0 = 2\pi f_0 = 2\pi/T_0 = 5\pi/12$ and the signal can be written as

$$x(t) = cos(8\frac{5\pi}{12}t) + sin(3\frac{5\pi}{12}t) = cos(8\omega_0 t) + sin(3\omega_0 t)$$

i.e., the two terms are the 3th and 8th harmonic of the fundamental frequency ω_0, respectively.

c. Explain any five types of elementary signals with mathematical equations and graphical plot.

Elementary Signals

Unit step signal

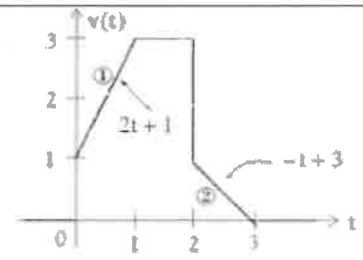
Ramp Signal

Unit impulse function: Amplitude of unit impulse approaches 1 as the width approaches zero and it has zero value at all other values.

Sinusoidal signal: A continuous time sinusoidal signal is given by,

Exponential signal:

d.i. 2



. Equations for the linear segments of Figure 1

Following the same procedure as in the previous examples, we obtain

$$v(t) = (2t+1)[u_0(t) - u_0(t-1)] + 3[u_0(t-1) - u_0(t-2)] + (-t+3)[u_0(t-2) - u_0(t-3)]$$

Multiplying the values in parentheses by the values in the brackets, we obtain

$$v(t) = (2t+1)u_0(t) - (2t+1)u_0(t-1) + 3u_0(t-1) - 3u_0(t-2) + (-t+3)u_0(t-2) - (-t+3)u_0(t-3)$$

$$v(t) = (2t+1)u_0(t) + [-(2t+1)+3]u_0(t-1) + [-3+(-t+3)]u_0(t-2) - (-t+3)u_0(t-3)$$

and combining terms inside the brackets, we obtain

$$v(t) = (2t+1)u_0(t) - 2(t-1)u_0(t-1) - tu_0(t-2) + (t-3)u_0(t-3)$$

3

ii.

Signal Energy & Power Comments

Usually, the limits are taken over an infinite time interval

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \quad P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

- We will encounter many types of signals
- Some have infinite average power, energy, or both
- A signal is called an **energy signal** if $E_\infty < \infty$
- A signal is called a **power signal** if $0 < P_\infty < \infty$
- A signal can be an energy signal, a power signal, or neither type
- A signal can not be both an energy signal and a power signal

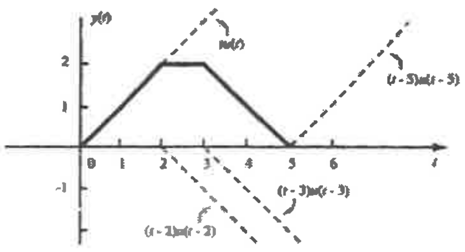
e.	Test the given system for linearity, causality, stability, memory and time variant. $y(t) = x(t^2)$ $y(t) = x(t^2)$ Linear Non Causal Stable Dynamic Time Variant	
f.	Explain the application of Signals and System in Multimedia Processing. **Multimedia** is content that uses a combination of different content forms such as text, audio, images, animations, video and interactivecontent. Multimedia contrasts with media that use only rudimentary computer displays such as text-only or traditional forms of printed or hand-produced material. Multimedia can be recorded and played, displayed, interacted with or accessed by information content processing devices, such as computerized and electronic devices, but can also be part of a live performance. Multimedia devices are electronic media devices used to store and experience multimedia content. Multimedia is distinguished from mixed media in fine art; for example, by including audio it has a broader scope. In the early years of multimedia the term "rich media" was synonymous with interactive multimedia, and "hypermedia" was a application of multimedia.	
Q2.a.	(a) We first express $x(t)$ and $h(t)$ in functional form: $x(t) = u(t) - u(t-3) \qquad h(t) = u(t) - u(t-2)$ $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$ $= \int_{-\infty}^{\infty} [u(\tau) - u(\tau-3)][u(t-\tau) - u(t-\tau-2)]\,d\tau$ $= \int_{-\infty}^{\infty} u(\tau)u(t-\tau)\,d\tau - \int_{-\infty}^{\infty} u(\tau)u(t-2-\tau)\,d\tau$ $- \int_{-\infty}^{\infty} u(\tau-3)u(t-\tau)\,d\tau + \int_{-\infty}^{\infty} u(\tau-3)u(t-2-\tau)\,d\tau$ Since $u(\tau)u(t-\tau) = \begin{cases} 1 & 0<\tau<t,\ t>0 \\ 0 & \text{otherwise} \end{cases}$ $u(\tau)u(t-2-\tau) = \begin{cases} 1 & 0<\tau<t-2,\ t>2 \\ 0 & \text{otherwise} \end{cases}$ $u(\tau-3)u(t-\tau) = \begin{cases} 1 & 3<\tau<t,\ t>3 \\ 0 & \text{otherwise} \end{cases}$ $u(\tau-3)u(t-2-\tau) = \begin{cases} 1 & 3<\tau<t-2,\ t>5 \\ 0 & \text{otherwise} \end{cases}$	10

we can express $y(t)$ as

$$y(t) = \left(\int_0^t d\tau\right)u(t) - \left(\int_0^{t-2} d\tau\right)u(t-2)$$

$$- \left(\int_3^t d\tau\right)u(t-3) + \left(\int_3^{t-2} d\tau\right)u(t-5)$$

$$= tu(t) - (t-2)u(t-2) - (t-3)u(t-3) + (t-5)u(t-5)$$

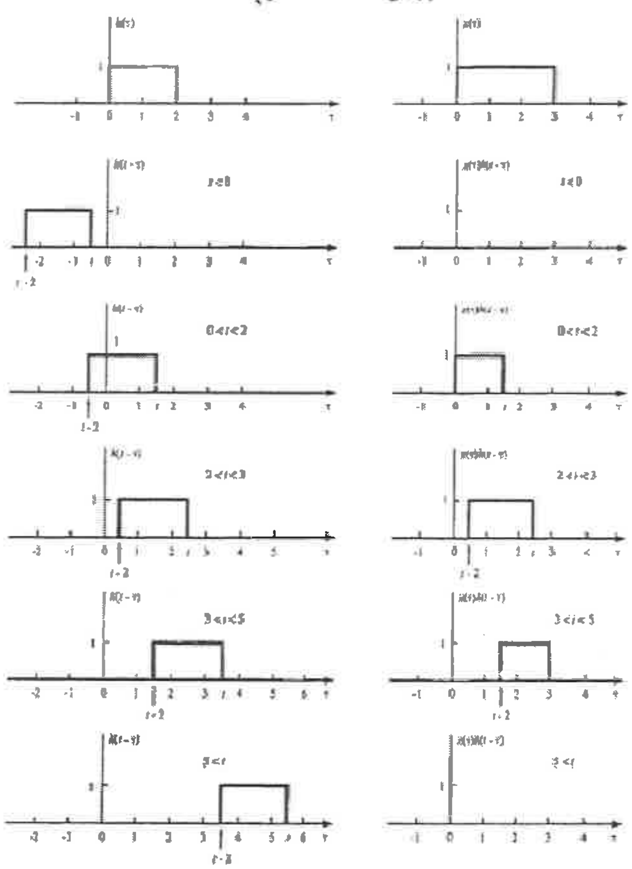


b. (b) by a graphical method. 10

Functions $h(\tau), x(\tau)$ and $h(t-\tau), x(\tau)h(t-\tau)$ for different values of t are sketched in figure below.

We see that $x(\tau)$ and $h(t-\tau)$ do not overlap for $t<0$ and $t>5$, and hence $y(t)=0$ for $t<0$ and $t>5$. For the other intervals, $x(\tau)$ and $h(t-\tau)$ overlap. Thus, computing the area under the rectangular pluses for these intervals, we obtain

$$y(t) = \begin{cases} 0 & t<0 \\ t & 0<t\le 2 \\ 2 & 2<t\le 3 \\ 5-t & 3<t\le 5 \\ 0 & 5<t \end{cases}$$



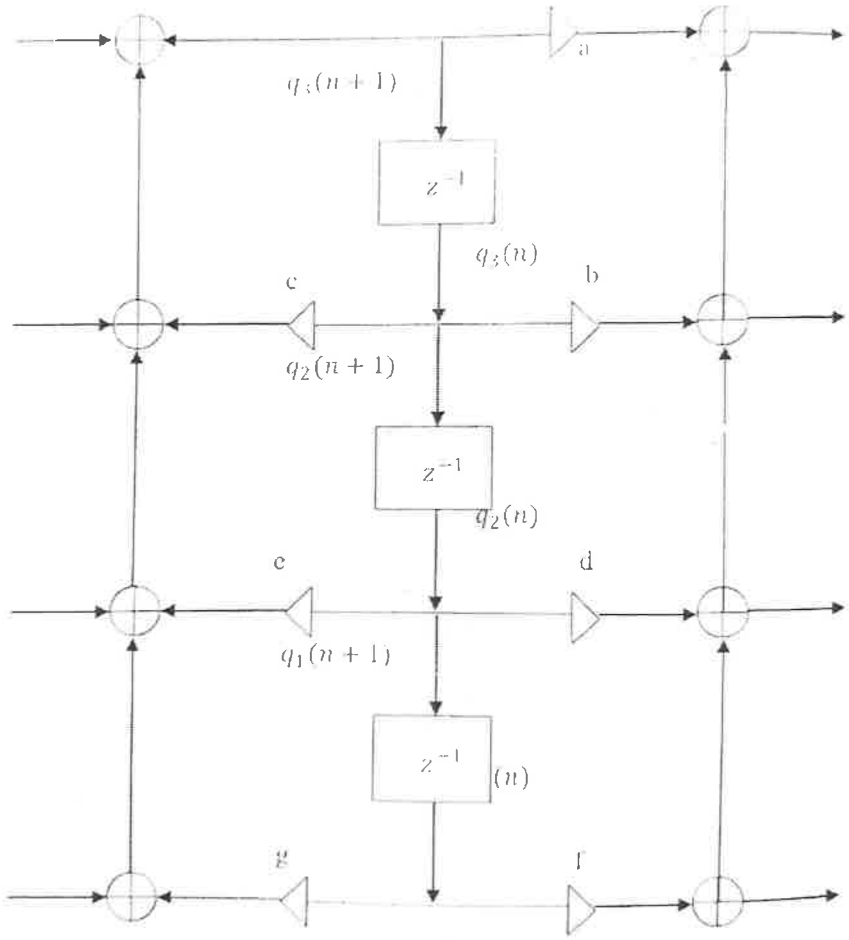
Q3.a.	Determine the sequence x[n] associated with Z-Transform using residue method. $$X(z) = \{\frac{(1 - e^{-a})z}{(z - 1)(z - e^{-a})}\}$$ $$x[n] = \frac{1}{2\pi j}\oint G(z)dz = \text{sum of residues of } G(z) \text{ corresponding to poles of } G(z)$$ Here, $G(z) = \frac{(1-e^{-a})z^n}{(z-1)(z-e^{-a})}$ $G(z)$ has two poles at $z = 1$ and $z = e^{-a}$ $$x[n] = \text{residue of } G(z) \text{ at } z = 1 + \text{Residue of } G(z) \text{ at } z = e^{-a}$$ Residue of $G(z)$ at $(z = 1) = R_{z=1} = (z-1)G(z)\|z = 1$ $= \frac{(1-e^{-a})z^n}{(z-e^{-a})}\|z = 1$ $= 1$ Residue of $G(z)$ at $(z = e^{-a}) = R_{z=e^{-a}} = (z - e^{-a})G(z)\|z = e^{-a}$ $= \frac{(1-e^{-a})z^n}{(z-1)}\|z = e^{-a}$ $= -e^{-an}$ Therefore, $x[n] = 1 - e^{-an}; n \geq 0$
b.	State and Prove Parseval's Theorem with respect to DTFT. **Parseval's Theorem:** Parseval's Theorem with respect to DTFT is exactly similar to the Parseval's theorem of continuous-time Fourier transform given in chapter 4. It states that if $x_1(n) \xleftrightarrow{DTFT} X_1(e^{j\omega})$ and $x_2(n) \xleftrightarrow{DTFT} X_2(e^{j\omega})$, then $$\sum_{n=-\infty}^{\infty} x_1(n)\cdot\overline{x_2(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\omega})\cdot\overline{X_2(e^{j\omega})}\,d\omega$$ Where the over-bar denotes the complex conjugation. **Proof:** $$\sum_{n=-\infty}^{\infty} x_1(n)\cdot\overline{x_2(n)} = \sum_{n=-\infty}^{\infty} x_1(n)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}\overline{X_2(e^{j\omega})\,e^{-j\omega n}\,d\omega}\right]$$ Interchanging the order of summation and integration on the right hand side $$\sum_{n=-\infty}^{\infty} x_1(n)\cdot\overline{x_2(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\overline{X_2(e^{j\omega})}\left[\sum_{n=-\infty}^{\infty} x_1(n)e^{-j\omega n}\right]\right\}d\omega$$ $$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\overline{X_2(e^{j\omega})}\,X_1(e^{j\omega})\,d\omega$$ Thus, $$\sum_{n=-\infty}^{\infty} x_1(n)\cdot\overline{x_2(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\omega})\cdot\overline{X_2(e^{j\omega})}\,d\omega \quad \ldots\ldots\ldots (6.12)$$ For the special case when $x_2(n) = x_1(n)$ for all n, we may write $$\sum_{n=-\infty}^{\infty} \|x(n)\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\|X(e^{j\omega})\right\|^2 d\omega \quad \ldots\ldots\ldots (6.13)$$ We know that LHS of the above equation is the energy E_x of the discrete-time signal $x(n)$. Hence analogous to the way we interpreted $\|X(f)\|^2$ in the case of Parseval's theorem, for continuous-time Fourier transform, here also, we call $\left\|X(e^{j\omega})\right\|^2$ it as Energy spectral density of the signal $x(n)$ and denote it by $S_{xx}(\omega)$. Therefore, $$E_x = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(\omega)\,d\omega \quad \ldots\ldots\ldots (6.14)$$ $S_{xx}(\omega)$, is the Energy spectral density, which contains the information about how the energy of $x(t)$ is distributed with respect to frequency.

Q4.a.	Determine the state model of the system governed by the equation y[n]= -2y[n-1]+3y[n-2]+0.5y[n-3]+2x[n]+1.5x[n]+1.5x[n-1]+2.5x[n-2]+4x[n-3]

Solution: The transfer function of the system described by above equation is,

$$H(z) = \frac{(2 + 1.5z^{-1} + 2.5z^{-2} + 4z^{-3})}{(1 + 2z^{-1} - 3z^{-2} - 0.5z^{-3})}$$

Therefore, direct form-II structure of the system is as follows:



Where, $a = 2,\ b = 1.5,\ c = -2,\ d = 2.5,\ e = 3,\ f = 4$ and $g = 0.5$

...choice of state variables is equal to the number of delay units. This direct form structure ... one input, one output and three delay elements. Therefore, let's choose three state ...ables for this structure.

...$[n]$, $q_2[n]$ and $q_3[n]$ be the three state variables. Assign these state variables at the ...t of each delay unit. Hence, $(n+1)^{th}$ value of state variable will be available at the input ...e delay element.

...e *equations* are formed by equating the sum of incoming signals of the delay unit to the ...1)th value of state variables as shown below:

$$q_1(n+1) = q_2(n)$$

$$q_2(n+1) = q_3(n)$$

$$q_3(n+1) = X(n) + 0.5q_1(n) + 3q_2(n) - 2q_3(n)$$

...nging equations in matrix form,

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \\ q_3(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 3 & -2 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \\ q_3(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x(n) \ldots\ldots\ldots$$

Output equation $y[n]$ is formed by equating incoming signals of output node point as shown ...ow:

$$y(n) = 4q_1(n) + 2.5q_2(n) + 1.5q_3(n) + 2q_3(n+1)$$

...e have $q_3(n+1)$ from

$$y(n) = 4q_1(n) + 2.5q_2(n) + 1.5q_3(n) + 2(x(n)$$

$$+0.5q_1(n) + 3q_2(n) - 2q_3(n)$$

$$y(n) = 5q_1(n) + 8.5q_2(n) - 2.5q_3(n) + 2X(n)$$

...riting in the matrix form,

$$y(n) = \begin{bmatrix} 5 & 8.5 & -2.5 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \\ q_3(n) \end{bmatrix} + [2]X(n)$$

State equations: $Q(n+1) = AQ(n) + BX(n)$

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \\ q_3(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 3 & -2 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \\ q_3(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x(n)$$

Where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Output equations: $Y(n) = C\,Q(n) + D\,X(n)$

b. Find Fourier series for $f(x)=x^3\,(-\pi,\pi)$

Solution: Given $f(x) = x^3\,(-\pi,\pi)$

Put $x = -x$

$$f(-x) = (-x)^3 = -x^3$$

Function is odd $a_0 = 0 \quad a_n = 0$

We have Fourier series as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos nx + b_n \sin nx]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} x^3 \sin nx\,dx$$

$$= \frac{2}{\pi}\left[\frac{(x^3)(-\cos nx)}{n} - \frac{(3x^2)(-\sin nx)}{n^2} + \frac{(6x)(\cos nx)}{n^3} - \frac{(6)(\sin nx)}{n^4}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-x^3\cos nx}{n} + \frac{6x\cos nx}{n^3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-\pi^3\cos n\pi}{n} + \frac{6\pi\cos n\pi}{n^3} - 0\right]$$

$$= \frac{2}{\pi}\frac{\pi\cos n\pi}{n}\left[-\pi^2 + \frac{6}{n^2} - 0\right]$$

$$= \frac{2(-1)^n}{n}\left[-\pi^2 + \frac{6}{n^2}\right]$$

Hence Fourier series is as follows,

$$f(x) = 0 + \sum_{n=1}^{\infty}\left[0 + \frac{2(-1)^n}{n}\left(-\pi^2 + \frac{6}{n^2}\right)\sin nx\right]$$

$$\therefore x^3 = \sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\left(-\pi^2 + \frac{6}{n^2}\right)\sin nx$$

Q5.a Determine DTFS for the sequence $x(n) = \cos^2((\pi/8)n)$

Solution:

$$x(n) = \frac{1 + \cos^2\left(\frac{\pi}{4}n\right)}{2}$$

$$x(n) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{4}n\right)$$

Here fundamental period is 8

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2}\right)$$

$$a_0 = \frac{1}{2},\quad a_1 = \frac{1}{4},\quad a_{-1} = \frac{1}{4},$$

$$a_0 = \frac{1}{2},\quad a_{-1} = a_{-1+8} = a_7 = \frac{1}{4},\quad a_1 = \frac{1}{4},$$

Rest

$$a_k = 0, \qquad 0 \le k \le 7$$

b. Find Laplace transform of $\frac{d}{dt} sin(t)\, u(t)$.

$$L\left\{\frac{d}{dt}\sin(t)u(t)\right\} \Longrightarrow L\{\cos(t)u(t)\}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$sL\{\sin(t)\,u(t)\} = s\frac{1}{s^2+1} = \frac{s}{s^2+1}$$

c. Find Inverse Laplace transform using convolution

$$L^{-1} = \{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\}$$

Solution: $F(s) = \frac{s^2}{(s^2+a^2)(s^2+b^2)}$

Splitting the given function as follows.

$$F_1(s) = \frac{s}{s^2+a^2}\ F_2(s) = \frac{s}{s^2+b^2}$$

Taking inverse individually as follows.

$$L^{-1}\{F_1(s)\} = L^{-1}\left\{\frac{s}{s^2+a^2}\right\}$$

$$= \cos at = F_1(t)$$

$$L^{-1}\{F_2(s)\} = L^{-1}\left\{\frac{s}{s^2+b^2}\right\}$$

$$= \cos bt = F_2(t)$$

Now by applying convolution Theorem,

$$L^{-1}\{F_1(s) * F_2(s)\} = \int_0^t F_1(u)\, F_2(t-u)du$$

$$= \int_0^t \cos au \cos b(t-u)\, du$$

$$= \int_0^t \cos au \cdot \cos(bt - bu)\, du$$

$$= \int_0^t \frac{1}{2}[\cos(au + bt - bu) + \cos(au - bt + bu)]du$$

$$= \frac{1}{2}\int_0^t [\cos\{(a-b)u + bt\} + \cos\{(a+b)u - bt\}]\, du$$

$$= \frac{1}{2}\left[\frac{\sin\{(a-b)u + bt\}}{a-b} + \frac{\sin\{(a+b)u - bt\}}{a+b}\right]_0^t$$

$$= \frac{1}{2}\left[\left[\frac{\sin(at - bt + bt)}{a-b} + \frac{\sin(at + bt - bt)}{a+b}\right] - \left[\frac{\sin bt}{a-b} + \frac{\sin(-bt)}{a+b}\right]\right]$$

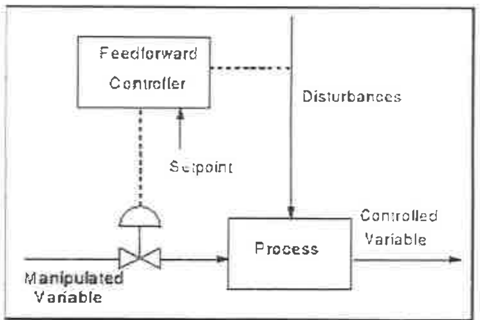
$$= \frac{1}{2}\left[\frac{\sin at}{a-b} + \frac{\sin(+at)}{a+b} - \frac{\sin bt}{a-b} + \frac{\sin bt}{a+b}\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{a-b} + \frac{1}{a+b}\right)\sin at - \left(\frac{1}{a-b} - \frac{1}{a+b}\right)\sin bt\right]$$

$$= \frac{1}{2}\left[\frac{2a}{a^2-b^2}\sin at - \left(\frac{2b}{a^2-b^2}\sin bt\right)\right]$$

$$= \frac{2}{2(a^2-b^2)}[a \sin at - b \sin bt]$$

$$\therefore f(t) = \frac{1}{a^2-b^2}(a \sin at - b \sin bt)$$

Q6.	Write short note on any two:
a.	Feedforward Control system A feedforward controller avoids the slowness of the feedback control. 

Using feedforward control the performance of control systems can be enhanced greatly.

Process variables such as pressure, level, flow, temperature are interrelated and so one variable may affect another as a disturbance in the process. Feedforward system measure important disturbance variables and take corrective action before they upset the process.

Here the setpoint is fixed in the feedforward controller after doing little complex mathematical derivations. The feedforward controller determines the needed change in the manipulated variable, so that, when the effect of the disturbance is combined with the effect of the change in the manipulated variable, there will be no change in the controlled variable at all. The disturbance is measured at the input side of the process and the manipulating variable also, so the controlling process is done before a disturbance affects the process.

b. ROC in Z-Transform and Laplace Transform 5

The **region of convergence** (**ROC**) is the set of points in the complex plane for which the **Z-transform** summation converges. Region of Convergence (ROC) Whether the Laplace transform of a signal exists or not depends on the complex variable as well as the signal itself. All complex values of for which the integral in the definition converges form a region of convergence (ROC) in the **s-plane**.

c. 5

Relation of ESD, PSD with auto-correlation

1. **Energy and Power Signals**

- An energy signal $x(t)$ has $0 < E < \infty$ for average energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

- A power signal $x(t)$ has $0 < P < \infty$ for average power

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.$$

- Can think of average power as average energy/time.
- An energy signal has zero average power. A power signal has infinite average energy. Power signals are generally not integrable so don't necessarily have a Fourier transform.
- We use power spectral density to characterize power signals that don't have a Fourier transform.

2. **Energy Spectral Density (ESD)**

- Defined as $\Psi_x(f) = |X(f)|^2$.
- Measures the distribution of signal energy $E = \int |x(t)|^2 dt = \int \Psi_x(f) df$ over frequency.
- Properties of ESD include $\Psi_x(f) \geq 0$, $\Psi_x(-f) = \Psi_x(f)$ for $x(t)$ real, and for $x(t)$ input to a filter with frequency response $H(f)$, the filter output $y(t)$ has ESD $\Psi_y(f) = |H(f)|^2 \Psi_x(f)$.

3. **Autocorrelation of Energy Signals**

- Defined for real signals as $R_x(\tau) = \int x(t)x(t-\tau)dt = x(\tau) * x(-\tau)$.
- Measures the similarity of a signal with a delayed version of itself.
- Autocorrelation defines signal energy: $E = R_x(0)$.
- Since $|R_x(\tau)| \leq R_x(0)$, can use autocorrelation for signal synchronization.
- The autocorrelation is symmetric: $R_x(\tau) = R_x(-\tau)$.
- The autocorrelation and ESD are Fourier Transform pairs: $R_x(\tau) \Leftrightarrow \Psi_x(f)$.

4. **Power Spectral Density (PSD)**

- Power signals have infinite energy: Fourier transform and ESD may not exist.
- Power signals need alternate spectral density definition with similar properties as ESD.
- Can obtain ESD for a power signal $x(t)$ that is time windowed with window size $2T$.
- PSD defined as the normalized limit of the ESD for the windowed signal $x_T(t)$: $S_x(f) = \lim_{T\to\infty} \frac{1}{2T}|X_T(f)|^2$.
- PSD measures the distribution of signal power $P = \lim_{T\to\infty} \frac{1}{2T} \int |x_T(t)|^2 dt = \int S_x(f)df$ over frequency domain.

5. **Properties of PSD**

- $S_x(f) \geq 0$
- $S_x(-f) = S_x(f)$

6. **Filtering and Modulation of Power Signals:**

- Let $x(t)$ be a power signal with PSD $S_x(f)$.
- If $x(t)$ is input to a filter with frequency response $H(f)$, then the filter output $y(t)$ has PSD $S_y(f) = |H(f)|^2 S_x(f)$.
- If $S_x(f)$ is bandlimited with bandwidth $B << f_c$, then for $z(t) = x(t)\cos(2\pi f_c t)$, $S_z(f) = .25[S_x(f - f_c) + S_x(f + f_c)]$.