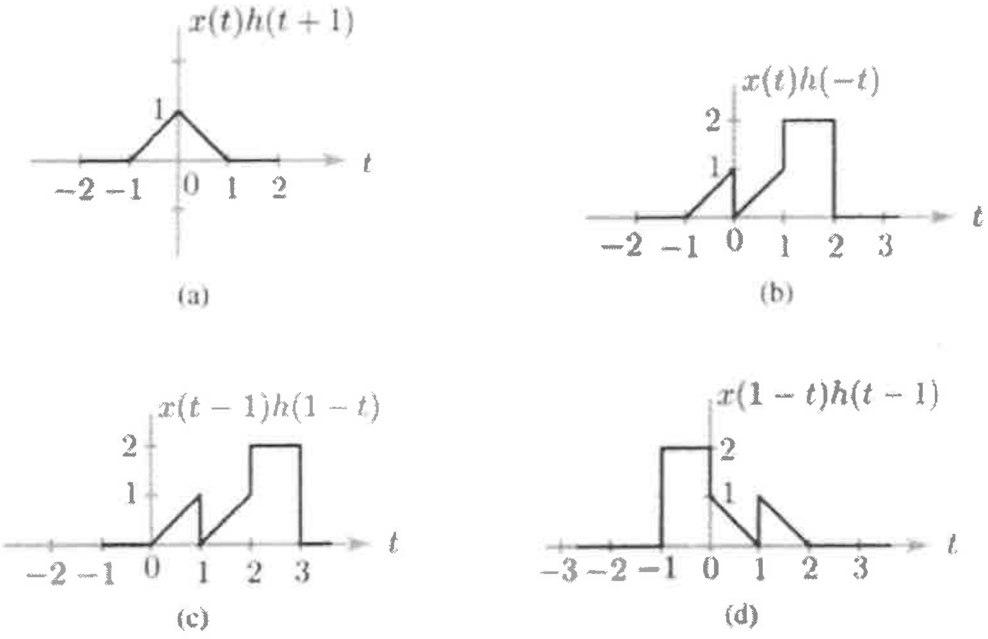
77135

SS old
Sol Set 1

Q1a)

$$x_{ev}(t) = \frac{1}{2}[x(t) + x(-t)]$$
$$= \frac{1}{2}[e^{-t}u(t) + e^{t}u(-t)]$$
$$x_{od}(t) = \frac{1}{2}[x(t) - x(-t)]$$
$$= \frac{1}{2}[e^{-t}u(t) - e^{t}u(-t)]$$

Q1d



(a) (b)

(c) (d)

Q1.e


Approximation of square wave in 5 steps

The three pictures on the right demonstrate the phenomenon for a square wave whose Fourier expansion is

$$\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \cdots$$

More precisely, this is the function *f* which equals $\pi / 4$ between $2n\pi$ and $(2n+1)\pi$ and $-\pi / 4$ between $(2n+1)\pi$ and $(2n+2)\pi$ for every integer n; thus this square wave has a jump discontinuity of height $\pi / 2$ at every integer multiple of π.

As can be seen, as the number of terms rises, the error of the approximation is reduced in width and energy, but converges to a fixed height. A calculation for the square wave (see Zygmund, chap. 8.5., or the computations at the end of this article) gives an explicit formula for the limit of the height of the error. It turns out that the Fourier series exceeds the height $\pi / 4$ of the square wave by

$$\frac{1}{2}\int_0^{\pi} \frac{\sin t}{t}\,dt - \frac{\pi}{4} = \frac{\pi}{2} \cdot (0.089490\ldots)$$

or about 9 percent. More generally, at any jump point of a piecewise continuously differentiable function with a jump of a, the nth partial Fourier series will (for n very large) overshoot this jump by approximately $a \cdot (0.089490\ldots)$ at one end and undershoot it by the same amount at the other end; thus the "jump" in the partial Fourier series will be about 18% larger than the jump in the original function. At the location of the discontinuity itself, the partial Fourier series will converge to the midpoint of the jump (regardless of what the

Q2a)Solution:

$$T = 5 \quad w_0 = \frac{2\pi}{5}$$

$$x(t) = \begin{cases} t+2 & -2 < t < -1 \\ 1.0 & -1 < t < 1 \\ 2-t & 1 < t < 2 \end{cases}$$

(a)

$$(-2,0)(-1,1)$$

$$(y-1) = \frac{-1}{-1}(t+1)$$

$$y = t+2$$

(b) $(1,1)(2,0)$

$$y - 0 = \frac{1}{-1}(x-2)$$

$$y = -x+2 = -t+2$$

$$C_0 = \frac{1}{5}\left[\int_{-2}^{-1}(t+2)dt + \int_{-1}^{1}dt + \int_{1}^{2}(2-t)dt\right]$$

$$C_0 = \frac{3}{5}$$

$$C_k = \frac{1}{5}\left[\underbrace{\int_{-2}^{-1}(t+2)e^{-j\frac{2n\pi}{5}t}dt}_{A} + \underbrace{\int_{-1}^{1}e^{-j\frac{2n\pi}{5}t}dt}_{B} + \underbrace{\int_{1}^{2}(2-t)e^{-j\frac{2n\pi}{5}t}dt}_{C}\right]$$

$$A = \int_{-2}^{-1} te^{-j\frac{2n\pi}{5}t}dt + \int_{-2}^{-1} 2e^{-j\frac{2n\pi}{5}t}dt$$

$$A = -\frac{1}{j\omega}\left\{te^{-j\omega t}\Big|_{-2}^{-1}\right\} + \frac{1}{\omega^2}e^{-j\omega t}\Big|_{-2}^{-1} + \frac{2}{-j\omega}e^{-j\frac{2n\pi}{5}t}\Big|_{-2}^{-1}$$

$$= \frac{5}{j2n\pi}\left(-e^{j\frac{2n\pi}{5}} + 2e^{j\frac{4n\pi}{5}}\right) + \frac{25}{4n^2\pi^2}\left(e^{j\frac{2n\pi}{5}} - e^{j\frac{4n\pi}{5}}\right) - \frac{10}{2n\pi j}$$

$$A = \frac{5}{j2n\pi}\left(-e^{j\frac{2n\pi}{5}} + 4e^{j\frac{4n\pi}{5}}\right) + \frac{25}{4n^2\pi^2}\left(e^{j\frac{2n\pi}{5}} - e^{j\frac{4n\pi}{5}}\right)$$

$$B = \frac{e^{j\frac{2n\pi}{5}} - e^{-j\frac{2n\pi}{5}}}{j\frac{2n\pi}{5}} = \frac{5}{j2n\pi}\left(e^{j\frac{2n\pi}{5}} - e^{-j\frac{2n\pi}{5}}\right)$$

$$C = \frac{-10}{j2n\pi}\left(e^{-j\frac{4n\pi}{5}} - e^{-j\frac{2n\pi}{5}}\right) + \frac{10}{j2n\pi}e^{-j\frac{4n\pi}{5}} - \frac{5}{j2n\pi}e^{-j\frac{2n\pi}{5}} - \frac{25}{4n^2\pi^2}e^{-j\frac{4n\pi}{5}} + \frac{25}{4n^2\pi^2}e^{-j\frac{2n\pi}{5}}$$

Q2b)

Solution: Given $x(t) = e^{2t}\,u(t) + e^{-2t}\,u(-t)$ ✓

$$\therefore \quad L[x(t)] = X(s) = \int_{-\infty}^{\infty} [e^{2t}\,u(t) + e^{-2t}\,u(-t)]\,e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} e^{2t}\,u(t)\,e^{-st}\,dt + \int_{-\infty}^{\infty} e^{-2t}\,u(-t)\,e^{-st}\,dt$$

$$= \int_{0}^{\infty} e^{2t}\,e^{-st}\,dt + \int_{-\infty}^{0} e^{-2t}\,e^{-st}\,dt$$

$$= \underbrace{\underbrace{\int_{0}^{\infty} e^{-(s-2)t}\,dt}_{\text{Converges if } \mathrm{Re}(s) > 2} + \underbrace{\int_{-\infty}^{0} e^{-(s+2)t}\,dt}_{\text{Converges if } \mathrm{Re}(s) < -2}}_{\text{does not converge for any real value of } s}$$

So there is no ROC and hence the Laplace transform of the given signal does not exist.

Q3a)

lution:

(a) Given $x(n) = u(n+2)\,u(-n+3)$

The signal $u(n + 2)\,u(-n + 3)$ can be obtained by first drawing the signal $u(n + 2)$ as shown in Figure 1.33(a), then drawing $u(-n + 3)$ as shown in Figure 1.33(b) and then multiplying these sequences element by element to obtain $u(n + 2)\,u(-n + 3)$ as shown in Figure 1.33(c).

$x(n) = 0$ for $n < -2$ and $n > 3$; $x(n) = 1$ for $-2 < n < 3$

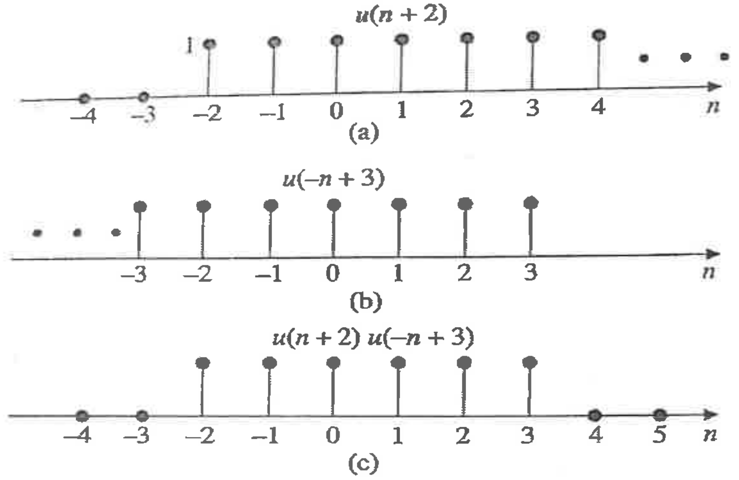
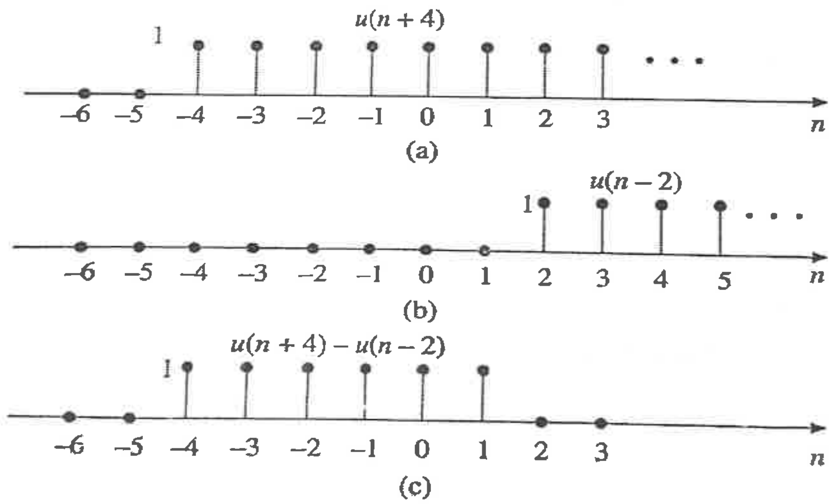


Figure 1.33 Plots of (a) $u(n + 2)$, (b) $u(-n + 3)$, (c) $u(n + 2)\,u(-n + 3)$.

(b) Given $x(n) = u(n + 4) - u(n - 2)$

The signal $u(n + 4) - u(n - 2)$ can be obtained by first plotting $u(n + 4)$ as shown in Figure 1.34(a), then plotting $u(n - 2)$ as shown in Figure 1.34(b) and then subtracting each element of $u(n - 2)$ from the corresponding element of $u(n + 4)$ to obtain the result shown in Figure 1.34(c).



Q3b)

The difference equation governing the system is,

$$y(n) = x(n) - 0.25\, y(n-2)$$

Let us take Z-transform of the difference equation governing the system with zero initial condition.

$$Z\{y(n)\} = Z\{x(n - 0.25\, y(n-2)\}$$

$$Z\{y(n)\} = Z\{x(n)\} - 0.25\, Z\{y(n-2)\}$$

$$Y(z) = X(z) - 0.25\, z^{-2}\, Y(z)$$

$$Y(z) + 0.25 z^{-2}\, Y(z) = X(z)$$

$$(1 + 0.25 z^{-2})\, Y(z) = X(z)$$

$Z\{x(n)\} = X(z)$

$Z\{y(n)\} = Y(z)$
$Z\{y(n-2)\} = z^{-2}\, Y(z)$
(Using shifting property)

$$\therefore \text{ Transter function, } \frac{Y(z)}{X(z)} = \frac{1}{1 + 0.25 z^{-2}}$$

We know that, $\dfrac{Y(z)}{X(z)} = H(z)$

$(a+b)(a-b) = a^2 - b^2$ | $j^2 = -1$

$$\therefore H(z) = \frac{1}{1 + 0.25 z^{-2}} = \frac{1}{z^{-2}(z^2 + 0.25)} = \frac{z^2}{(z + j\,0.5)(z - j\,0.5)}$$

By partial fraction expansion we can write,

$$\frac{H(z)}{z} = \frac{z}{(z + j0.5)(z - j0.5)} = \frac{A}{z + j0.5} + \frac{A^*}{z - j0.5}$$; where A^* is conjugate of A.

$$A = (z + j0.5)\left.\frac{H(z)}{z}\right|_{z=-j0.5} = (z + j0.5)\left.\frac{z}{(z + j0.5)(z - j0.5)}\right|_{z=-j0.5}$$

$$= \left.\frac{z}{z - j0.5}\right|_{z=-j0.5} = \frac{-j0.5}{-j0.5 - j0.5} = \frac{-j0.5}{2(-j0.5)} = \frac{1}{2} = 0.5$$

$$\therefore A^* = 0.5$$

$$\frac{H(z)}{z} = \frac{A}{z + j0.5} + \frac{A^*}{z - j0.5} = \frac{0.5}{z + j0.5} + \frac{0.5}{z - j0.5}$$

$$\therefore H(z) = \frac{0.5z}{z + j0.5} + \frac{0.5z}{z - j0.5} = \frac{0.5z}{z - (-j0.5)} + \frac{0.5z}{z - j0.5}$$

The impulse response is obtained by taking inverse Z-transform of H(z).

$$\therefore \text{ Impulse response, } h(n) = Z^{-1}\{H(z)\} = Z^{-1}\left\{\frac{0.5z}{z - (-j0.5)} + \frac{0.5z}{z - j0.5}\right\}$$

$$= 0.5\left[Z^{-1}\left\{\frac{z}{z - (-j0.5)}\right\} + Z^{-1}\left\{\frac{z}{z - j0.5}\right\}\right]$$

$$= 0.5\left[(-j0.5)^n\, u(n) + (j0.5)^n\, u(n)\right]$$

$$Z\{a^n\, u(n)\} = \frac{z}{z - a}$$

Alternatively the impulse response can be expressed as shown below.

Here, $-j0.5 = 0.5\angle -90^\circ = 0.5\angle -\pi/2 = 0.5\angle -0.5\pi$

$+j0.5 = 0.5\angle 90^\circ = 0.5\angle \pi/2 = 0.5\angle 0.5\pi$

$$\therefore \quad h(n) = 0.5\left[(0.5\angle -0.5\pi)^n + (0.5\angle 0.5\pi)^n\right] u(n)$$

$$= 0.5\left[0.5^n\angle -0.5n\pi + 0.5^n\angle 0.5n\pi\right] u(n)$$

$$= 0.5\,(0.5)^n\left[\cos 0.5n\pi - j\sin 0.5n\pi + \cos 0.5n\pi + j\sin 0.5n\pi\right] u(n)$$

$$= 0.5\,(0.5)^n\; 2\cos 0.5n\pi\; u(n)$$

$$= 0.5^n \cos(0.5n\pi)\; u(n)$$

Q4a)

a) Impulse response, $h(t) = (2 + t)\, e^{-3t}\, u(t)$

The transfer function is given by Laplace transform of Impulse response.

$\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}$, $\mathcal{L}\{t\, u(t)\} = \frac{1}{s^2}$

$\therefore$ Transfer function, $H(s) = \mathcal{L}\{h(t)\}$

if, $\mathcal{L}\{x(t)\} = X(s)$,
then, $\mathcal{L}\{e^{-at}x(t)\} = X(s - a)$

$$= \mathcal{L}\{(2 + t)\, e^{-3t}\, u(t)\} = \mathcal{L}\{2\, e^{-3t}\, u(t)\} + \mathcal{L}\{t\, e^{-3t} u(t)\}$$

$$= 2\mathcal{L}\{e^{-3t}\, u(t)\} + \mathcal{L}\{t\, u(t)\}\Big|_{s=s+3} = 2 \times \frac{1}{s+3} + \frac{1}{s^2}\Big|_{s=s-3}$$

$$= \frac{2}{s+3} + \frac{1}{(s+3)^2} = \frac{2(s+3)+1}{(s+3)^2} = \frac{2s+7}{s^2+6s+9}$$

Given $x(t) = e^{at}\, u(-t)$

$$X(\omega) = \mathrm{F}[e^{at}\, u(-t)] = \int_{-\infty}^{\infty} e^{at}\, u(-t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt$$

$\therefore$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt = \int_{0}^{\infty} e^{-(a-j\omega)t}\, dt = \left[\frac{e^{-(a-j\omega)t}}{-(a-j\omega)}\right]_0^{\infty} = \frac{1}{a-j\omega}$$

Q4b)

Given $x(t) = te^{-at}\, u(t)$

$\therefore$

$$X(\omega) = \mathrm{F}[te^{-at}\, u(t)] = \int_{-\infty}^{\infty} te^{-at}\, u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} te^{-(a+j\omega)t}\, dt = \left[t\,\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\, dt$$

$$= \left[t\,\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} - \left[\frac{e^{-(a+j\omega)t}}{(a+j\omega)^2}\right]_0^{\infty} = 0 - 0 - 0 + \frac{1}{(a+j\omega)^2}$$

$$= \frac{1}{(a+j\omega)^2}$$

Q5a)

Given $x(n) = \left(\frac{1}{4}\right)^n u(n+1)$

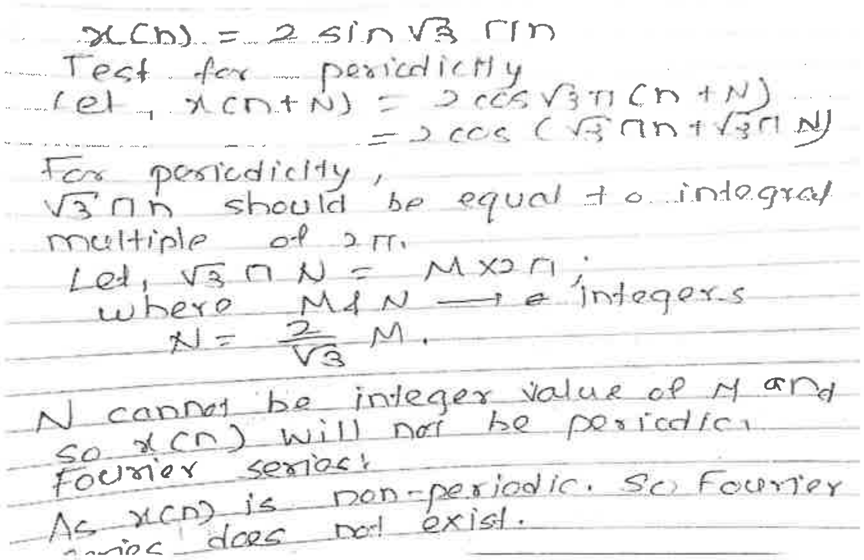
$$X(\omega) = \mathrm{F}\left\{\left(\frac{1}{4}\right)^n u(n+1)\right\} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n+1)\, e^{-j\omega n}$$

$$= \sum_{n=-1}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n} = \sum_{n=-1}^{\infty} \left(\frac{1}{4}e^{-j\omega}\right)^n$$

$$= \left(\frac{1}{4}e^{-j\omega}\right)^{-1} \left[\sum_{n=0}^{\infty} \frac{1}{4} e^{-j\omega}\right] = 4e^{j\omega}\left[\frac{1}{1-(1/4)\, e^{-j\omega}}\right]$$

$$= \frac{4e^{j\omega}}{1-(1/4)\, e^{-j\omega}}$$

Q5b)



$x(n) = 2\sin\sqrt{3}\pi n$

Test for periodicity

let, $x(n+N) = 2\cos\sqrt{3}\pi(n+N)$

$= 2\cos(\sqrt{3}\pi n + \sqrt{3}\pi N)$

For periodicity,

$\sqrt{3}\pi n$ should be equal to integral multiple of 2π.

Let, $\sqrt{3}\pi N = M \times 2\pi$;

where $M \& N \rightarrow$ integers

$N = \frac{2}{\sqrt{3}} M$.

N cannot be integer value of M and

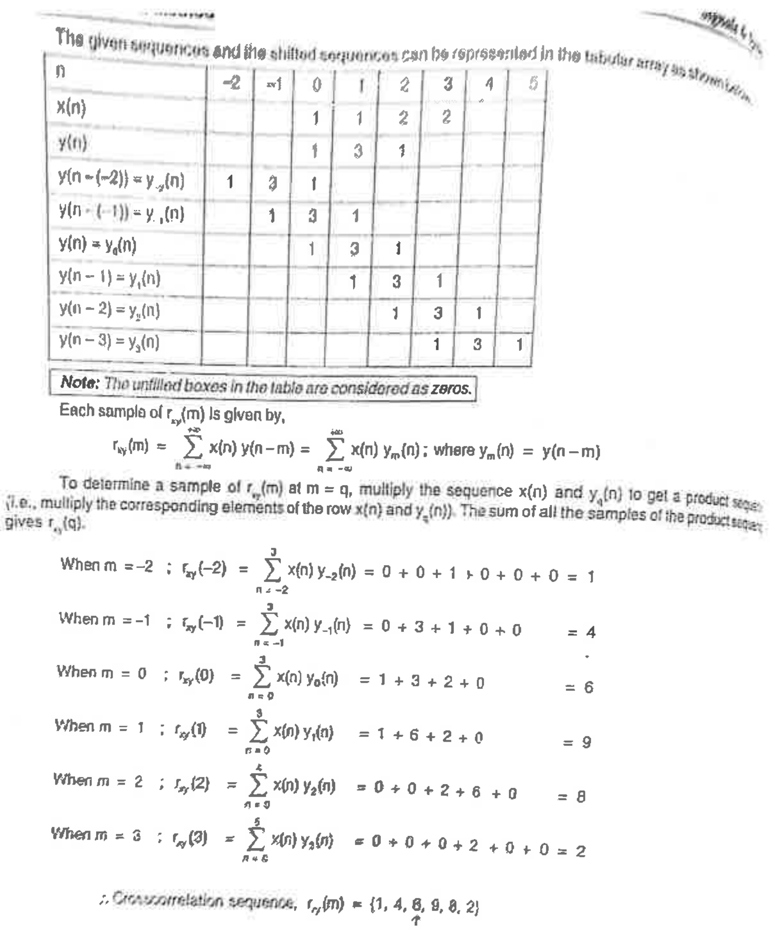
so $x(n)$ will not be periodic.

Fourier series:

As $x(n)$ is non-periodic. So Fourier

series does not exist.

Q5c)



The given sequences and the shifted sequences can be represented in the tabular array as shown below.

n	-2	-1	0	1	2	3	4	5
x(n)			1	1	2	2		
y(n)			1	3	1			
$y(n-(-2)) = y_{-2}(n)$	1	3	1					
$y(n-(-1)) = y_{-1}(n)$		1	3	1				
$y(n) = y_0(n)$			1	3	1			
$y(n-1) = y_1(n)$				1	3	1		
$y(n-2) = y_2(n)$					1	3	1	
$y(n-3) = y_3(n)$						1	3	1

Note: *The unfilled boxes in the table are considered as zeros.*

Each sample of $r_{xy}(m)$ is given by,

$$r_{xy}(m) = \sum_{n=-\infty}^{+\infty} x(n)\,y(n-m) = \sum_{n=-\infty}^{+\infty} x(n)\,y_m(n)\;;\text{ where } y_m(n) = y(n-m)$$

To determine a sample of $r_{xy}(m)$ at $m = q$, multiply the sequence $x(n)$ and $y_q(n)$ to get a product seque (i.e., multiply the corresponding elements of the row $x(n)$ and $y_q(n)$). The sum of all the samples of the product seque gives $r_{xy}(q)$.

$$\text{When } m = -2\;;\; r_{xy}(-2) = \sum_{n=-2}^{3} x(n)\,y_{-2}(n) = 0+0+1+0+0+0 = 1$$

$$\text{When } m = -1\;;\; r_{xy}(-1) = \sum_{n=-1}^{3} x(n)\,y_{-1}(n) = 0+3+1+0+0 = 4$$

$$\text{When } m = 0\;;\; r_{xy}(0) = \sum_{n=0}^{3} x(n)\,y_0(n) = 1+3+2+0 = 6$$

$$\text{When } m = 1\;;\; r_{xy}(1) = \sum_{n=0}^{3} x(n)\,y_1(n) = 1+6+2+0 = 9$$

$$\text{When } m = 2\;;\; r_{xy}(2) = \sum_{n=0}^{4} x(n)\,y_2(n) = 0+0+2+6+0 = 8$$

$$\text{When } m = 3\;;\; r_{xy}(3) = \sum_{n=3}^{5} x(n)\,y_3(n) = 0+0+0+2+0+0 = 2$$

$$\therefore \text{Crosscorrelation sequence, } r_{xy}(m) = \{1, 4, \underset{\uparrow}{6}, 9, 8, 2\}$$

Q6a)

(iii) Given $x_1(t) = \cos t\ u(t);\ x_2(t) = u(t)$

We know that

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$\therefore \quad x_1(t) * x_2(t) = \int_{-\infty}^{\infty} \cos \tau\ u(\tau)\, u(t-\tau)\, d\tau$$

$u(\tau) = 1$ for $\tau > 0$ and $u(t - \tau) = 1$ for $(t - \tau) \geq 0$ or for $\tau < t$.
Hence $u(\tau)\ u(t - \tau) = 1$ only for $0 < \tau < t$. For all other values of τ, $u(\tau)\, u(t-\tau)$ $= 0$.

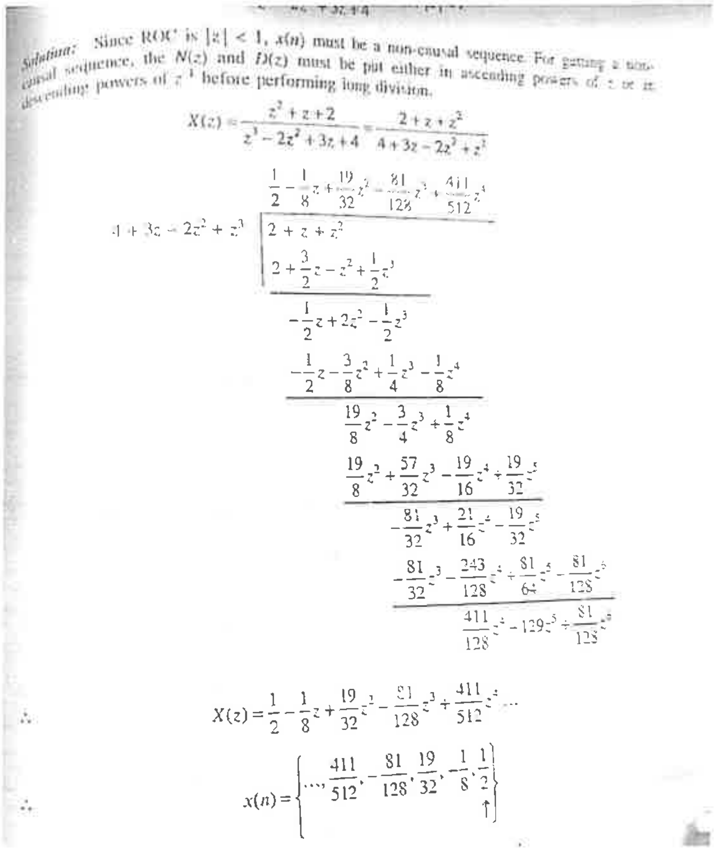
$$\therefore \quad x_1(t) * x_2(t) = \int_{0}^{t} \cos \tau\, d\tau$$

$$= \left[\sin \tau\right]_0^t$$

$$= \sin t \qquad \text{for } t \geq 0$$

$$\therefore \quad x_1(t) * x_2(t) = \sin t\, u(t)$$

Q6b)



Solution: Since ROC is $|z| < 1$, $x(n)$ must be a non-causal sequence. For getting a non-causal sequence, the $N(z)$ and $D(z)$ must be put either in ascending powers of z or in descending powers of z^{-1} before performing long division.

$$X(z) = \frac{z^2 + z + 2}{z^3 - 2z^2 + 3z + 4} = \frac{2 + z + z^2}{4 + 3z - 2z^2 + z^3}$$

$$\begin{array}{r|l} & \frac{1}{2} - \frac{1}{8}z + \frac{19}{32}z^2 - \frac{81}{128}z^3 + \frac{411}{512}z^4 \\ \hline 4 + 3z - 2z^2 + z^3 & 2 + z + z^2 \\ & 2 + \frac{3}{2}z - z^2 + \frac{1}{2}z^3 \\ \hline & -\frac{1}{2}z + 2z^2 - \frac{1}{2}z^3 \\ & -\frac{1}{2}z - \frac{3}{8}z^2 + \frac{1}{4}z^3 - \frac{1}{8}z^4 \\ \hline & \frac{19}{8}z^2 - \frac{3}{4}z^3 + \frac{1}{8}z^4 \\ & \frac{19}{8}z^2 + \frac{57}{32}z^3 - \frac{19}{16}z^4 + \frac{19}{32}z^5 \\ \hline & -\frac{81}{32}z^3 + \frac{21}{16}z^4 - \frac{19}{32}z^5 \\ & -\frac{81}{32}z^3 - \frac{243}{128}z^4 + \frac{81}{64}z^5 - \frac{81}{128}z^6 \\ \hline & \frac{411}{128}z^4 - 129z^5 + \frac{81}{128}z^6 \end{array}$$

$$\therefore \quad X(z) = \frac{1}{2} - \frac{1}{8}z + \frac{19}{32}z^2 - \frac{81}{128}z^3 + \frac{411}{512}z^4 \ldots$$

$$\therefore \quad x(n) = \left\{ \ldots, \frac{411}{512}, -\frac{81}{128}, \frac{19}{32}, -\frac{1}{8}, \underset{\uparrow}{\frac{1}{2}} \right\}$$