68146

May 2019 WTP paper solution and Marking Scheme.

Q1b. $\rho_v = r^2 z^2 \sin(0.6\phi)$

The charge within the volume is

$$Q = \int \rho_v \, dV = \int_2^4 \int_0^{\pi} \int_0^{0.1} r^2 z^2 \sin(0.6\phi) \, r\,dr\,d\phi\,dz \quad (2)$$

$$= \frac{r^4}{4}\Big|_0^{0.1} \; \frac{z^3}{3}\Big|_2^4 \; \frac{-\cos 0.6\phi}{0.6}\Big|_0^{\pi} \quad (2)$$

$Q = 1.018 \times 10^{-3}$

$Q = 1.018\ mC$ (1)

Cancelled

Q3b. $\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_0 \mu_{r1}}{\epsilon_0 \epsilon_{r1}}}$ For first media (1)

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 1}{(10^{-9}/36\pi) \times 8.5}} = 129.31\ \Omega$$

For second media, (1)

$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = 377\ \Omega$ ∵ Region 2 is free space

Transmission coefficient of E-field

$$\Gamma_T = \frac{E_T}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad (2)$$

$E_t = 2.23 \times 10^{-3}\ V/m$

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Reflection coefficient of E-field

$$\Gamma_R = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \quad (2)$$

$$E_r = 0.733 \times 10^{-3} \; (V/m)$$

To find reflected and transmitted $\overline{H}$

$$\frac{E_i}{H_i} = \eta_1 \quad \therefore H_i = 11.6 \; \mu A/m \quad (1)$$

The transmission coefficient of magnetic field is (2)

$$\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

$$H_t = H_i \left(\frac{2\eta_1}{\eta_1 + \eta_2} \right) = 5.93 \times 10^{-6} \; A/m$$

The reflection coeff of magnetic field (1)

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = -5.67 \times 10^{-6} \; A/m$$

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Q4b $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8$ m/s (1)

$\lambda = \frac{v}{f} = 30\times10^{-3}$ m (1)

$\beta = \frac{\omega}{v} = \frac{2\pi\times10\times10^9}{3\times108}$ (1)

$= 0.209\times10^3$ rad/m

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$ (1)

Let us assume that wave is travelling in +z direction (1)

$\frac{E_x}{H_y} = \eta$

∴ Component of H in ay direction is

$H_y = \frac{E_x}{\eta} = \frac{10}{120\pi}$

$H_y = 26.53$ mA/m

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Q6b. To check Cu is a good conductor at highest frequency 30 GHz.

$$\frac{\sigma}{\omega \epsilon} = \frac{58 \times 10^6}{2\pi \times 30 \times 10^9 \times (10^{-9}/36\pi)}$$ (2)

$$= 34.8 \times 10^5 \geq 1$$

Thus Cu is a good conductor.

$$\therefore \delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{2}{2\pi f \times 4\pi \times 10^{-7} \times 58 \times 10^6}}$$ (2)

$$= \frac{66 \times 10^{-3}}{\sqrt{f}}$$

at $f = 60\,Hz$ $\delta = 8.5 \times 10^{-3}$ m (1)

$f = 1\,MHz$ $\delta = 6.6 \times 10^{-5}$ m

$f = 30\,GHz$ $\delta = 3.8 \times 10^{-7}$ m

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