Q.2a) $G(s) = \dfrac{K}{s(s+1+j)(s+1-j)}$

$P = 3, Z = 0$

$N = P = 3$

3 branches approach to ∞.

(1mk.) $s = 0, -1-j, -1+j$.

No breakaway point as per general prediction.

Angles of asymptotes:-

$\theta_0 = 60^\circ$

$\theta_1 = 180^\circ$

(1mk) $\theta_2 = 300^\circ$

Centroid:-

$\sigma = \dfrac{0-1-1}{3} = \dfrac{-2}{3} = -0.67$

(1mk)

Intersection with imaginary axis:-

$s^3 + 2s^2 + 2s + K = 0$

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 2 & K \\ s^1 & \frac{4-K}{2} & 0 \\ s^0 & K & \end{array}$$

$K_{mar} = 4$

$A(s) = 2s^2 + K = 0$

$2s^2 = -4$

(2mks) $s^2 = -2$

$s = \pm j\sqrt{2} = \pm j1.414$

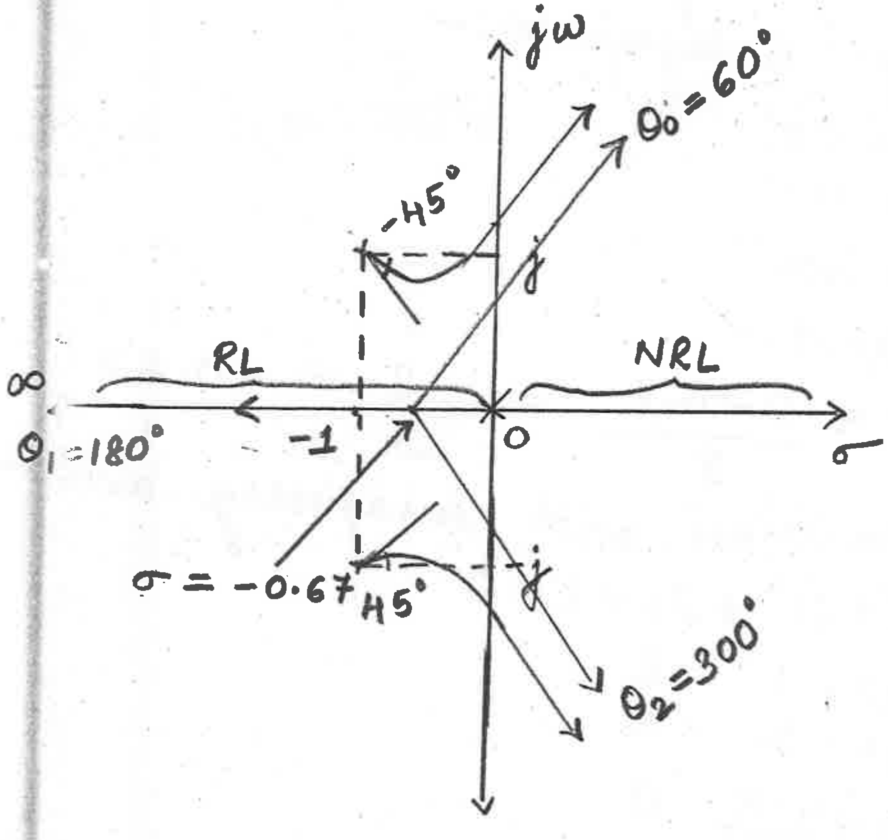
Angle of departures:-

$\phi_{d_1} = -45^\circ$ at $-1+j$

(2mks) $\phi_{d_2} = 45^\circ$ at $-1-j$

(2)

(3mks)



(3)

Step 1: The summing point that exists after the block $G_1(s)$ and the take-off point that exists after the block $G_3(s)$ as indicated by dotted lines in Fig. E3.10(b) are shifted before the block $G_1(s)$ and after the block $G_4(s)$ respectively and the resultant block diagram is shown in Fig. E3.10(c).

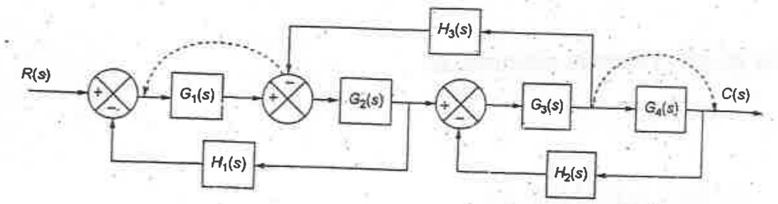


Fig. E3.10(b)

Step 2: The blocks in series as indicated by dotted lines in Fig. E3.10(c) are combined to form a single block. The resultant block diagram is shown in Fig. E3.10(d).

(2mks.)

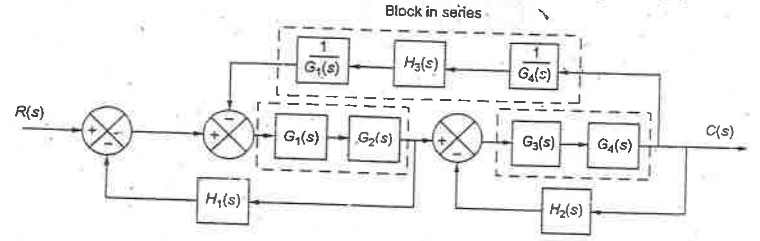
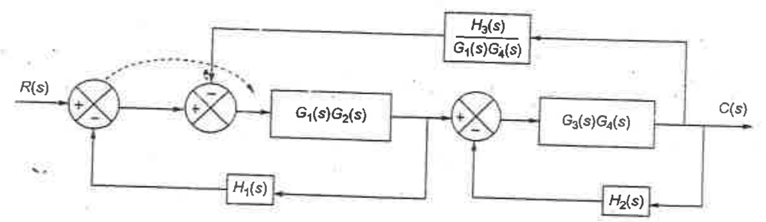


Fig. E3.10(c)

Step 3: The summing points that exist between the input signal and the block $G_1(s)G_2(s)$ as indicated by dotted lines in Fig. E3.10(d) are interchanged and the resultant block diagram is shown in Fig. E3.10(e).

(1mk)



Step 4: The feedback path as represented by dotted lines in Fig. E3.10(e) can be reduced and the resultant block diagram is shown in Fig. E3.10(f).

(2mks)

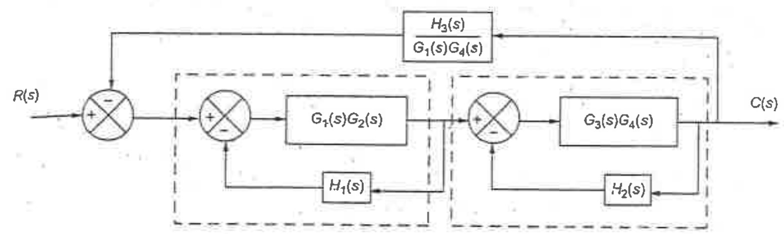


Fig. E3.10(e)

Step 5: The blocks in series as indicated by dotted lines in Fig. E3.10(f) are combined to form a single block. The resultant block diagram is shown in Fig. E3.10(g).

(2mks)

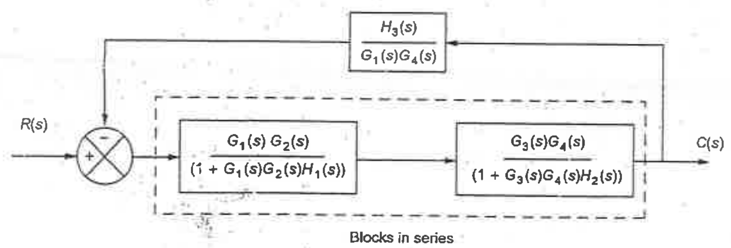


Fig. E3.10(f)

Step 6: The feedback path as indicated by dotted lines in Fig. E3.10(g) can be reduced in order to determine the transfer function of the system.

(2mks)

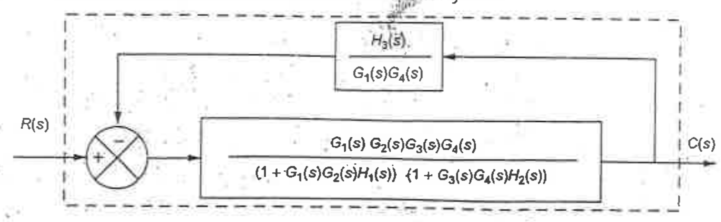


Fig. E3.10(g)

(4)

Hence, the transfer function of the given system is obtained as

$$\frac{C(s)}{R(s)}=$$

$$\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{\{[1+G_1(s)G_2(s)H_1(s)]\times[1+G_3(s)G_4(s)H_2(s)]\times[G_1(s)G_4(s)]\}+G_1(s)G_2(s)G_3(s)G_4(s)H_3(s)}$$

(1mk.)

Q4 b) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s^2+10s+K}$ — (1mk.)

$\omega_n^2 = K$

$\therefore \omega_n = \sqrt{K}$ — (1mk)

$2\xi\omega_n = 10$ — (1mk.)

$\therefore \xi = \frac{5}{\sqrt{K}}$ — (1mk.)

For $\xi = 0.5$, $K = 100$

$\therefore \omega_n = 10\ rad/sec$ — (1mk.)

$\omega_d = \omega_n\sqrt{1-\xi^2} = 8.66\ rad/sec$ — (1mk.)

$t_s = \frac{4}{\xi\omega_n} = 0.8\ sec.$ — (1mk.)

$\%M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 16.303\%$ — (2mks)

$T_p = \frac{\pi}{\omega_d} = 0.3627\ sec.$ — (1mk.)

Q.5a) $1+G(s)H(s) = 0$

$1+\frac{K}{s(1+0.4s)(1+0.25s)} = 0$

$s(1+0.4s)(1+0.25s)+K=0$

$0.1s^3+0.65s^2+s+K=0$

$$\begin{array}{c|cc} s^3 & 0.1 & 1 \\ s^2 & 0.65 & K \\ s^1 & \frac{0.65-0.1K}{0.65} & 0 \\ s^0 & K & \end{array}$$

(3mks)

For system to be stable, all terms in 1st column must be +ve.

from s^0, $K>0$

from s^1,

$$\frac{0.65-0.1K}{0.65} > 0$$

$\therefore 0.65 > 0.1K$

$K < 6.5$

$\therefore$ range of values of K,

(3mks) $0 < K < 6.5$

$$\frac{0.65-0.1K_{mar}}{0.65} = 0$$

(2mks) $\therefore K_{mar} = 6.5$

$A(s) = 0.65s^2 + K = 0$

At $K = K_{mar}$,

$A(s) = 0.65s^2 + K_{mar} = 0$

$\therefore 0.65s^2 + 6.5 = 0$

$\therefore s^2 = -10$

$\therefore s = \pm j\,3.162$

(2mks) $\therefore w = 3.162$ rad/sec

Q.5b) $G(s)H(s) = \dfrac{80}{s(s+2)(s+20)}$

$$= \frac{2}{s(1+s/2)(1+s/20)}$$

$$G(j\omega)H(j\omega) = \frac{2}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)}$$

Two corner frequencies,

$\omega_{c_1} = 2$ rad/sec.

(2mks.) $\omega_{c_2} = 20$ rad/sec.

Term	Corner freq.	slope of the term (dB/dec)	change in slope (dB/dec)
$\frac{1}{j\omega}$	—	−20 dB/dec.	−
$\frac{1}{1+\frac{j\omega}{2}}$	$\omega_{c_1} = 2$	−20	−20−20=−4
$\frac{1}{1+\frac{j\omega}{20}}$	$\omega_{c_2} = 20$	−20	−40−20=−6

(2mks.)

Phase Angle Plot can be obtained as,

$$\phi = -90^\circ - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/20)$$

(8)

(2 mks)

Freq. ω (rad/sec)	$-\tan^{-1}(\omega/2)$	$-\tan^{-1}(\omega/20)$	ϕ
0.2	$-5.7°$	$-0.57°$	$-96.27°$
2	$-4.5°$	$-5.7°$	$-140.7°$
8	$-75°$	$-21.8°$	$-187.76°$
10	$-78.69°$	$-26.56°$	$-195.29°$
20	$-84.28°$	$-45°$	$-219.28°$
40	$-87.13°$	$-63.43°$	$-240.58°$
∞	$-90°$	$-90°$	$-270°$

(4 mks)