6. b) $\cancel{E(s)} = F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx$ (6)

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \left(a e^{-\alpha x} + b e^{-\beta x}\right) \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[a \cdot \frac{\alpha}{s^2+\alpha^2} + b \cdot \frac{\beta}{s^2+\beta^2} \right]$$

c) a) $f(x) = \frac{2}{\pi} \int_0^\infty \sin \lambda x \, d\lambda \int_0^\infty f(t) \sin \lambda t \, dt$ (4)

$$= \frac{2}{\pi} \int_0^\infty \sin \lambda x \, d\lambda \int_0^\pi \sin t \sin \lambda t \, dt$$

$$= \frac{2}{\pi} \int_0^\infty \sin \lambda x \left(-\frac{1}{2}\right) \left[\frac{\sin t(1+\lambda)}{1+\lambda} - \frac{\sin t(1-\lambda)}{1-\lambda} \right]_0^\pi d\lambda$$

$$= \frac{2}{\pi} \int_0^\infty \frac{\sin \lambda x \sin \lambda \pi}{1-\lambda^2} \, d\lambda$$

(4).

b) $f(x) = \frac{2}{\pi} \int_0^\infty \cos \lambda x \, d\lambda \int_0^\infty f(t) \cos \lambda t \, dt$

$$= \frac{2}{\pi} \int_0^\infty \cos \lambda x \int_0^\infty e^{-s} \cos s \cos \lambda t \, dt \, d\lambda$$

$$= \frac{1}{\pi} \int_0^\infty \cos \lambda x \; \frac{\lambda^2 + 2}{\lambda^4 + 4} \, d\lambda .$$

—— x ——