

Basic statistics

(3 Hours)

[Total Marks : 80

- N.B.** (1) All questions carry 20 marks.
 (2) Q.1 is compulsory and attempt any 3 questions from Q.2 to Q.5.
 (3) Scientific non-programmable handheld calculator is allowed.
 (4) Graph Paper will be provided on request.
 (5) Use of mathematical and statistical tables is permitted.
 (6) Assume relevant data wherever necessary.

1. (a) Fill in the blanks : 10

- (i) If the value of correlation coefficient is 0.93, it can be concluded that there is _____ correlation.
 (a) High degree positive (b) No
 (c) Negative (d) Perfect positive
- (ii) If two regression coefficients are positive, the value of correlation coefficient is _____.
 (a) Negative (b) Positive
 (c) Zero (d) Unknown
- (iii) In decision making problems there is only one _____.
 (a) Policy maker (b) Policy
 (c) State of nature (d) Payoff
- (iv) Maximax criterion is a decision making under _____.
 (a) Certainty (b) Uncertainty
 (c) risk (d) control
- (v) Correlation coefficient is _____ mean of regression coefficient.
 (a) Arithmetic (b) Geometric
 (c) Harmonic (d) Weighted
- (vi) For testing goodness of fit, _____ test is used.
 (a) chisquare (b) F
 (c) ANOVA (d) None of these
- (vii) Chi-squar test is _____ test.
 (a) Parametric (b) Non parametric
 (c) Non statistical (d) None
- (viii) ANOVA is used if means of _____ sample are to be tested.
 (a) One (b) Two
 (c) More than two (d) less than 2
- (ix) _____ chart can be used for testing the defective in a lot.
 (a) \bar{x} (b) R
 (c) C (d) P
- (x) Correlation coefficient lies between _____ ?
 (a) 0 & 1 (b) -1 & 0
 (c) -1 & 1 (d) 2 & 5



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1. (b) State whether following statements are **True** or **False** :

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- (i) In decision theory probabilities are associated with the state of nature.
- (ii) If two variables are independent then they are correlated.
- (iii) If both the regression coefficients are 0.7 each, then the value of correlation coefficient is 0.35.
- (iv) Decision tree calculations begins from right to left.
- (v) While calculating rank correlation coefficient, if variable X is ranked in increasing order then variable Y also has to be ranked in increasing order.
- (vi) Calculated value of F is always greater than one.
- (vii) F test can be used for testing independence of attributes.
- (viii) Chi square test is used for testing means of two samples.
- (ix) Chi square test is a non parametric test.
- (x) F test is used for testing means of two large samples.

2. Solve any **two** :

- (a) The following table gives indices of industrial production of registered unemployed (in hundred thousand). Calculate the value of the correlation coefficient so obtained. 10

Year	1991	1992	1993	1994	1995	1996	1997	1998
Index of Production	100	102	104	107	105	112	103	99
Number Unemployed	15	12	13	11	12	12	19	26

- (b) Calculate Spearman's coefficient of correlation between marks assigned to ten students by judges X and Y in a certain competitive test as shown below : 10

S. No.	1	2	3	4	5	6	7	8	9	10
Marks by judge X	52	53	42	60	45	41	37	38	25	27
Marks by judge Y	65	68	43	38	77	48	35	30	25	50

- (c) The following table shows the ages (X) and blood pressure (Y) of 8 person : 10

X	52	63	45	36	72	65	47	25
Y	62	53	51	25	79	43	60	33

Obtain the regression equation of Y on X and find the expected blood pressure of a person who is 49 years old.

- (d) Write a note on 'Types of Correlations'.

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3. Solve any two :

(a) In a correlation study the following values are obtained :

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	X	Y
Mean	65	67
Standard Deviation	2.5	3.5
Coefficient of Correlation	0.8	

Find the two regression equation that are associated with the above values.

(b) On the basis of observations made on 39 cotton plants the total of yield of cotton (X_1), number of bolls, i.e. seed vessels (X_2) and height (X_3) are :

$$r_{12} = 0.8, r_{13} = 0.65 \text{ and } r_{23} = 0.7$$

Comment on the partial correlation between yield of cotton and the number of bolls, and the effect of height.

(c) Calculate (a) $R_{1,23}$, (b) $R_{3,12}$ and (c) $R_{2,13}$ for the following data :

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$$\begin{aligned} \bar{X}_1 &= 6.8 & \bar{X}_2 &= 7.0 & \bar{X}_3 &= 74 \\ S_1 &= 1.0 & S_2 &= 0.8 & S_3 &= 9 \\ r_{12} &= 0.6 & r_{13} &= 0.7 & r_{23} &= 0.65 \end{aligned}$$

(d) Use exponential smoothing method with $\alpha = 0.6$ and forecast exports for 2011

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Exports	51	53	50	57	60	55	59	62	68	70	72

4. Solve any two :

(a) A nation-wide mail order house desires to verify the accuracy of its clerical work in completing invoices. Sub-groups of 200 are taken each day for 30 consecutive days for inspection. A defective is defined as an invoice containing at least one of a number of possible errors. The number of defectives found in each of these 30 groups are as follows :

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7	5	6	3	4	10	3	2	5	8
6	1	2	3	5	4	8	9	7	10
3	5	4	3	3	7	6	4	3	5

Construct control chart of the data given above and comment on the state of control.

- (b) The table below gives the (coded) measurements obtained in 20 sample (sub-groups). Construct control charts based on the mean and the range. The values of these statistics are given below for the respective samples. 10

Sub-groups																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-1	2	1	2	1	1	-1	1	2	-2	0	2	0	0	-1	1	2	2	0	1
2	0	1	1	-1	-1	1	1	1	1	1	1	1	0	2	-1	1	0	2	-3
1	1	0	0	0	2	0	2	-1	-2	-3	-1	-3	-1	1	2	-1	1	1	-1
0	0	0	-1	0	0	-2	-1	0	2	2	0	2	0	1	0	0	0	-1	1
1	1	1	0	-1	-2	1	0	0	1	1	0	1	1	2	-2	0	1	1	2

Construct suitable control chart & check if the process is in control

- (c) The following table gives the number of errors of alignment observed at final inspection of a certain model of bus. Prepare a C-chart and comment on the state of control. 10

Bus Number	Number of alignment defects	Bus No.	No. of alignment defects
1001	6	1011	8
1002	10	1012	6
1003	8	1013	10
1004	7	1014	10
1005	12	1015	6
1006	9	1016	12
1007	5	1017	3
1008	7	1018	11
1009	3	1019	2
1010	4	1020	1

- (d) A company has to choose one of the four types of Biscuits-Glucose, Multigrain, Coconut and Cream. Sales expected during the next year are highly uncertain. Marketing department estimates the profits considering manufacturing cost, promotional efforts and distribution set up etc as given in table below :

Profits on estimated level of sales (in ₹ Lakhs) for Quantities

Course of Action	S1	S2	S3
	10,000 Quantity	20,000 Quantity	30,000 Quantity
Glucose (A1)	10	30	45
Multigrain (A2)	15	60	80
Coconut (A3)	20	35	60
Cream (A4)	30	55	70

What will be the company manager's decision if following criterion is applied ?

- (i) Maximin (ii) Maximax (iii) Hurwitz (alpha = 0.7) (iv) Minimax regret.

5. Solve any two :

- (a) The following table gives the number of units of production per day turned out by four different types of machines : 10

Employee	Type of machines			
	M ₁	M ₂	M ₃	M ₄
E ₁	40	36	45	30
E ₂	38	42	50	41
E ₃	36	30	48	35
E ₄	46	47	52	44

Using Analysis of Variance (i) test the hypothesis that the mean production is the same for the four machines, and (ii) test the hypothesis that the employees do not differ with respect to mean productivity.

- (b) Kerala Traders Co. Ltd. wishes to test the whether its three salesmen A, B and C tend to make sales of the same size or whether they differ in their selling ability as measured by the average size of their sales. During the last week there have been 14 sale calls-A made 5 calls, B made 4 calls and C made 5 calls. Following are the weekly sales record of the three salesmen : 10

A ₹	B ₹	C ₹
300	600	700
400	300	300
300	300	400
500	400	600
0	-	500

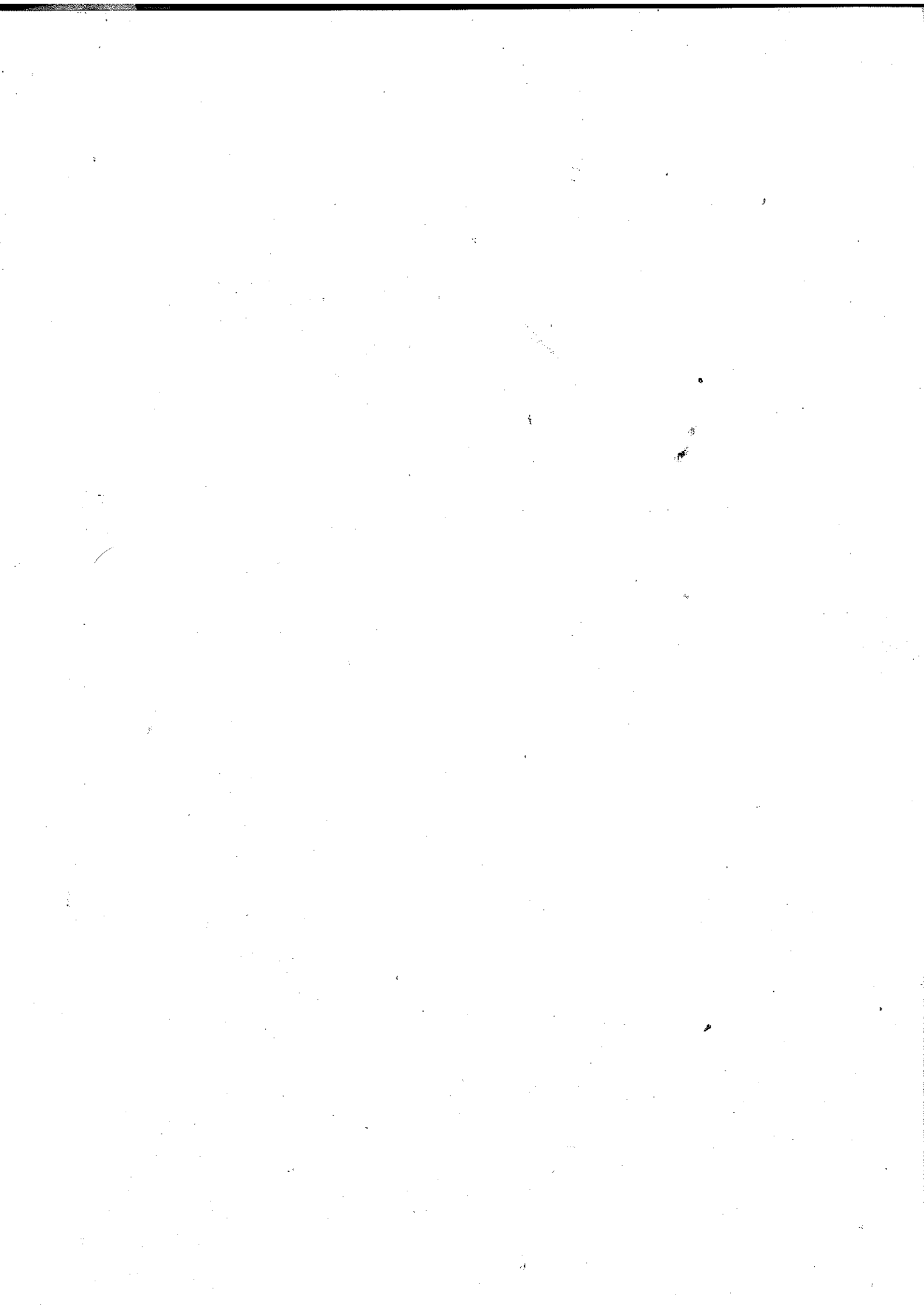
Perform the analysis of variance and draw you conclusions.

- (c) A controlled experiment was conducted to test the effectiveness of drug. Under this experiment 300 patients were treated with new drug and 200 not treated with the drug. The results of the experiment are given below : 10

Details	Cured	Condition worsened	No effect	Total
Treated with the drug	200	40	60	300
Not treated with the drug	120	30	50	200
Total	320	70	110	500

Use chi square and comment on the effectiveness of the drug

- (d) A manager has to make a choice from 3 available courses of action A₁, A₂ and A₃. There are two possible states of nature S₁ and S₂ with probabilities of occurrence as 0.7 and 0.3 respectively. For state S₁, the pay offs for three actions are ₹ 25,000/-, ₹ 35,000/- and ₹ 20,000/- respectively. While for state S₂ the pay offs are ₹ 45,000/- ₹ 50,000/- and ₹ 35,000/- respectively. Represent the problem with the help of a Decision Tree and suggest the most preferred decision and corresponding expected value. 10



Optimisation Models

- N.B.** (1) All questions carry 20 marks.
(2) Q. 1 is **compulsory** and attempt any 3 questions from Q.2 to Q.5.
(3) **Figures** to the **right** indicate marks.
(4) Use of **Non Programmable calculator** only is **allowed**.
(5) Use of **Mobile Phones** in the Exam Hall is **prohibited**.
(6) Support your answers with **diagram/illustration** wherever is **required**.
(7) Refer **Statistical Table** if **required**.

1. Choose the Correct Alternative (solve any 20) :

20

- (a) The assignment model is a special case of the _____ model.
(i) Maximum-flow (ii) Transportation
(iii) Networking (iv) Linear Programming
- (b) An assignment problem is a special form of transportation problem where all supply and demand values equal to _____.
(i) 0 (ii) 1
(iii) 2 (iv) 3
- (c) The Assignment problem is said to be balanced if total rows and total capacities are _____.
(i) Not equal (ii) Equal
(iii) Symmetric (iv) Not defined
- (d) Alternate solution in Assignment problem exist if _____.
(i) There is no single zero available for assignment row or column wise
(ii) Dominance principle is satisfied
(iii) Hungarian is not applicable
(iv) None of these
- (e) The purpose of a dummy row or column in an assignment problem is to _____.
(i) Obtain balance between total activities and total resources
(ii) Prevent a solution from becoming degenerate
(iii) Provide a means of representing a dummy problem
(iv) None of the above
- (f) An optimal of an assignment problem can be obtained only if _____.
(i) Each row and column has only one zero element
(ii) Each row and column has at least one zero element
(iii) The data are arrangement in a square matrix
(iv) None of the above
- (g) The method used for solving an assignment problem is called _____.
(i) Reduced matrix method (ii) Hungarian method
(iii) MODI method (iv) None of the above
- (h) The purpose of the transportation approach for locational analysis is to minimize _____.
(i) Total profit (ii) Total shipping costs
(iii) Total variable costs (iv) Total fixed costs

- (i) The initial solution to a transportation problem can be generated in any manner, so long as _____.
- (i) It minimizes cost
 - (ii) It ignores cost
 - (iii) All supply and demand are satisfied
 - (iv) Degeneracy does not exist
- (j) Which of the following statements about the northwest corner rule is false ?
- (i) One must exhaust the supply for each row before moving down to the next row
 - (ii) One must exhaust the demand requirements of each column before moving to the next column.
 - (iii) When moving to a new row or column, one must select the cell with the lowest cost
 - (iv) One must check that all supply and demand constraints are met.
- (k) A transportation problem has a feasible solution when _____.
- (i) The number of filled cells is one less than the number of rows plus the number of columns.
 - (ii) All the squares are used
 - (iii) The solution yields the lowest possible cost
 - (iv) All demand and supply constraints are satisfied
- (l) When the number of shipments in a feasible solution is less than the number of rows plus the number of columns minus one then _____.
- (i) The solution is optimal
 - (ii) There is degeneracy and an artificial allocation must be created
 - (iii) Dummy source must be created
 - (iv) A dummy destination must be created
- (m) The total cost of the optimal solution to a transportation problem is _____.
- (i) Cannot be calculated from the information given
 - (ii) Can be calculated based only on the entries in the filled cells of the solution
 - (iii) Is calculated by multiplying the total supply (including any dummy values) by the average cost of the cells.
 - (iv) None of these
- (n) In a minimization problem, a negative improvement index (Cell Evaluation) in a cell indicates that the _____.
- (i) Total cost will increase if units are reallocated to that cell
 - (ii) Total cost will decrease if units are reallocated to that cell
 - (iii) Current iteration is worse than the previous one
 - (iv) Problem has no feasible solution.
- (o) The critical path refers to _____.
- (i) Longest path in terms of Duration
 - (ii) Shortest path in terms of Duration
 - (iii) Longest path in terms of length
 - (iv) Shortest path in terms of length

- (p) The full form of CPM is _____
(i) Critical Project Method (ii) Critical Path Method
(iii) Critical Path Management (iv) None of these
- (q) PERT stands for _____
(i) Project Evaluation and Review Technique
(ii) Program Evaluation and Review Technique
(iii) Both (a) and (b)
(iv) None of these
- (r) _____ is deterministic.
(i) PERT (ii) CPM
(iii) Both (iv) None of these
- (s) A two person game is said to be zero-sum, if
(i) gain of one player is exactly matched by a loss to the other so that their sum is equal to zero.
(ii) gain of one player does not match the loss to the other
(iii) both the players must have an equal number of strategies
(iv) diagonal entries of the pay-off matrix are zero.
- (t) A game is said to be fair, if
(i) upper value is more than lower value of the game
(ii) upper and lower values of the game are not equal
(iii) upper and lower values of the game are same and zero
(iv) none of the above
- (u) When maximum and minimax values of the game are same, then
(i) there is a saddle point
(ii) solution does not exist
(iii) strategies are mixed
(iv) none of the above
- (v) Game theory models are classified by the
(i) number of players (ii) sum of all payoffs
(iii) number of strategies (iv) all of the above
- (w) Markov analyse is
(i) helpful in evaluating alternative maintenance policies in production department
(ii) used to determine the optimal course of action in a given problem
(iii) used in the situations involving multiple time periods
(iv) none of the above
- (x) Which of the following is not correct ?
(i) Transition probabilities can also be represented by a probability tree diagram
(ii) Each row of the transition matrix represents and one-step transition probability distribution over all states
(iii) In a system, recurrent states are those which are not transient

2. (a) A company has three factories F_1, F_2 and F_3 with supply of 800, 600 and 1000 units respectively. There are four warehouses W_1, W_2, W_3 and W_4 with demand of 400, 500, 700 and 800 units respectively. A feasible solution is given below. (With allocations and unit cost data).
- Test the solution for optimality using modified distribution method.
 - If the solution is not optimal, find optimal solution by modifying it.
 - Find minimum transportation cost.

From \ To	W_1	W_2	W_3	W_4	Supply
F_1	12 (300)	6 (500)	20	25	800
F_2	6 (100)	11	15 (500)	12	600
F_3	9	15	17 (200)	7 (800)	1000
Demand	400	500	700	800	2400 2400

2. (b) For the following two person, zero sum game, find saddle point using principles of dominance. Also find optimal strategies for each player and value of the game.

Player B

	B-I	B-II	B-III	B-IV	B-V
A-I	20	30	50	60	80
A-II	100	120	110	140	150
A-III	30	40	70	50	90
A-IV	90	40	80	50	30
A-V	60	70	30	80	10

3. (a) M/s. ICT has 3 plants situated at Chennai, Nagpur and Udaipur having monthly manufacturing capacity of 1600 units, 1200 units and 1700 units respectively. These plants supply finished products to their 4 Warehouses situated at Mumbai, Delhi, Hyderabad and Kolkata having their monthly demand of 1000 units, 1200 units, 800 units and 1200 units respectively.

The transportation cost per unit (in Rupees) from plants to warehouses is as follows:

Plants	Warehouses			
	Mumabi	Delhi	Hydrabad	Kolkota
Channai	17	28	15	8
Nagpur	68	14	38	58
Udaipur	38	6	68	18

The shipments from Chennai to Hyderabad and Nagpur to Delhi are not possible due to certain operational problems.

Determine the Initial Feasible Solution for the above problem by supplying Vogel's Approximation Method (VAM) and calculate the Total Transportation costs.

3. (b) A company solicits bids on each of the four projects from five contractors. Only one project may be assigned to any contractor. The bids received (in thousands of rupees) are given in the accompanying table. Contractor D feels unable to carry out project 3 and therefore, submits no bid. 10

Jobs	Contractor				
	A	B	C	D	E
1	18	25	22	26	25
2	26	29	26	27	24
3	28	31	30	-	31
4	26	28	27	26	19

- (i) Use the Hungarian method to find the set of assignment.
(ii) What is the minimum total achievable cost ?

4. (a) The data relating to a project are given below : 10

Activity	Optimistic Time (Days) (a)	Most Likely Time (Days) (m)	Pessimistic Time (Days) (b)
1-2	1	2	3
1-3	2	3	4
1-4	1	4	7
2-4	4	8	12
2-5	2	3	10
3-6	1	2	9
4-6	4	5	6
5-6	3	3	3

- (i) Tabulate expected time and variance of all activities.
(ii) Draw network diagram and find expected completion time.
(iii) Find project completion time for 85% confidence level.

4. (b) A small assembly plant assembles PC's through nine activities. 10

The time duration for which is given below :

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Duration (hrs.)	2	2	1	4	8	5	3	1	5	4	3

- (i) Draw a network diagram for it
- (ii) Calculate and tabulate for each activity :
Earliest Start (ES), Earliest Finish (EF), Latest Start (LS) and Latest Finish (LF) times.
- (iii) Find the critical and sub critical paths.
- (iv) Calculate and tabulate Total Float, Free Float, Interfering Float and Independent Float.

5. (a) Solve the following game by using the principle of dominance : 10

		Player B					
		I	II	III	IV	V	VI
Player A	I	4	2	0	2	1	2
	II	4	3	1	3	2	2
	III	4	3	7	-5	1	2
	IV	4	3	4	-1	2	2
	V	4	3	3	-2	2	2

5. (b) The 'School of International Studies for Population' found out by its survey that the mobility of the population (in per cent) of a state to a village, town and city is in the following percentages. 10

		To		
		Village	Town	City
From	Village	50	30	20
	Town	10	70	20
	City	10	40	50

What will be the proportion of population in village, town and city after two years, given that the present has population of 0.7, 0.2 and 0.1 in the village, town and city respectively ?



Advance linear Programming

N.B. (1) Q.1 is compulsory.

(2) Answer any three questions from Q. 2 to Q.5.

1. (a) Fill in the blanks :

10

- (i) The variables in a linear program are a set of _____ that need to be determined in order to solve the problem.
- (ii) A function which is to be maximized or minimized subject to the given constraints is called as _____ function.
- (iii) The feasible set defined by the constraint set $\{x \geq 0, y \geq 0\}$ is _____.
- (iv) A combination of inputs which all cost the same amount is represented by an _____ line.
- (v) The set of all possible points of an optimization problem that satisfy the problem's constraints is called as the _____ region.
- (vi) Vertices of a feasible region are also referred to as _____ points.
- (vii) If one of the basic variables takes on a zero value, then that LPP is _____.
- (viii) _____ method is used to solve balanced assignment problem.
- (ix) _____ variables may have a positive or a negative sign.
- (x) _____ variables are for \geq constraints.

(b) Match Column A with Column B :

10

Column A	Column B
1. Hungarian Method	a. A redundant constraint
2. No change in feasible region	b. Gomery cut
3. Integer Programming	c. Assignment Problem
4. A Hamiltonian Graph	d. No effect on feasible solution
5. Northwest Corner Method	e. Unbalanced Assignment Problem
6. Redundant Constraint	f. Travelling salesman problem
7. Shortest Path	g. Transportation problem
8. Non Square Cost Matrix	h. Dijkstra's algorithm
9. Non linear constraints	i. Non linear programming
10. Goal Programming	j. Charnes, Cooper and Ferguson

2. (a) Solve the following transportation problem :

10

- (i) North west corner method
- (ii) Vogel's approximation method.

From	To				Available
	A	B	C	D	
I	2	3	11	7	6
II	1	0	6	1	1
III	5	8	15	9	10
Requirement	7	5	3	2	

2. (b) Solve the LPP using Simplex Method :

10

Maximize $Z = 3X_1 + 5X_2 + 4X_3$

Subject to the constraints :

$2X_1 + X_2 \leq 8$

$2X_2 + 5X_3 \leq 10$

$3X_1 + 2X_2 + 4X_3 \leq 15$

$X_1, X_2, X_3 \geq 0$

OR

2. (c) Solve the following transportation problem :

10

- (i) North West corner method
- (ii) Vogel's approximation method

Sites

		To			Supply (Availability)
		A	B	C	
Plants	1	4	3	8	300
	2	7	5	9	300
	3	4	5	5	100
Demand (requirement)		200	200	300	

2. (d) Minimize $Z = 3X_1 + 2X_2$

10

Subject to the constraints

$5X_1 + X_2 \leq 10$

$X_1 + X_2 \geq 6$

$X_1 + 4X_2 \geq 12$

$X_1, X_2 \geq 0$

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3. (a) Maximize $X_1 + 2X_2$
Subject to the constraints

$$2X_2 \leq 7$$

$$X_1 + X_2 \geq 7$$

$$2X_1 \geq 11$$

And $X_1, X_2 \geq 0$ and integers

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- (b) Four jobs (J1, J2, J3 and J4) need to be executed by four workers (W1, W2, W3 and W4), one job per worker. The matrix below shows the cost of assigning a certain worker to a certain job. Find the optimal assignment and the minimum cost. 10

	J1	J2	J3	J4
W1	82	83	69	92
W2	77	37	49	92
W3	11	69	5	86
W4	8	9	98	23

OR

- (c) Maximize $2X_1 + 1.7X_2$
Subject to the constraints

$$4X_1 + 3X_2 \leq 7$$

$$X_1 + X_2 \leq 4$$

$$2X_1 \geq 11$$

And $X_1, X_2 \geq 0$ and integers

10

- (d) At the head office of a company there are five registration counters. Five persons are available for service. How should the counters be assigned to persons so as to maximize the profit ? 10

Person

Counter	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

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4. (a) Use the Wolf's method to solve the following quadratic programming problem : **10**

$$\text{Maximize } Z = 2X_1 + 3X_2 - 2X_1^2$$

$$\text{Subject to } X_1 + 4X_2 \leq 4$$

$$X_1 + X_2 \leq 2$$

$$X_1, X_2 \geq 0$$

- (b) How does solver add-in is used in solving OR problems ? **10**

OR

- (c) Use the Wolf's method to solve the following quadratic programming problem : **10**

$$\text{Minimize } Z = X_1^2 + X_2^2 + X_3^2$$

$$\text{Subject to } X_1 + X_2 + 3X_3 = 2$$

$$5X_1 + 2X_2 + X_3 = 5$$

$$X_1, X_2, X_3 \geq 0$$

- (d) Explain any TWO softwares available for solving different Operations Research problems. **10**

5. (a) Using Graphical Method Maximize $Z = 50x + 18y$ **10**

Subject to the constraints

$$2x + y \leq 100$$

$$x + y \leq 80$$

$$x \geq 0, y \geq 0$$

- (b) What is SPSS and how does it work ? **10**

OR

- (c) Using Graphical Method Maximize $Z = f(x, y) = 3x + 2y$ **10**

$$2x + y \leq 18$$

$$2x + 3y \leq 42$$

$$3x + y \leq 24$$

$$x \geq 0, y \geq 0$$

- (d) Explain the working of LINDO software. **10**