(3 Hours)   [Total Marks : 80

N.B. : (1) All questions carry **20** marks.
   (2) Question No.1 is **compulsory** & attempt 3 questions from Q. N. 2 to 5.
   (3) **Each** question carry **20** marks.
   (4) **Figures** to the **right** indicate **marks**.

1. Select the correct option. Write the number of sub-question & the alphabet corresponding to the correct option.

   1. Inventories are necessary
      (a) to satify demand   (c) avoid shortages
      (b) carry reserve stocks   (d) All of above

   2. Optimum quantity increases. if
      (a) Unit cost decreases   (c) Unit cost is constant
      (b) Unit cost increases   (d) none

   3. As order quantity increases carrying cost.
      (a) increases   (c) remains same
      (b) decreases   (d) none

   4. Which is not an assumption in EOQ
      (a) Ordering cost per order is constant
      (b) Unit price is same
      (c) Inventory carrying cost per annum changes
      (d) none

   5. What is the effect on ordering cost if order quantity increases.
      (a) No effect   (c) decreases
      (b) increases   (d) can not determine.

   6. Queue can be formed only when
      (a) arrival rate is more than service rate
      (b) arrival rate is lesser than service rate
      (c) arrival rate is same as service rate
      (d) none

   7. Result of simulation must be viewed as
      (a) exactly same as reality   (c) approximation
      (b) unrealistic   (d) none

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8. Which application is applicable for sequencing of 2 machines & n jobs.
    (a) LIFO   (c) LILO
    (b) FIFO   (d) none

9. In 'n' jobs & 'm' machines. $m > 2$, the condition to reduce it to 2 machine problem is
    (a) minimum 1st machine $\geq$ maximum of machines in between
    (b) minimum of last machine $\geq$ maximum of machines in between
    (c) both a & b
    (d) either a or b

10. Replacement is concerned with
    (a) items that deteriorates
    (b) items that fail suddenly
    (c) man power
    (d) all above.

11. In case of Time value of money
    (a) Present value factor serves as weight
    (b) Time is important
    (c) Cost is to be discounted
    (d) all above

12. Which is the key characteristic of queneing system.
    (a) Utilization factor   (c) arrival rate
    (b) idle time of server   (d) none.

13. Multiple servers may be
    (a) Parallel   (c) in combination with a & b
    (b) series   (d) all of the above.

14. Process of simulation
    (a) refered as 'Monte Carlo' simulation
    (b) powerful mathematical tool
    (c) does not depend on simulation run
    (d) all above.

15. Simulation can be carried out
    (a) Only when mathematical formula is possible for the situation
    (b) irrespective of mathematical formula
    (c) both a & b
    (d) none.

16. The important step required for simulation
   (a) Test & validate the model   (c) both a & b
   (b) Conduct an experiment   (d) none.

17. Simulation is carried out
   (a) To simulate real life situation
   (b) To realize about complexity of real life problems
   (c) both a & b
   (d) none.

18. In case of single server.
   (a) Multiple queues are possible
   (b) Only one queue is possible
   (c) both a & b
   (d) none

19. In case of multiple server
   (a) Multiple queues are possible   (c) both a & b
   (b) One queue is possible   (d) none

20. In retrogressive failure, the failure probability.
   (a) increases   (c) remains constant
   (b) decreases   (d) none

2. (a) We have six jobs, each of which must go through machines A, B and C in the order ABC. Processing time (in hours) are given in the following table :

Job	:	1	2	3	4	5	6
Machine A	:	8	3	7	2	5	1
Machine B	:	3	4	5	2	1	6
Machine C	:	8	7	6	9	10	9

Determine a sequence for the six jobs that will minimize the elapsed time & find total time.

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(b) A television repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution approximately with an average rate of 10 per 8-hour day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought in ?

3. (a) Annual Demand for product is 10,000 units with its unit price of ₹ 1/- If the ordering cost is ₹ 25/- per order & inventory carrying cost is 12.5% per annum Calculat (Assume 1 year = 300 working days)

    (i)   EOQ

    (ii)   How many orders will be placed in a year

    (iii) Time between two orders

    (iv) Total cost of inventory.

(b) A warehouse has only one loading dock manned by a three person crew. Trucks arrive at the loading dock at an average rate of 4 trucks per hour and the arrival rate is Poisson distributed. The loading of a truck takes 10 minutes on an average and can be assumed to be exponentially distributed. The operating cost of a truck is ₹ 20 per hour and the members of the loading crew are paid @ ₹ 6 each per hour. Would you advise the truck owner to add another crew of three persons ?

4. The following mortality rates have been observed for a certain type of fuse :

Week	:	1	2	3	4	5
Per cent failing by the end of week :		5	15	35	75	100

There are 1,000 fuses in use and it costs ₹ 5 to replace an individual fuse. If all fuses were replaced simultaneously it would cost ₹ 1.25 per fuse. It is proposed to replace all fuses at fixed intervals of time, whether or not they have burnt out, and to continue replacing burnt out fuses as they fail. At what intervals the group replacement should be made ? Also prove that this optimal policy is superior to the straight forward policy of replacing each fuse only when it fails.

5. Observations of past data show the following patterns in respect of inter-arrival durations and service durations in a single channel queuing system. Using the random number table below, simulate the queue behaviour for a period of 60 minutes and estimate the probability of the service being idle and the mean time spent by a customer waiting for service.

Inter-arrival Time		Service Time	
Minutes	Probability	Minutes	Probability
2	0.15	1	0.10
4	0.23	3	0.22
6	0.35	5	0.35
8	0.17	7	0.23
10	0.10	9	0.10

Random numbers (start at North-West corner and proceed along the row)

93	14	72	10	21
81	87	90	38	10
29	17	11	68	99
51	40	30	52	71

(3 Hours)   [Total Marks : 80

N.B. :(1) Question No. 1 is **compulsory** carrying 20 marks.
    (2) Attempt any 3 out of Question 2 to Question 5 carrying **20 marks** each.
    (3) Figures to the **right** indicate marks.
    (4) Use of **Non-programmable** calculator only is **allowed**.
    (5) Use of **Mobile Phones** in the Exam Hall is **prohibited**.
    (6) **Support** your answers with diagram / illustration wherever is **required**.
    (7) Graph Paper/s & Statistical table will be **provided** on demand.

1. (a) Select the appropriate alternative and write the proper sub-question number and   **20**
the correct sub-sub-question alphabet corresponding to the appropriate answer.
    (1) The LPP means the set of various functions having the relationship represented by
        (a) Curved Lines   (c) Diagonal Lines
        (b) Spiral Lines   (d) Geometrical Straight Lines
    (2) The Basic Principle used in obtaining Dual from given Primal is ------
        (a) Inverse of a Matrix .   (c) Identity Matrix
        (b) Transpose of a Matrix   (d) Diagonal of a Matrix
    (3) For any LPP, The Optimum Solution obtained by Graphical Method is always ----------obtained by Simplex Method.
        (a) Greater than that
        (b) Lesser than that
        (c) Either greater or lesser than that
        (d) Neither greater nor lesser than that
    (4) Each and every Constraint in any LP Model is expressed by------
        (a) $\leq$ (c) $=$
        (b) $\geq$ (d) All of the above
    (5) An optimum solution to any LPP is always Proper Sub-set of --------
        (a) Incomplete Solution   (c) Vague Solution
        (b) Complete Solution   (d) Feasible Solution
    (6) Out of the following the most Correct Algorithm that you feel is ------
        (a) Simplex; Tabulation; Formulation; Reformulation.
        (b) Formulation; Tabulation; Reformulation; Simplex.
        (c) Tabulation; Formulation; Reformulation; Simplex.
        (d) Tabulation; Reformulation; Formulation; Simplex.

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(7) In Simplex Method for Minimization there will always be an existence of _____
   (a) Slack Variables   (c) Additional Variables
   (b) Surplus Variables   (d) b) along with c).

(8) Which of the variable/s is/are used in the case of Pure Minimization of LPP to be solved by Simplex.
   (a) Only Surplus   (c) Only Ancilliary
   (b) Only Slack   (d) None of the above

(9) The Shadow Price is obtained in the
   (a) Shadow Feasible Region in Graphical Solution.
   (b) Non -shadow region of Feasible Solution in GM for LPP.
   (c) NER part of Simplex table.
   (d) in the Min. Ratio Colum

(10) Sesitivity Analysis is a process applicable to
   (a) Only Coefficients in Objective function.
   (b) Only Coefficients in the Constraints.
   (c) Numerals on RHS of each constraint.
   (d) All the above three.

(11) The Degeneracy can be detected in SIMPLEX by
   (a) Only Basic Variable Quantity is Zero.
   (b) There is a tie in Replacement Ratio Colum.
   (c) Both of the above a) and b).
   (d) None of the above a), b) and c).

(12) If there exist Δ value of Non Basis variable to be zero then there occurs
   (a) Alternate Solution.   (c) Degenerate Solution.
   (b) Unique Solution.   (d) Infesible Solution.

(13) The value under Slack Variable in the Row of Z_j is termed as-------
   (a) Degenerate Value   (c) Unique Value
   (b) Alternate Solution   (d) Shadow Price

(14) In Simplex Process, the order of x) Entering Variable and y) Departing Variable is
   (a) x then y   (c) Only x and not y
   (b) y then x   (d) Only y and not x.

(15) Sensitivity Analysis.
  (a) provides the range within which parameter may change without affecting optimality.
  (b) Allows the decision-maker more meaningful information about the changes in LP model parameters.
  (c) is also called as the Post Optimality Analysis as it is carried out after the Optimal Solution is obtained.
  (d) is all of the above.

(16) The Graphical Solution Set to LPP may have
  (a) Single Point   (c) Convex Polygon
  (b) A Line Segment   (d) Either of the above three a), b) or c)

(17) Dual of Dual Simplex in LPP is termed as
  (a) Trial   (c) Primal itself
  (b) Dual only   (d) None of the above

(18) Collecting and Scrutinising the data values given in Word form are first to be written in typical form called as
  (a) Formulation only   (c) Tabulation only
  (b) Reformulation only   (d) All the above

(19) Operations Research Approach is
  (a) Innovative   (c) Multi-disciplinary
  (b) Scientific   (d) All of the above

(20) LP Model is based on the assumption of
  (a) Certainty   (c) Proportionality
  (b) Additivity   (d) All of the above.

2. Attempt both :
  (a) Solve graphically   10
      Minimize $Z = 4x + 8y$
      Subject to $2x + 2y \leq 8$
      $3x + 2y \geq 30$
      $4x + 2y \leq 36$
      $x, y \geq 0$

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(b) Solve by Simplex Method 10

Maxmize $Z = 40x + 60y$

Subject $4x + 5y \leq 120$

$5x + 3y \leq 150$

$x, y \geq 0$

3. Attempt both :

(a) The problem is represented as follows 10

Maximize $Z = 200x_1 + 300x_2$

Subject to $2X_1 + 4X_2 \leq 17$

$3X_1 + 3X_2 \leq 15$

$x_1, x_2, \geq 0$

C_j		200	300	0	0	Qty (B)	Min Ratio OR Replacement Ratio
Basis		X_1	X_2	S_1	S_2		
X_2	300	0	1	1/2	-1/3	7/2	??
X_1	200	1	0	-1/2	2/3	3/2	??
Z_j		??	??	??	??	??	??

(i) Find the Values at the places of ??

(ii) Find if the solution is Optimum or not. State why ?

(iii) What are the Shadow Prices of the Resources ?

(b) The problem is represented as follows 10

Minimize $Z = 60X_1 + 22X_2$

Subject : $6X_1 + X_2 \leq 3$ ---------- (R1)

$4X_1 + 2X_2 \leq 4$ -------- (R2)

$X_1, X_2, \geq 0$

(i) Write the Dual of the above Primal.

(ii) Solve the Dual and get the Solution for Primal.

4. A business problem is formulated and expressed below as an LPP.   **20**

X_1 & X_2 are the production volumes of products A & B.

R_1 & R_2 are the resources of the Objective function.

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

Subject to

(1) $X_1 + X_2 + X_3 \leq 3$

(2) $X_1 + 4X_2 + 7X_3 \leq 9$

$X_1, X_2, X_3 \geq 0$

The Simplex Tableau for LPP is given as

Profit coefficient C_B	Basis Variables X_B	Solution Values B	C: X:	4 X_1	6 X_2	2 X_3	0 S_1	0 S_2
4	X_1	1		1.	0	-1	4/3	-1/3
6	X_2	2		0	1	2	-1/3	1/3
	Z_i = ?		X_2	2	6	8	10/3	2/3
	$\Delta = C_j - Z_j$			0	0	-6	- 10/3	-2/3

(i) Find the value of ? State if the solution is optimum. Why ?

(ii) Find the Range of Basic variable coefficient C_1, such that the current optimum product mix does not get changed.

(iii) If C_3 is raised from 2 to 12 what effect will you get ?

(iv) If the Coefficients in the objective function are changed from 4, 6 and 2 respectively to 2, 8 and 4 what effect you will get on current optimal product mix.

5. A Solution to LPP is given as under. Study it carefully from each nook n corner and   20
   answer the Questions it follows.

Maximize $Z = 2X_1 + 5X_2 + 8X_3$
Subject to

(1) $6X_1 + 8X_2 + 4X_3 \le 96$

(2) $2X_1 + X_2 + 2X_3 \le 40$

(3) $5X_1 + 3X_2 + 2X_3 \le 60$

   $X_1, X_2, X_3 \ge 0$

Profit coefficient C_B	Basis Variables X_B	Solution Values B	2 X_1	5 X_2	8 X_3	0 S_1	0 S_2	0 S_3
5	X_1	8/3	1/3	1	0	1/6	-1/3	0
8	X_2	56/3	5/6	0	1..	-1/12	2/3	0
0	S_3	44/3	7/3	0	0	-1/3	-1/3	1
	Z_j	488/3	25/3	5	8	1/6	11/3	0
NER	$\Delta_j = C_j - Z_j$		-19/3	0	0	-1/6	-11/3	0

Answer the following Questions :

(i)   Is it a Feasible Solution ? Why ? ; Is it an optimum solution ? Why ?

(ii) Do you think the problem has an Alternate solution ? If Yes, Find it
      OR else state why so ?

(iii) Are you getting Optimum Product Mix ? If so, Show the same along
      with Profit Contribution.

(iv) Suppose, Company has decided to produce 6 units of X_1, will X_2 and
      X_3 have to be reduced ? If so why ?

(v)   Keeping the Company's total profit unchanged, how much will the
      price of product X_1 have to be hiked if any customer wishes to pay
      more price for the same product X_1 ?

(3 Hours)   [Total Marks : 80

N.B. :(1) All questions carry 20 marks.

(2) Q. 1 is compulsory and attempt any **three** questions out of Q. 2, Q. 3, Q. 4 and Q. 5.

(3) Scientific **non-programmable** handheld calculator is **allowed**.

(4) Graph paper will be **provided** on request.

1. (a) Multiple choice questions.   10

   (1) Which of the following is positional average ?

       (a) Geometric Mean   (c) Harmonic Mean

       (b) Arithmetic Mean   (d) Median

   (2) Which of the following Measures of dispersion dependent on all observations ?

       (a) Standard Deviation   (c) Range

       (b) Quartile Deviation   (d) None of them

   (3) The value of probability is :

       (a) Less than 0   (c) more than 0

       (b) between 1 and 0   (d) From 0 to 1

   (4) If $A \cap B = \phi$ then A and B are said to be

       (a) Exhausive   (c) both A and B

       (b) Mutually Exclusive   (d) none

   (5) For which of the following distribution Mean=Median=Mode.

       (a) Binomial distribution   (c) Normal distribution

       (b) Poisson distribution   (d) none

   (6) Which of the following distribution is continuous distribution ?

       (a) Binomial distribution   (c) Normal distribution

       (b) Poisson distribution   (d) none

   (7) If two fair dice are rolled down. What is the probability that the sum of the numbers appeared on upper most faces of the dice is even number ?

       (a) 1/2   (c) 1/4

       (b) 3/4   (d) 1

   (8) Which of the following is a Non-parametric test ?

       (a) t test   (c) Chi square test

       (b) F test   (d) Z test

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(9) In which of the following Non-Random Sampling methods is also known as purposive sampling ?

    (a) Judgement Sampling   (c) Cluster Sampling

    (b) Convenience Sampling   (d) Sequential Sampling

(10) Which of the following can be found out by using Histogram ?

    (a) Mean   (c) Mode

    (b) Median   (d) All of them

(b) Do as directed :   **10**

(1) Which of the following is the **Odd** one out ? Explain briefly.

    Standard deviation, Mode, Range, Quartile deviation

(2) Say whether the following statement is **possible** or **not possible** :

    The mean of the data set is greater than the median.

(3) Whether it is true or false : If the value of arithmetic mean and median are 34.5 and 34.1 respectively, the value of mode can be 33.3.

(4) In experiment of throwing a fair die :

    If the event $A = \{ 3, 4 \}$ then $A^c = $ _____

(5) Write sample space for the following random experiment

    Tossing of two coins

(6) ------------- may be defined as the logical and systematic organisation of statistical data into rows and columns.

(7) For a Normal distribution, mean and standard deviation are equal, whether the statement is **true** or **false**.

(8) Any statistical measure computed from population is known as _____ .

(9) F-test is a non-parametric test, where the statement is **true** or **false**.

(10) --------------- hypothesis is complementary to the null hypothesis.

2.   **20**

(a) Locate mode from the following data :

Rainfall in cms	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No. of Years	2	5	8	12	10	7	6

(b) Explain the importance of classification and tabulation.

(13)

(c) How many four digits numbers can be made by using the digits 1,2,3 and 4 ? When (i) repetition is allowed and (ii) repetition is not allowed.

(d) In how many ways can 5 men, 3 women and 2 children be arranged for photograph so that (i) all women are together and so are all men and children. (ii) all men are together and so are all women.

20

3.

(a) Calculate Quartile Deviation and its co-efficient from the following data.

Class-interval	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	4	12	16	22	10	8

(b) From the following data find Mean and standard deviation.

Income (Rs.)	800	1000	1500	1800	2000	2500
No. of persons	16	24	26	30	20	6

(c) The mean salary of 1000 employee was Rs. 541. Later on after payment of salary, it was found that the salary of two employees was wrongly entered as Rs. 891 and Rs. 495. Their correct salary was Rs. 591 and Rs. 555 Find correct A.M.

(d) What is meant by a "measure of central tendency" ? Discuss merits and demerits of median and mode.

4. (a) Two fair dice are rolled. X denotes the sum of the numbers appearing on the **20** uppermost faces of the dice. From the probability distribution of X find

(a) P (X is a multiple of 3) (b) P (X < 5) (c) P (5<X<10).

(b) Students of a class were given a test in Economics & Accountancy of these students 20% failed in Economics, 15% failed in Accountancy and 5% failed in both. Find the chance that a student selected at random (a) failed in at least one of the two subjects (b) failed in only Accountancy (c) passed in both the subjects.

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(c) A bag contains 7 white balls, 5 black balls and 4 red balls. If two balls are drawn at random from the bag. find the probability that

(a) both the balls are white (b) One is black and other is red

(d) Define : (i) Mutually Exclusive events (ii) Independent events (iii) Conditional Probabiity (iv) Probability Distribution of a random variable X.

5. (a) Explain the terms : (1) Standard Error (2) Null Hypothesis   20
                   (3) Cluster Sampling (4) Simple Random Sampling

(b) The mean life time of sample of 100 fluorescent light tubs produces by a company is computed to be 1,570 hours with a standard deviation of 1200 hours. The company claims that the average life of the tubes produced by the company is 16000 hours. Using the 5 % level of significance is the claim acceptable ?

(c) In a survey of 70 business firms it was found that 45 were planning to expand their capacities next year. Does the sample information contradict the hypothesis that 70% of the firms in general are planning to expand next year ?

(d) Distinguish between a population and a sample. What is random sampling ? Describe some methods of drawing a sample from a finite population.

(3 Hours)   [Total Marks : 80

N.B. :(1) Question No. 1 is compulsory.
   (2) Attempt any **three** questions from Q.2 to Q.5.
   (3) Scientific **non-programmable** handheld calculator is **allowed.**

Q.1 Rewrite the following statements with correct option (any **Ten**) :–   20

   (i) Find the amount after 3 years on a Principal of Rs. 15000 at 10% p.a. –
      (a) 19000   (b) 18295   (c) 19965   (d) 18000

   (ii) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ is equal to

      (a) 25   (b) 27   (c) 20   (d) 10

   (iii) The solution of the set of equation $3x + 4y - = 7, 4x - y = 3$ is
      (a) (1, –1)   (b) (1, 1)   (c) (2, 1)   (d) (1, –2)

   (iv) $\lim\limits_{x \to 0} \dfrac{3^x - 6^x}{x}$ is equal to

      (a) log 1/2   (b) log 2   (c) log 4   (d) log 1/4

   (v) If the average revenue is 45 and the marginal revenue is 36. The elasticity of demand is –
      (a) 6   (b) 7   (c) 5   (d) 3

   (vi) The matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3 \times 3}$ is a _____.

      (a) Scalar matrix   (b) Identity matrix   (c) Null matrix   (d) None of these

   (vii) The equation $x^2 - (p + 4) x + 2p + 5 = 0$ has equal roots, the value of p will be –
      (a) ± 1   (b) 2   (c) ± 2   (d) –2

   (viii) If $f'(x) = \dfrac{x^2}{2} - ax + 1$, $f(0) = 2$ than integration constant is _____

      (a) 3   (b) –3   (c) 4   (d) 2

   (ix) Find $\dfrac{d^2y}{dx^2}$, if $y = \log_5 x$

      (a) $\dfrac{\log x}{\log 5}$ (b) $\dfrac{1}{x \log 5}$ (c) $\dfrac{1}{x^2 \log 5}$ (d) $\dfrac{\log x}{(\log 5)^2}$

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(x) If marginal revenue is MR = 7–4x–x², the revenue function is –

(a) $7x - 2x^2 - \dfrac{x^3}{3} + c$ (b) $-4-2x$

(c) $7x - 4x^2 - \dfrac{x^3}{3}$ (d) $7 - 4x - x^2$

(xi) Simple interest on Rs. 2,000 for 5 years at 6% p.a. is –
     (a) 600   (b) 880   (c) 648   (d) 6000

(xii) The value of determinant A = $\begin{vmatrix} 2 & 1 & 1 \\ 4 & -3 & 1 \\ -2 & 5 & 1 \end{vmatrix}$ is

(a) 8   (b) –8   (c) 10   (d) –10

Q.2 (a) Find the values of x and y for equations x + 5y = 36 and $\dfrac{x+y}{x-y} = \dfrac{5}{3}$ **20**

(b) Solve $3x^2 - 18 + \sqrt{3x^2 - 4x + 6} = 4x$

(c) Find the inverse of the matrix A where $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

(d) Find product of the following two determinants :–

$A = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 5 & 8 \\ 1 & 3 & 2 \\ 0 & 4 & 6 \end{vmatrix}$

Q.3 (a) A man borrowed a certain amount for 2 years from his friend at 3% and had to pay   **20** a simple interest of Rs. 120. He once again took a loan of the same amount for 4 years from a bank at 15% interest, compounded quarterly. Find the interest he will have to pay to the bank.

(b) What sum of money will amount to Rs. 6050 in 3 years at 7% p.a. simple interest ?

(c) Consider –

$f(x)$ = $x + 1$ $-1 < x < 0$
       = 1 $0 \le x < 1$
       = x $1 \le x \le 2$

Examine whethr f(x) is continuous at x = 1

(d) Evaluate $\displaystyle\lim_{n \to \infty} \dfrac{n^2(n+5)}{n(n+1)(n+2)}$

(17)

4. (a) If $z = \log\left(\dfrac{x^3 + y^3}{x+y}\right)$. Show that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 2$ **20**

   (b) The following is a linear homogenous production function $x = \sqrt{aL^2 + 2hLK + bk^2}$ where X, L, K represent output, labour and capital respectively. Show that

$$L.\frac{\partial X}{\partial L} + K.\frac{\partial X}{\partial L} = X$$

   (c) The manufacturer x units of articles at a cost $(12x + 95)$ per units and the demand function if $P = 47x - 45$, when P is price and x is demand. Find x for which the total profit is increasing.

   (d) Find $\dfrac{dy}{dx}$: (i) $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ (ii) If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

5. (a) Evaluate $\displaystyle\int \dfrac{e^{5x}}{\sqrt{e^{5x}+1}}\,dx$ **20**

   (b) The marginal cost of a production of a firm is given as $c'(x) = 5 + 0.13x$. Further, the marginal revenue is $R'(x) = 18$. Also it is given that $C(0) = Rs.120$. Compute the total profit.

   (c) Find the consumer surplus and producer surplus under pure competition for demand function $p = \dfrac{8}{x+1} - 2$ and supply function $p = \dfrac{x+3}{2}$, where p is price and x is quantity.

   (d) Evaluate $\displaystyle\int_{1}^{3} \dfrac{1}{x(1+x^2)}\,dx$.