M.SC. (MATHS) PART-I
Algebra
(Rev) (JAN - 2019)

Q. P. Code: 35869

Time: 3 Hours **Total Marks: 80**

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

Q1. (a) If $V\ (F)$ is a finite dimensional vector space then prove that any two bases of V will have the same number of element.

(b) Show that the vectors (1,2,1) , (2,1,0) ,(1 ,-1,2) form a basis of R^3.

Q2. (a) If W be a subspace of a finite dimensional vector space $V\ (F)$, then show that

$\dim\ V/W = \dim V - \dim W$.

(b) Find the matrix of the linear transformation T on $V_3\ (R)$ defined as

$T(a, b, c) = (2a + c, a - 4b\ , 3a)$ with respect to ordered basis B' where

$B' = \{ (1,1,1), (1,1,0), (1,0,0)\}$.

Q3. (a) If a $n \times n$ matrix A has n distinct Eigen values, then prove that A is diagonalizable.

(b) Find all (complex) characteristic values and characteristic vectors of the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q4. (a) If $\alpha\ and\ \beta$ are vectors in an inner product space $V\ (F)$. Then prove that

$\| \alpha + \beta \| \leq \| \alpha \| + \| \beta \|$

(b) Prove that two vectors $\alpha\ and\ \beta$ in a real inner product space are orthogonal if and only if

$\| \alpha + \beta \|^2 \leq \| \alpha \|^2 + \| \beta \|^2$

SECTION II (Attempt any two Questions)

Q5. (a) Show that a finite semi group G in which cancellation law holds is a group.

(b) Show that the order of a subgroup of a finite group divides the order of the group.

Q6. (a) Show that every homomorphic image of a group G is isomorphic to a quotient group.

(b) Find $Aut\ (G)$, if G is a finite cyclic group of order n .

Q7. (a) Show that the centre of a ring R is a subring of R .

(b) Prove that a division ring is a simple ring.

Q8. (a) Let R be a commutative ring. Prove that an ideal P of R is prime ideal if and only if $R\ /P$ is an integral domain.

(b) If R is a UFD. , then prove that the product of two primitive polynomial in $R[X]$ is a primitive polynomial in $R[X]$.

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M.SC. (MATHS) PART-I
Analysis & Topology
(Rev) (JAN - 2019)

Q. P. Code: 11745

Time: 3 Hours **Total Marks: 80**

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

Q.1. (a) State and prove Lebesgue covering lemma.
(b) State and prove Heine Borel theorem.

Q.2. (a) Define a compact set. Show that the continuous image of a compact set is compact.
(b) Define a connected set. If A and B are connected sets is A∪ B and A ∩ B connected? Justify your answer.

Q.3. (a) When is a function $f : S \to \mathbb{R}$ (where S is an open subset of $\mathbb{R}^n$) said to be differentiable at $a \in S$. Show that if f is differentiable at a, then the total derivative of f at a is unique.
(b) Let S be an open subset of $\mathbb{R}^n$ and I be an open interval in $\mathbb{R}$. Let $f: S \to \mathbb{R}$ and $r: I \to \mathbb{R}^n$ such that $r(I) \subset S$. Let $g = f\ o\ r$. If $r'(t)$ exists at $t \in I$ and f is differentiable at $r(t)$, then show that q is differentiable at $r(t)$ and $q'(t) = \nabla f(a).r'(t)$ where $a = r(t)$.

Q.4. (a) State and prove Implicit function theorem
(b) State the Taylor's theorem and find the Taylor's expansion upto third order a for the function $f(x,y)$ at (x_0, y_0)

$$f(x,y) = e^x \cos y, x_0 = 0, y_0 = \frac{\pi}{2}.$$

SECTION II (Attempt any two Questions)

Q.5. (a) Let X be a topological space and let A be a subset of X. Then show that A is closed if and only if D(A) ⊂ A .
(b) Let X and Y be topological spaces. Then show that a mapping f: X →Y is continuous if and only if the inverse image under f of every open set in Y is also open in X.

Q.6. (a) Let (X, τ) be a topological space and let E be a connected subset of X such that E ⊂ A ∪ B where A and B are separated sets. Then show that E ⊂ A or E ⊂ B that is E cannot intersects A and B.
(b) Show that the image of a locally connected space under a mapping which is both continuous and open is locally connected.

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Q.7. (a) Show that a topological space X is compact if and only if every basic open cover of X has a finite subcover.

(b) Show that every closed subspace of a locally compact space is locally compact.

Q.8. (a) Show that a metric space is compact if and only if it is complete and totally bounded.

(b) Show that the metric space (□ ,d) is complete where d denotes the usual metric of R.

M.SC. (MATHS) PART-I
Complex Analysis
(Rev) (JAN - 2019)

Q.P. Code: 37369

Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

1) (a) Let $\sum_{n=0}^{\infty} a_n z^n$ be a power series and let R be given by $\frac{1}{R} = \lim\sup |a_n|^{\frac{1}{n}}$, $0 \le R < \infty$.

Prove that,

(i) If $|z| < R$, the series converges absolutely.

(ii) If $0 < r < R$ then the series converges uniformly on $\{z : |z| \le r\}$.

(iii) If $|z| > R$, then the series diverges. (10)

(b) Find the roots common to $x^4 + 1 = 0$ and $x^6 - i = 0$. (10)

2) (a) Prove that the Mobius Transformation takes circles onto circles. (10)

(b) Show that under the transformation $w = \frac{z-1}{z+1}$, the straight line $y = x$ is a circle and find its center and radius. (10)

3) (a) If G is an open connected set and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0, \forall z \in G$, then prove that f is constant. (10)

(b) Find an analytic function $f(z) = u + iv$, where $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$. (10)

4) (a) State and prove Cauchy Goursat Theorem. (10)

(b) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)\, dz$ along

(i) The straight line joining (1 - i) to (2 + i).

(ii) Along the curve whose parametric equation is $x = t + 1, y = 2t^2 - 1$. (10)

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Q.P. Code: 37369

SECTION-II (Attempt any two questions)

5) (a) Define: Winding number of a curve. Hence or otherwise prove that if $x:[0,1]\to C$ is a rectifiable closed curve and $\alpha \notin \{x\}$ then $\eta(x;\alpha)$ is an integer, where $\eta(x;\alpha)$ denotes winding number of x w.r.t. the point α. (10)

(b) State Cauchy Integral Formula. Hence or otherwise evaluate

i) $\int_c \frac{z+3}{2z^2+3z-2}dz$, where C is the circle $|z-i|=2$.

ii) $\int_c \frac{z+2}{z^3-2z^2}dz$, where C is the circle $|z-2-i|=2$ (10)

6) (a) State and prove Schwartz Lemma. (10)

(b) Let G be any subset of $\mathbb{C}$. Suppose f is non-constant and analytic function in a domain of G. If $|f|$ attains its local minimum in G at α, then $f(\alpha)=0$. (10)

7) (a) State and prove Casorati - Weierstrass theorem. (10)

(b) Find all the possible Laurent Series expansions of $f(z)=\frac{1}{z(z+1)(z-2)}$. (10)

8) (a) State and prove Cauchy's Residue theorem. (10)

(b) Using contour integration evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2}d\theta$ where $-1 < a < 1$. (10)

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M.SC. (MATHS) PART-I
Discrete Mathematics & Differential Equations
(Rev) (JAN - 2019)

Q. P. Code: 28232

(3 Hours) [Total marks: 80]

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt **any two** questions)

1. A) Prove that "The equation $ax + by = c$ has integer solution if and only if $(a, b)|c$. If x_0, y_0 is a solution , then all integer solutions are given by
$$x = x_0 + \frac{b}{(a,b)}n, \quad y = y_0 - \frac{a}{(a,b)}n, n \in Z."$$ [10]

   B) Solve the congruence $296 \equiv 176\ (mod\ 114)$. [10]

2. A)Determine the number of ways of placing r indistinguishable balls into n indistinguishable boxes, i) with the restriction that no box is empty. ii) with no restriction. [10]

   B)Show that the number of derangements of a set of n elements is
$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots \ldots + (-1)^n \frac{1}{n!}\right]$$ [10]

3. A) Show that among any n+1 positive integers not exactly 2n there must be an integers that divides one of the other integers. [10]

   B) A class of 32 students is organized in 33 teams. Every team consists of three students and there are no identical teams. Show that there are two teams with exactly one common student. [10]

4. A) Let G be a graph with $\|G\| \geq 1$. then show that G has a subgraph H with $\delta(H) > \varepsilon(H) \geq \varepsilon(G)$. [10]

   B) Prove that ' the Petersen graph is not Hamiltonian.' [10]

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Q. P. Code: 28232

SECTION-II (Attempt **any two** questions)

5. A)Prove that a continuously differentiable $f:\mathbb{R}^2 \to \mathbb{R}^2$ has the locally Lipschitz property. [10]

   B) Obtain approximate solution to with in t^3 of the initial value problem: [10]

$$\frac{dx}{dt} = 2x + 3y, x(0) = 1.$$

$$\frac{dy}{dt} = t + y, \quad y(0) = 2.$$

6. A) If $\emptyset_1(x)$ is a solution of $L_2(y) = 0$ on an interval I and $\emptyset_1(x) \neq 0$ on I then show that the other linearly independent solution of $L_2(y) = 0$ is $\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \left[\frac{1}{\emptyset_1(t)^2} e^{-\int a_1 t dt}\right] dt.$ [10]

   B) One solution of $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$ of $x > 0$ is $\emptyset_1(x) = x$. Find a basis for the solutions for $x > 0$. [10]

7. A) Show that $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$. where $P_n(x)$ is Legendre polynomial of degree *n*. [10]

   B) Find the normal form of the Strum-Liouville equation $\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + \lambda q(x)y = 0.$ [10]

8. A) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$, which passes through the line $x_0(t) = 1$, $y_0(t) = 0$, $z_0(t) = t$. [10]

   B) Solve $u_y = {u_x}^4$ $u(x,0) = x^2$, $x \in \mathbb{R}$. [10]

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M.SC. (MATHS) PART-I
Set Theory, Logic & Elementary Probability Theory
(Rev) (JAN - 2019)

Q. P. Code : 40991

[Marks: 80

N.B. 1) Attempt any two questions from section - I and any two questions from section - II.
2) All questions carry equal marks.
2) Answers of section I and section-II should be written in **different** answer books.

SECTION-I (Attempt any two questions)

1. (a) Define partition of a nonempty set S. Show that every partition of S defines unique equivalence relation on S and conversely. (10)

   (b) Prove or disprove:
   i. $P(A \cup B) = P(A) \cup P(B)$ (5)
   ii. $P(A \cap B) = P(A) \cap P(B)$ where $P(X)$ denotes power set of X. (5)

2. (a) Show that open interval $(0, 1)$ and set of real numbers $\mathbb{R}$ have same cardinality. (5)

   (b) Show that open interval $(0, 1)$ is uncountable. (5)

   (c) If B is non-empty set then show that following statements are equivalent. (10)
   1) B is countable
   2) There is surjective function $f : \mathbb{Z}_+ \to B$
   3) There is injective function $g : B \to \mathbb{Z}_+$.

3. (a) State Axiom of Choice and use it to show that given a collection $\mathcal{B}$ of nonempty sets (not necessary disjoint) there exists a function $c : \mathcal{B} \longrightarrow \cup_{B\in\mathcal{B}} B$ such that $c(B)$ is the element of B for each $B \in \mathcal{B}$. (10)

   (b) Define the following terms in partially ordered set: (5)
   1) Maximal element
   2) Lower bound
   3) Chain (totally ordered set)

   (c) If $S \subset \mathbb{Z}_+$ such that $1 \in S$ and $k + 1 \in S$, whenever $k \in S$ then show that $S = \mathbb{Z}_+$ where $\mathbb{Z}_+$ is set of natural numbers. (5)

4. (a) Show that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles. (10)

   (b) i. Define symmetric group S_n. How many elelments of order 5 in S_7. (5)
   ii. Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form. (5)

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SECTION-II (Attempt any two questions)

5. (a) Give any two definitions of probability. State their limitations. (05)

(b) State and prove monotone and subtractive property of probability. (05)

(c) A class C of subsets of Σ, such that either A or $\overline{A}$ are countable. Is C a sigma field? (05)

(d) A boy is throwing stones at a target, if the probability of hitting the target is 3/5. (i) Write the probability distribution of X = no. of attempts required before hitting the target. (ii) What is the probability that target it hit on the 5th attempt ? (iii) Find the mean and variance of X. (05)

6. (a) Define pair wise independence and mutual independence. Examine for pair wise and mutual independence of events K, R, and S which are respectively getting of a king, red and spade card in a random draw from a well shuffled pack of 52 cards. (05)

(b) Explain the concepts of (i) Partition of the sample space, (ii) Borel sigma field, (iii) Dirac measure, (iv) Limit superior of events and (v) Lebesgue measure. (05)

(c) Find constant K, if following is density function: $f(x) = k(1 - x^2), \quad 0 < x < 1$. Hence find (a))$P(-1, 0.75]$, (b) mean of X. (05)

(d) Define a distribution function (d.f) of a continuous random variable and state and prove its any two properties. (05)

7. (a) State (a)Central limit theorem, (b) Weak law of large numbers, (c) Any two properties of Characteristic function, (d) Chebyshev's inequality. (08)

(b) Xi has Binomial with $(n = 3i, p = 1/4), i = 1, 2$. Assuming independence of X_1, X_2, state the distribution of $X_1 + X_2$. Find mean and variance of $X_1 + X_2$. (06)

(c) The joint p.d.f of X, Y is $f(x, y) = 2$ for $0 < x < y < 1$; find conditional p.d.f of X given y. (06)

8. (a) Find K if following is joint pmf of X, Y: $P(x, y) = K(3^{(x-1)}4^y)^{-1}; \quad x, y = 1, 2, \ldots$, are they independent? (06)

(b) For any r.v.'s X, Y show that (a) $E^2[XY] \leq E[X^2]E[Y^2]$ (b)$EyE[X/Y = y] = E[X]$. (06)

(c) Examine whether the Strong law of large numbers holds for sequence of independent r.v.'s (04)

$$\{X_k\}\text{: } X_k = \begin{cases} \pm k & \text{with probability } \dfrac{1}{2\sqrt{k}} \\ 0 & \text{with probability } 1 - \dfrac{1}{\sqrt{k}} \end{cases}.$$

(d) A truck can load 50 containers of identical size and shape. If average weight of the container is 50 kg. and s.d 15 kg. What is the probability that 50 containers will overload the truck. It is given that truck can carry safely 2800 kg? Given $P[Z < 2.83] = 0.9977, P[Z < 0.4] = 0.6554$ where Z has $N(0, 1)$. (04)

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