

(3 Hours)

[Total Marks: 100]

Note: (i) All questions are compulsory.

(ii) Figures to the right indicate marks for respective parts.

Q.1 Choose correct alternative in each of the following (20)

i. From following which is not the characteristic of algorithm?

- (a) Finiteness (b) ambiguity
(c) Input (d) Effectiveness

ii. I. In the best case analysis, we calculate lower bound on running time of an algorithm.
II. In the worst case analysis, we calculate upper bound on running time of an algorithm.

Which of the following is true?

- (a) Only I is correct. (b) Only II is correct.
(c) Both I and II are correct. (d) Both I and II are incorrect.

iii. S_1 : Any algorithm is a program.
 S_2 : Any program is an algorithm.

- (a) Both S_1 and S_2 are true. (b) Both S_1 and S_2 are false.
(c) S_1 is true and S_2 is false. (d) S_2 is true and S_1 is false.

iv. A connected graph, with how many vertices of odd degree has Euler path.

- (a) 5 (b) 1
(c) 2 (d) 3

v. $K_{2,3}$ is a ----- graph.

- (a) planar (b) non-planar
(c) Euler (d) None of these.

vi. The adjacency matrix of a simple graph is

- (a) Identity (b) Skew-symmetric
(c) Symmetric (d) Triangular

vii. The sum of the degrees of the vertices of an undirected graph is always

- (a) Odd (b) even
(c) Prime (d) None of these

viii. To represent hierarchical relationship between elements which structure is suitable?

- (a) Graph
- (b) Tree
- (c) Binary Tree
- (d) None of these

ix. is a tree in which every node has no or two children nodes

- (a) Forest
- (b) Bipartite Graph
- (c) Binary Tree
- (d) Unary Tree

x. What is a full binary tree?

- (a) Each node has exactly zero or two children
- (b) Each node has exactly two children
- (c) All the leaves are at the same level
- (d) Each node has exactly one or two children

Q2. Attempt any **ONE** question from the following: (08)

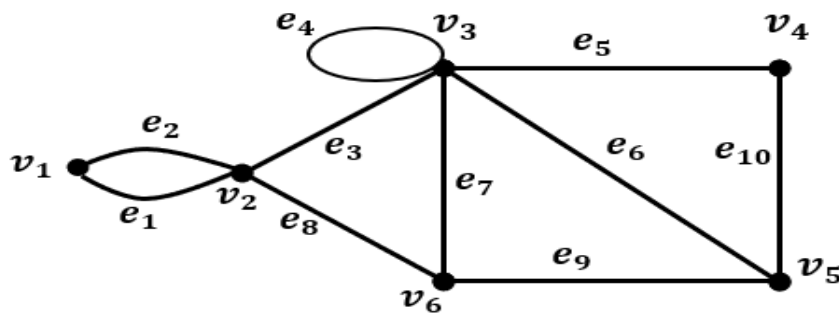
- a) i. Write an algorithm to find sum of n terms of the sequences 3,7,11,15.... Trace your algorithm for $n = 3$. Also calculate worst case time complexity for the above algorithm.
- ii. I. Explain the concept big-O.
II. Check whether $T(n) = n^3 + 20n + 1$ is $O(n^3)$.
III. Check whether $T(n) = n^3 + 20n + 1$ is $O(n^2)$.

Q.2 Attempt any **TWO** questions from the following: (12)

- b) i. Write a recursive algorithm to find 'n!' for given non-negative integer n.
- ii. Design an algorithm that swaps values of a and b
I. Using temporary variable
II. without using temporary variable.
- iii. Describe the Tower of Hanoi problem. Discuss the solution for 3 discs and design the recursive algorithm for n disc.
- iv. Write an algorithm to count total number of positive and negative values from given set.
Take a dry run for set A, where $A = \{20, -17, -5, 0, 5\}$

Q3. Attempt any **ONE** question from the following: (08)

- a) i. Explain which of the following walks are trail, path or simple circuit, where v_i represents vertex and e_i edge of graph.



- (a) $v_1 e_1 v_2 e_3 v_3 e_4 v_3 e_5 v_4$
- (b) $e_1 e_3 e_5 e_5 e_6$
- (c) $v_2 v_3 v_4 v_5 v_3 v_6 v_2$
- (d) $v_2 v_3 v_4 v_5 v_6 v_2$

- ii. Give an example of a graph which contains:
- (i) An Eulerian circuit that is also a Hamilton cycle.
 - (ii) An Eulerian circuit and a Hamilton cycle that are distinct.
 - (iii) An Eulerian circuit, but not a Hamilton cycle.
 - (iv) A Hamilton cycle but not an Eulerian circuit.
- Justify your answer in each case.

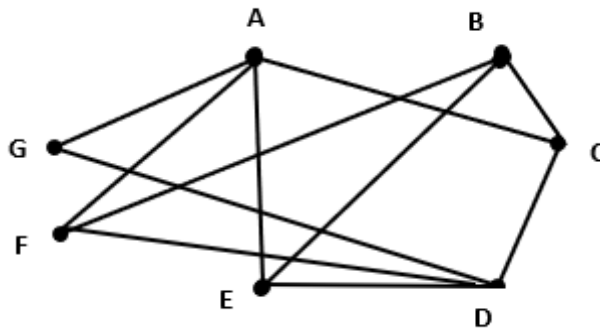
Q3. Attempt any **TWO** questions from the following: (12)

- b) i. Define Cycle C_n , Wheel W_n Draw C_4, C_5, W_4 and W_5 .
- ii. Draw the graph using the following Adjacency matrix.

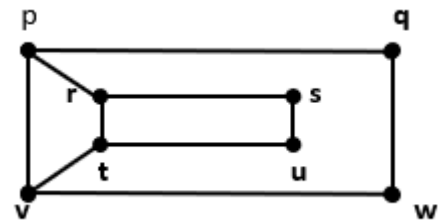
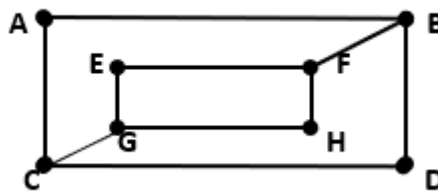
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Also write Incidence Matrix for the same graph.

- iii. Define bipartite graph. Is the following graph bipartite? Justify your answer.

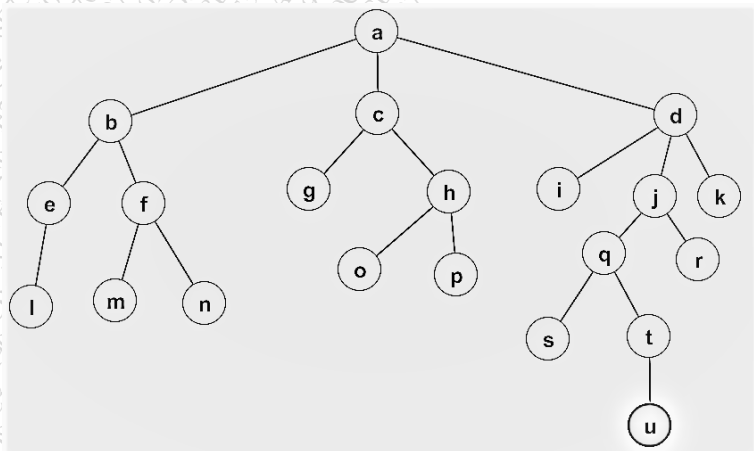


iv. Define isomorphism of two graphs. Determine whether following graphs are Isomorphic. Justify your answer.



Q4. Attempt any **ONE** question from the following: (08)

- a) i. Explain the concepts
- root
 - internal vertices
 - leaves
 - children of f
 - parent of h
 - siblings of o
 - Ancestors of m
 - descendants of b



Write the above parts of the given rooted tree.

- ii. Construct Binary search tree for following key sequence banana, peach, apple, pear, coconut, mango and papaya using alphabetical order.

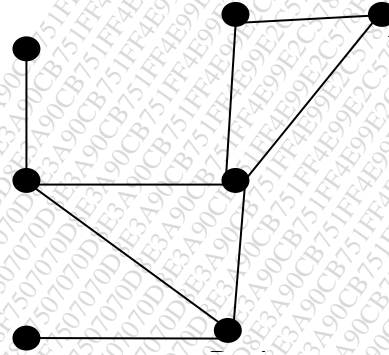
Q4. Attempt any **TWO** questions from the following: (12)

- b) i. Use Huffman coding to encode the following symbols with the frequencies listed.

A: 0.08 , B:0.10 , C:0.12, D:0.15, E:0.20, F:0.35

What is the average number of bits used to encode a character?

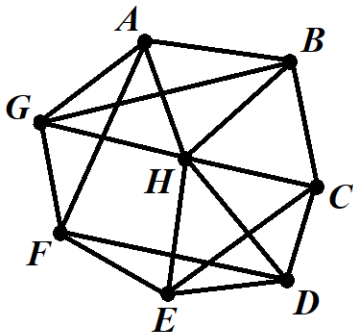
- ii. Use Kruskal's Algorithm to find minimum spanning tree in weighted graph shown in given figure.



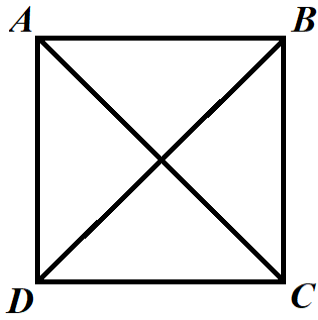
- iii. How many weighings of a balance scale are needed to find a lighter counterfeit coin among four coins? Describe an algorithm to find the lighter coin using the number of weighings.
 iv. How many weighings of a balance scale are needed to find a lighter counterfeit coin among four coins? Describe an algorithm to find the lighter coin using the number of weighings.

Q5. Attempt any **FOUR** questions from the following: (20)

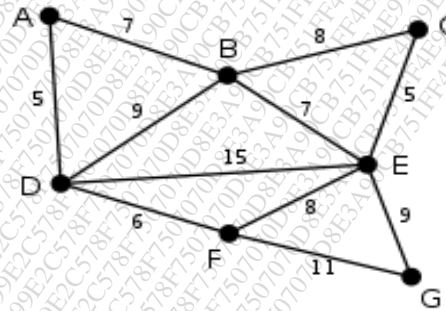
- a) Design an algorithm to finding the greatest common divisor of two positive integers a and b. Trace your algorithm for a=42, b=35
 b) Describe the characteristic of an Algorithm.
 c) State Dirac's theorem. Find the degree of each of the vertices in the following graph and hence prove that the graph has a Hamilton circuit.



- d) Define Planar Graph. State the Euler's formula with respect to Planar Graph. Verify the same for the following graph.



- e) Use Prim's Algorithm to find minimum spanning tree in weighted graph shown in given figure.



- f) a) How many edges does a tree with 10000 vertices have?
 b) How many edges does a full binary tree with 1000 internal vertices have?
