Duration : 2 ½ Hrs. Total marks: 75

N.B. :
1) All questions are compulsory
2) Figures to the right indicate full marks.

Q.1 (a) I) Define an unbiased estimator. [2]

II) i) Give an example of a consistent as well as anunbiased estimator. [2]

ii) Not consistent but unbiased estimator. [2]

(b) State Neyman Factorization Theorem. In each of the following cases, find sufficient estimator. [9]

i) X follows binomial distribution with parameters k and p.

ii) X follows Normal (μ, σ^2) , for σ^2 (μ known)

OR

Q.1 (p) Explain using suitable examples in each case. [4]

i) An unbiased estimator is unique.

ii) An unbiased estimator is always consistent.

(q) Define Relative Efficiency of an estimator. [3]

A r.v X follows Rectangular distribution over [0,θ]

Let $T_1 = 2\bar{X}$ and $T_2 = \left(\frac{n+1}{n}\right) Y_n$ be the two estimators of θ. Y_n is the nth order statistics.

i) check for their unbiasedness [4]

ii) Which estimator is more efficient ? [4]

Q.2 (a) Define MVUE. Prove that it's unique if it exists. [8]

(b) Let $X_1 \ldots X_n$ be a r.s. from binomial distribution with parameters k and p. Obtain CRLB for the variance of an unbiased estimator of p. Also check whether it's attained [7]

OR

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Q.2a (p) State and prove Cramer-Rao Inequality stating clearly the Regularity conditions. [9]

(q) A r.v. X has the following pdf. [6]

$f(x,\theta) = \theta(1-\theta)^{x-1}, x = 1,2,\ldots\infty$

Obtain CRLB of $\frac{1}{\theta}$, $0 < \theta < 1$

Q.3 (a) Write a note on i) method of Maximum Likelihood (MLE). [5]

(b) Obtain the estimarors of the unknown Parameters using MLE. [10]

i) $f(x;\theta) = \frac{\theta^a}{\Gamma a} e^{-x\theta}, x^{a-1}, x > 0$

a known

$\theta > 0$

= 0 , otherwise (o.w)

ii) $f(x,\theta) = \frac{2x}{\theta} e^{-x^2/\theta} e$, $x > 0, \theta . 0$

= 0, o.w.

OR

Q.3 (p) Describe i) 'Method of Moments'. [8]

ii) Method of minimum chi-square and modified minimum Chi-square.

(q) Obtain MLE of θ_1 and θ_2, if pdf of a r.v. X is given by, [7]

$f(x, \theta_1, \theta_2) = \frac{1}{\theta_2} e^{\frac{-(x-\theta_1)}{\theta_2}}, x \geq \theta_1$

$\Theta_2 > 0$

= 0, o.w.

Q.4 (a) Explain the following terms used in case of Bayesian estimation. [8]

i) Prior and Posterior distribution

ii) 'Squered error Loss Function'. (SELF) and Bayes' estimator.

(b) Obtain 100 (1-α) % Confidence Interval (C.I) for population (Normal) variance [7]

σ^2 when μ is unknown.

<u>OR</u>

Q.4 (p) Explain the term 'Pivot Quantity' in obtaining C.I. Obtain 100 (1-α) % C.I. for [8]

$\frac{\sigma_1^2}{\sigma_2^2}$, when two independent samples are drawn from Normal populations with means

μ_1 and μ_2 known.

(q) A r.s. of size n is drawn from binomial distribution with parameters, k and θ. The prior distribution of θ is Beta with parameters a and b. Assuming SELF, obtain Bayes' estimator of θ. [7]

Q.5 (a) i) State Properties of MLE. [8]

ii) Explain exponential Family of probability distributions.

(b) A r.v. X follows exponential distribution with mean θ. Check for its consistency. [7]

<u>OR</u>

Q.5 (p) Prove that two distinct unbiased estimators of unknown Parameter θ give rise to infinitely many unbiased estimators. Illustrate with the help of an example. [8]

(q) Comment on the following [7]

i) MLE is unique.

ii) Binomial distribution with parameters k and p belongs to Exponential Family of distributions.
