QP Code : 40169

Time: 2:30 Hours | Total Marks: 75

N.B: 1) questions are compulsory
2) From questions 1, 2 and 3 attempt any **One** from part (a) and any **Two** from part (b)
3) Attempt any **Three** from question 4
4) Figures to the right indicate marks

1. (a) i) Let U be an open set in $\mathbb{R}^2$ containing the rectangle [a, b] X [c, d]. Suppose f: $U \to \mathbb{R}$ is continuously differential function. Show that g'(x) = $\int_c^d \frac{\partial f(x,y)}{\partial x} dy$ where g(x) = $\int_c^d f(x,y)dy \ \forall x \in [a,b]$. 8

ii) Define the double integral of a bounded function $f : S \to \mathbb{R}$ where $S = [a,b] \times [c,d]$ is a rectangle in $\mathbb{R}^2$. Further show with usual notations $m(b-a)(d-c) \leq \iint_S f \leq M(b-a)(d-c)$ 8

(b) i) Prove that a continuous function is integrable for a rectangular domain in $\mathbb{R}^2$. 6

ii) Evaluate the iterated integral $\int_0^1 \int_{\sqrt{3}y}^{\sqrt{4-y^2}} \sqrt{x^2+y^2}\, dxdy$ by converting to polar co-ordinates. 6

iii) Evaluate $\int_0^9 \int_{\sqrt{x}}^3 \sqrt{1+y^3}\, dydx$ by reversing the order of integration. Sketch the region of integration. 6

iv) Evaluate $\iint_S x^2 y dA$ where S is the region bounded by the lines $2x - y = 1,\ 2x - y = -2,\ x + 3y = 0,\ x + 3y = 1$. 6

2. (a) i) Suppose F is a continuous vector field defined on an open connected set U in $\mathbb{R}^n$. Define a function $\phi: U \to \mathbb{R}$ by $\phi(v) = \int_{v_0}^{v} F$ where v_0 is a fixed point in U and F is conservative. Show that $\nabla\phi(v) = F(v).\ \forall v \in U$. 8

ii) State and prove Green's Theorem for a rectangle.
Evaluate$\oint_c (3y - e^{\sin x})dx + (7x + \sqrt{y^4+1}\,)\, dy$ where $C$is the circle $x^2 + y^2 = 9$. 8

(b) i) Evaluate $\int_C [\frac{1+y^2}{x^3} dx - \frac{1+y^2}{x^3} ydy]$ Where C is the straight line path joining (1,0) to (2,0) and (2,0) to (2,1). 6

ii) $F = (P,Q)$ is a continuously differentiable function defined on a simply connected region D in $\mathbb{R}^2$. Show that $\int_c Pdx + Qdy = 0$ around every closed curve $C\ in\ D$ if and only if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}\ \ \forall\,(x,y) \in D$ 6

iii) Using Green's theorem evaluate the line integral $\oint_C 2x \cos y\, dx + x^2 \sin y\, dy$, where C is the boundary of the region R enclosed between $y = x^2 and y = x$ oriented positively. 6

iv) Calculate the work done in moving the particle from the point $P = (2,-1)$ to $Q = (-4,2)$ by the force field $F(x,y) = (x^2 + 4xy + 4y^2,\ 2x^2 + 8xy + 8y^2)$, by showing first that F is conservative. 6

[Turn over

3 (a) i) Define smoothly equivalent parameterization of a surface S in $\mathbb{R}^3$. Let S be a smooth parametric surface describe by R and r which are smoothly equivalent functions with R(s, t) = r(G(s, t)) where G(s, t) = u(s, t) i+ v(s, t) j being continuously differentiable, then show that $\iint_{r(A)} f ds = \iint_{R(B)} f ds$ Where G(B) = A. 8

ii) State Divergence Theorem for a solid in 3-space bounded by an orientable closed surface with positive orientation and prove the divergence Theorem for cubical region. 8

(b) i) Evaluate $\iint_S F.ndS$ where S is the hemisphere above the XY plane with unit radius and $F(x, y, z) = (x, y, 0)$. 6

ii) Using Stoke's theorem evaluate $\oint_C F.dr$ where $F(x, y, z) = \mathrm{x\,i} + \mathrm{y\,j} + (\mathrm{x}^2 + \mathrm{y}^2)\,\mathrm{k}$, C is the boundary of the part of the paraboloid $z = 1 - x^2 - y^2$ in the first octant . 6

iii) Using Stoke's theorem evaluate $\iint_S curl\, F.n\, dS$ where $F(x, y, z) = (xy, yz, zx)$ where S is the triangular surface with vertices $(1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$. 6

iv) Assuming $S$and V satisfy the conditions of the Gauss Divergence theorem and scalar fieldsf, g have continuous partial derivatives, n is an unit outward normal to surface S and f is a harmonic function. Then prove the following 6

(1) $\iint_S (f\nabla g - g\nabla f).ndS = \iiint_V (f\nabla^2 g - g\nabla^2 f)dV$

(2) $\iint_S \nabla f.ndS = 0$

4 i) $f: [0,1] \times [0,1] \to \mathbb{R}$ is defined by $f(x, y) = \begin{cases} 1, & x \in \mathbb{Q} \\ -1, & x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$ 5

Show that f is not integrable on the given domain.

ii) Use spherical co-ordinates to evaluate $\iiint_S z\, dxdydz$ where S is the solid enclosed by $x^2 + y^2 + z^2$=1, $z \geq 0$. 5

iii) Evaluate the $\int_C f(r)dr$, where $f(x, y, z) = (xz,\ y + z,\ x)$ and $C: x(t) = e^t, y(t) = e^{-t}, z(t) = e^{2t},\ 0 \leq t \leq 1$. 5

iv) Using Green's Theorem, find the area of the region D whose boundary is positively oriented simple closed curve bounded by the lines $y = 1, y = 3, x = 0$ and the parabola $y^2 = x$ oriented positively. 5

v) Using divergence theorem evaluate $\iint_S F.n\, ds$ where $F(x, y, z) = (x^2, y^2, z^2)$ and S is a surface of the sphere $x^2 + y^2 + z^2 = 25$ above the plane $z = 3$. 5

vi) Find surface area of S, where S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 9$. 5