Q. P. Code: 38805

(3 hours) **Total Marks: 80**

N.B: (1) Question no.**1** is **compulsory.**

(2)Attempt any **three** questions from remaining **five** questions.

(3)**Figures** to the **right** indicate **full** marks.

(4)Assume suitable data if necessary.

1. (a) Find the Laplace Transform of $\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}dt$ **(5)**

(b)If $A=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$, find $2A^4-5A^3-7A+6I$. **(5)**

(c) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4}dz$ where C is the circle $|z-1|=3$ **(5)**

(d) The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be (i) between 60 and 75 (ii) more than 75. **(5)**

2. (a)Using the residue theorem evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$. **(6)**

(b)From the following data calculate Spearman's rank correlation coefficient between X and Y

X: 18, 20, 34, 52, 12.

Y: 39, 23, 35, 18, 46. **(6)**

(c)Reduce the following quadratic form to canonical form. Also find its rank and signature

$x^2+2y^2+2z^2-2xy-2yz+zx.$ **(8)**

3. (a)Find the Eigen values and Eigen vectors of $A=\begin{bmatrix}2 & -1 & 1\\ 1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$. **(6)**

(b)Evaluate $\int_C \frac{1}{4(z^2+1)}dz$ where C is the circle $|z|=2$. **(6)**

(c)Find the inverse Laplace Transform of $\frac{1}{(s-3)(s+4)^2}$ using convolution theorem. **(8)**

4.(a)Find the bilinear transformation which maps the points z= 1,i,-1 onto the points w=0,1,∞ . **(6)**

(b)Find the orthogonal trajectory of the family of curves given by $e^{-x}\cos y + xy = \alpha$ **(6)**

(c)Solve the following NLPP using Lagrange's multipliers method **(8)**

$$\begin{aligned}&Optimise\ Z = 196 - 24x_1 - 8x_2 - 12x_3 + 2x_1^2 + 2x_2^2 + 2x_3^2 \\ &subject\ to\ x_1 + x_2 + x_3 = 11;\ x_1, x_2, x_3 \geq 0.\end{aligned}$$

5. (a)Find the inverse Laplace transform of $\log\left[\dfrac{s^2+a^2}{s^2+b^2}\right]$. **(6)**

(b) Show that the matrix A is diagonisable. Also find the transforming matrix and the diagonal matrix where $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ **(6)**

(c)Using Kuhn-Tucker conditions solve the following NLPP **(8)**

$$\begin{aligned}&Maximise\ Z = 2x^2{}_1 - 7x_2{}^2 + 12x_1x_2 \\ &subject\ to\ 2x_1 + 5x_2 \leq 98;\ x_{1,}x_2 \geq 0.\end{aligned}$$

6. (a) Find an analytic function whose real part is $e^{-x}\left(x\sin y - y\cos y\right)$. **(6)**

(b)Find the Laplace transform of $\int_0^t e^{-u}\cos^2 u\, du$. **(6)**

(c) Verify the Cayley-Hamilton Theorem for matrix A and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. **(8)**
