

**[Time: Three Hours]**

**[ Marks:80]**

Please check whether you have got the right question paper.

- N.B:
1. Question.No.1 is compulsory.
  2. Attempt any three questions out of the remaining five questions.
  3. Figures to the right indicate full marks.

- Q.1**
- a) Find the Laplace Transform of  $\frac{\cos\sqrt{t}}{\sqrt{t}}$  **05**
  - b) Verify Cayley-Hamilton Theorem for  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$  and deduce that  $A^8 = 625 I$  **05**
  - c) with usual notation find p of  $9P(X = 4) = P(X=2)$  **05**
  - d) Evaluate  $\int_c \bar{z} dz$ , where c is the upper half of the circle  $r = 1$ . **05**
- Q.2**
- a) Find the inverse Laplace-Transform by using convolution theorem  $\frac{1}{s^2(s+a)^2}$ . **06**
  - b) If  $f(z) = u + iv$  is analytic and  $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ . find  $f(z)$  **06**
  - c) Find a matrix P that diagonalises the matrix  $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$  and hence. Find the diagonal matrix and transforming matrix. **08**
- Q.3**
- a) Find the inverse Laplace Transform of  $\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$ . **06**
  - b) find the Bilinear Transformation which maps the points  $z = -1, 1, \infty$  onto the points  $w = -i, -i, i$ . **06**
  - c) Using the Kuhn-Tucker conditions solve the following N.L.P.P. **08**  
 Maximise  $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$   
 Subject to  $x_1 + x_2 \leq 2$   
 $2x_1 + 3x_2 \leq 12$   
 $x_1, x_2, x_3 \geq 0$

**Q.4** a) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1)dz$ , along the curve  $x = t + 1$ ,  $y = 2t^2 - 1$ . **06**

b) Find the coefficient of correlation between x and y for the following data: **06**

x:	62	64	65	69	70	71	72	74
y:	126	125	139	145	165	152	180	208

c) Reduce the following quadratic form  $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$  to normal form through congruent transformation. Also find its rank, signature and value-class. **08**

**Q.5** a) Evaluate  $\int_0^{2\pi} \frac{\cos 2\theta}{5+4 \cos \theta} d\theta$  **06**

b) The marks obtained by students in a college are normally distributed with mean 65 and variance 25. if 3 students are selected at random from this college what is the probability that at least one of them would have scored more then 75 marks? **06**

c) Find the orthogonal matrix which diagonalises the real symmetrix matrix.  $\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$  **08**

**Q.6** a) Using Lagrange's multiplier method solve the following N.L.P.P. **06**

Optimise  $Z = 4x_1 + 8x_2 - x_1^2 - x_2^2$   
 Subject to  $x_1 + x_2 = 4$   
 $x_1, x_2 \geq 0$

b) Find the eigen values and eigen vectors for **06**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

c) find inverse Laplace Transform of **08**

i)  $\frac{e^{4-3s}}{(s+4)^{5/2}}$       ii)  $\tan^{-1} \left( \frac{s+a}{b} \right)$