M.SC (MATHS) PART-II (JUNE-2018)
ALGEBRA & FIELD THEORY (REV)
(PAPER-I) (JUNE- 2018)

QP Code : 21951

Revised] **(3 Hours)** **[Total Marks:80**

Instructions:

- Attempt **any two** questions from **Section I** and **any two** questions from **Section II**
- **All questions** carry **equal marks**
- **Answers** to **Section I and II** should be written in **same answer book**

SECTION I (Attempt Any Two)

1. (a) Define a simple group. Prove that if G is a finite group and H is a proper normal subgroup of largest order, then G/H is simple.

   (b) State and prove Jordan-Hölder theorem.

2. (a) Show that the character table of D_8 and the quaternion group Q_8 is same.

   (b) State Maschke's theorem. Hence prove that if G is a finite group and F is a field whose characteristic does not divide $|G|$ then every finitely generated FG-module is completely reducible.

3. (a) Define Module and submodule. Show that (i) $\mathbb{Z}$-modules are same as abelian groups and (ii) $\mathbb{Z}$-submodules are same as subgroups.

   (b) Let M, N be R-modules. Let ϕ, ψ be elements of $\text{Hom}_R(M, N)$. Define $\phi + \psi$ by $(\phi + \psi)(m) = \phi(m) + \psi(m)$ for all $m \in M$ and multiplication defined as function composition. Show that w.r.t. above addition and multiplication $\text{Hom}_R(M, M)$ is a ring with 1.

4. (a) LEt S and T be linear transformations of a finite dimensional vector space V over field F. Prove that following are equivalent:
   (i) S and T are similar linear transformations.
   (ii) the $F[x]$-modules obtained from V via S and T are isomorphic $F[x]$-modules
   (iii) S and T have same rational canonical form

   (b) Compute the Jordan canonical form for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}$

SECTION II (Attempt Any Two)

5. (a) Let K be a finite extension field of the field E and let E be a finite extension field of the field F. The show that K is a finite extension field of F and $[K : F] = [K : E][E : F]$.

   (b) Let E be an extension field of the field F. Show that the set of all elements of E that are algebraic over F is a subfield of E.

[TURN OVER

6. (a) Define Separable extension of a field. Show that an irreducible polynomial $f(x)$ over a field F of characteristic $p > 0$ is inseparable iff $f(x) = g(x^p)$, i.e. $f(x)$ is polynomial in x^p.

   (b) Prove that the multiplicative group of a finite field is cyclic.

7. (a) Define Galois group. Let $\tau : \mathbb{C} \to \mathbb{C}$ defined by $\tau(a + bi) = a - bi$. Prove that τ is an automorphism of $\mathbb{C}$.

   (b) Define character χ of a group. If $\chi_1, \chi_2, \ldots, \chi_n$ are distinct characters of group G with values in a field L then show that they are linearly independent over L.

8. (a) Define constructible number. If a and b are constructible numbers then prove that ab is constructible.

   (b) Define solvable group. Show that factor group of solvable group is solvable.

M.SC (MATHS) PART-II (JUNE-2018)
ALGEBRA - II (OLD)
(PAPER-I) (JUNE- 2018)

Q. P. Code: 50376

(3 Hours) [Total Marks: 100

N.B.: (1) **Attempt any FIVE questions**.
(2) Figures to the right indicate marks for respective subquestions.

1. (a) Let G be a group of order $p^k m$, where p is a prime and p does not divide m. Show that G has a subgroup of order p^k. (10)
   (b) Prove that a group of order p^2 is either a cyclic group or isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$. (10)

2. (a) (i) Show that any subgroup of a solvable group is solvable. (5)
   (ii) Find all composition series of cyclic group G of order 18 and verify Jordan-Holder theorem for this group G. (5)
   (b) Let G be a nilpotent group and H a proper normal subgroup of G. Show that
$$H \cap Z(G) \neq \{e\},$$
   where $Z(G)$ denote the center of G. (10)

3. (a) Let R be a ring with 1 and M be an R-module. If A, B are submodules of M, then show that
$$(A + B)/B \cong A/(AB).$$
   Further if $A \subseteq B$, then show that $(M/A)/(B/A) \cong M/B$. (10)
   (b) (i) Let R be a ring with 1 and $F = R^n$ be a free module with basis $\{e_1, \ldots, e_n\}$. Let M be an R-module and let $m_1, \ldots, m_n \in M$. Show that there exist a unique R-module homomorphism $\varphi : F \to M$ such that $\varphi(e_i) = m_i$. (5)
   (ii) Let M be an R-module. For any subset A of M, let
$$RA = \{r_1 a_1 + \cdots + r_n a_n : r_i \in R, a_i \in A, 1 \leq i \leq n, n \in \mathbb{Z}^+\}$$
   Show that RA is a submodule of M and is the smallest submodule of M which contains A (5)

4. (a) Show that a ring R is Noetherian if and only if every increasing sequence of ideals:
$$I_1 \subset I_2 \subset I_3 \subset \cdots$$
   is eventually constant. (10)
   (b) Let R be a PID and M be a finitely generated free module over R, of rank n. Show that every submodule of M is also free of rank $\leq n$. (10)

[TURN OVER]

5. (a) Let K/F be a field extension and $\alpha \in K$ be algebraic over F. Show that there is a unique monic irreducible polynomial $m(x) \in F[x]$, which has α as a root. (10)

   (b) Let $F \subseteq K \subseteq L$ be fields. Show that $[L : F] = [L : K][K : F]$, where if one side of the equation is infinite, the other side is also infinite. (10)

6. (a) (i) If an angle of magnitude θ is constructible, then show that $\sin\theta$ and $\cos\theta$ are constructible real numbers. (5)

   (ii) Let α, β be a constructible real numbers. Show that $\alpha + \beta$ is constructible. (5)

   (b) (i) Let $\tau : \mathbb{C} \to \mathbb{C}$ be defined by $\tau(a + bi) = a - bi$. Prove that τ is an automorphism of $\mathbb{C}$. Also find the fixed field of τ. (5)

   (ii) Let F be a field of characteristic $p > 0$. If K is a finite extension of F such that $[K : F]$ is relatively prime to p, then show that K is separable over F. (5)

7. (a) Let K/F be a Galois extension and let $G = G(K/F)$. Show that there is a bijection between the set of subfields E of K containing F and set of subgroups H of G. (10)

   (b) Find the Galois group of the polynomial $x^4 + x^3 + x^2 + x + 1$ over $\mathbb{Q}$. (10)

8. (a) Let F be a field of characteristic 0, and let $a \in F$. If K is the splitting field of $x^n - a$ over F, then show that $G(K/F)$ is a solvable group. (10)

   (b) Show that there exist polynomials of degree 5 in $\mathbb{Q}[x]$ that are not solvable by radicals over $\mathbb{Q}$. (10)

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QP CODE : 28879

Duration:[3 Hours] [Marks: 80]

N.B. 1) All questions carry equal marks.
2) Solve any **Two** questions from section A.
3) Solve any **Two** questions from section B.

Section A

1. (a) (i) Let A be a rectangle in $I\!R^n$. If $f : A \to I\!R$ is a continuous function then prove that f is integrable on A. (5)

(ii) When a subset A of $I\!R^n$ is said to have a measure zero? Show that the closed interval $[a, b]$ does not have measure zero. (5)

(b) State and prove Fubini's theorem. (10)

2. (a) (i) If $E \subseteq I\!R^n$ and $x \in I\!R^n$ then prove that $m^*(x + E) = m^*(E)$. (5)

(ii) Show that every closed subset of $I\!R^n$ is measurable. (5)

(b) Show that there is a non-measurable subset in $I\!R$. (10)

3. (a) (i) Show by an example that Lebesgue integrable function may not be Riemann integrable. (5)

(ii) Let f be a non-negative measurable function on E. Show that $\int_E f = 0$ if and only if $f = 0$ a.e. (5)

(b) State and prove Egoroff's theorem. (10)

4. (a) (i) If f and g are non-negative measurable functions on E then prove that $\int_E (f + g) = \int_E f + \int_E g$. (5)

(ii) State and prove Monotone convergence theorem. (5)

(b) (i) Let $\{f_n\}$ be a sequence of non-negative measurable function on E. If $f_n \to f$ pointwise a.e. on E then prove that $\int_E f \leqslant \lim_{n\to\infty} \inf \int_E f_n$ (5)

(ii) Let f be a measurable function on E. Prove that f^+ and f^- are integrable over E if and only if $| f |$ is integrable over E. (5)

[TURN OVER

Section B

5. (a) (i) Suppose that if f is integrable periodic function. Given a in R, let f_a be the translation such that $f_a(x) = f(x-a)$ then show that $\hat{f_a}(n) = e^{-ina}\hat{f}(n)$, for all $n \in Z$. (5)

(ii) If f is 2π periodic function and continuous at θ then show that (5)

$$\lim_{N \longrightarrow \infty} S_N(f)(\theta) \longrightarrow f(\theta)$$

where $S_N(f)(\theta)$ is N-th partial sum of Fourier series of function f.

(b) (i) Define the Nth Dirichlet kernel $D_N(x)$ and show that $D_N(x) = \dfrac{\sin(N+1/2)x}{sin(x/2)}$. (5)

(ii) Consider the function f defined on the interval $[-\pi, \pi]$ as (5)

$$f(\theta) = \begin{cases} 1 - \dfrac{|\theta|}{\delta}, & \text{if} \quad |\theta| \leq \delta \\ 0, & \text{if} \quad |\theta| > \delta \end{cases}$$

then Show that $f(\theta) = \dfrac{\delta}{2\pi} + 2\sum_{n=1}^{\infty}(\dfrac{1-\cos n\delta}{n^2\pi\delta})\cos n\theta$.

6. (a) (i) Define an infinite dimensional sequence space $l^2(z)$. Is $l^2(z)$ Hilbert space? Justify. (5)

(ii) Let $\{e_k\}_{k=1}^{\infty}$ be an orthonormal set. Let $f \in H$ and $(f, e_j) = 0$ for all j. Then show that if finite linear combination of elements in $\{e_k\}$ are dense in H then $f = 0$. (5)

(b) (i) Let $\{e_k\}_{k=1}^{\infty}$ be an orthonormal set in Hilbert space H then show that for any $f \in H$, $\sum_{k=1}^{\infty}|(f, e_k)|^2 \leq \|f\|^2$. (5)

(ii) Prove or disprove: Any Hilbert space has an orthonormal basis. (5)

7. (a) (i) Show that the space $L^2[-\pi, \pi]$ is complete. (5)

(ii) State and prove the Riesz-Fischer theorem. (5)

(b) (i) If f is integrable function defined on the circle then show that (5)

$$\|f - S_N(f)\| \leq \|f - \sum_{|n| \leq N} c_n e_n\|$$

for any complex number c_n, where $S_N(f)$ is the N-th partial sum of Fourier series of f and $\{e_n\}$ be orthonormal set.

[TURN OVER

(ii) Show that any two infinite dimensional Hilbert spaces are unitarily equivalent. (5)

8. (a) (i) Let f be a continuous function defined on the unit circle. Let a_n be the n-th Fourier coefficient of f. Let $u(r, \theta)$ be defined on open unit disc by (5)

$$u(r, \theta) = \sum_{n=-\infty}^{\infty} a_n r^{|n|} e^{in\theta}$$

Show that $u(r, \theta)$ is Laplacian function.

(ii) Define the Poisson kernel is given by $P_r(\theta)$. Is the Poisson kernel $P_r(\theta)$ a good kernel? Justify. (5)

(b) (i) State the Dirichlet problem in the unit disc. Find general solution of the Dirichlet problem in the unit disc. (5)

(ii) Let $D = \{(r, \theta)/0 \leq r \leq 1, 0 \leq \theta \leq 2\pi\}$ be the unit disc. Find the solution of Dirichlet problem $\triangle\ u = 0$ on D subject to the constant temperature u_0 along the circumference. (5)

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M.SC (MATHS) PART-II (JUNE-2018)

ANALYSIS -II (OLD)

(PAPER -II) (JUNE- 2018)

Q. P. Code: 50664

[3 hours –Scheme A **Idol** students] Total Marks : 100

[2 hours –Scheme B] Total Marks : 40

N.B (1)Scheme A (IDOL) students will attempt any Five questions.

Scheme B students will attempt any Three questions

(2) **All** Questions Carry Equal Marks. Justify the answers.

Write the scheme , under which you are appearing on page 1 of answerbook .

Q.1.

(a)Prove or disprove for Lebesgue outer measure, if (A_n) is an decreasing sequence of sets ,with respect to set inclusion having finite outer measure an $\varnothing = \cap_{n\in\mathbb{N}} A_n$, then $\lim_{n\to\infty} m^* (A_n) = 0$.

(b) Define a lebesgue measurable set. Show that the collection of the lebesgue measurable sets form an σ algebra of sets .

Q. 2

(a) Show that, $lim\,\sup(f_n)$; $n \in \mathbb{N}$ is a measurable function if each (f_m) is a measurable function.

(b) Show that the measurable functions over ℝ , form a vector space.

Q. 3

(a) State and Prove dominated convergence theorem

(b) Show that a Riemann integrable function is Lebesgue integrable and the integrals have same . value.

Q. 4

(a) Show that $£^1$ [a, b] is not contained in $£^2$ [a, b]. Is $£^2$ [a, b]] $\subset$ $£^1$ [a, b]? Justify..

(b) Give an example of an unbounded function which is lebesgue integrable over ℝ.

[Turn Over

Q. 5

(a) State and prove Tonneli's theorem for lebesue integrable functions over $\mathbb{R}^{2.}$

b) Give an example to illustrate $\int_a^b(\int_a^b f(x,y)dx)\, dy = \int_a^b(\int_a^b f(x,y)dy)\, dx$ but f is not lebesgue integrable over [a, b] × [a, b]

Q. 6

(a) i) Give an example of a lebesgue integrable function which is not Riemann integrable

ii) Give an example of a function so that the improper Riemann integral of f exists but the Lebesgue integral of f does not exist.

b) State and prove Egoroffs theorem.

Q. 7

a) Show that $£^2 [\mathbb{R}]$ is a normed linear space and it is complete.

b) Define Fourier transform. State and prove Plancharel's theorem for $£^2$.

Q. 8

(a) State and prove Fejer's theorem about convergence Fourier series.

(b) Define a complete orthonormal set in $£^2[a, b]$. Illustrate.

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M.SC (MATHS) PART-II (JUNE-2018)
DIFFERENTIAL GEOMETRY & FUNCTIONAL ANALYSIS (REV)
(PAPER -III) (JUNE- 2018)

Q. P. Code : 40989

Duration:[3 Hours] [Marks: 80]

N.B.
1) All questions carry equal marks.
2) Solve any **Two** questions from section A.
3) Solve any **Two** questions from section B.

Section A

1. (a) (i) Prove that every line is intersection of two planes. (5)
   (ii) If A is 3×3 orthogonal matrix with $det(A) = 1$ then show that A has an eigenvalue equals to 1. (5)

   (b) (i) Let $m : I\!R^n \longrightarrow I\!R^n$. Show that m is an isometry which fixes the origin if and only if $\langle m(x), m(y)\rangle = \langle x, y\rangle$ for all $x, y \in I\!R^n$. (5)
   (ii) Prove that a linear operator on $I\!R^2$ is a reflection if its eigenvalues are 1 , -1 and the eigenvectors with these eigenvalues are orthogonal. (5)

2. (a) (i) Define signed curvature of a plane curve and show that the signed curvature is the rate at which the tangent vector of curve rotates. (5)
   (ii) Let $\gamma(t)$ be a regular curve and s be its arc length starting at any point of γ. Then show that γ is smooth function of t. (5)

   (b) (i) Let $\gamma : I \to I\!R^3$ be a parametrized curve and $v \in I\!R^3$ be a fixed vector. If $\dot{\gamma}(t)$ is orthogonal to v for all $t \in I$ and $\gamma(0)$ is also orthogonal to v then show that $\gamma(t)$ is orthogonal to v for all $t \in I$. (5)
   (ii) Let $\gamma : (-1, \infty) \to I\!R^2$. The parametric equation of Folium of Descartes is given by $\gamma(t) = (\frac{3at}{1+t^3}, \frac{3at^2}{1+t^3})$. Show that $\lim\limits_{t\to\infty} \dot{\gamma} = (0, 0)$. (5)

3. (a) (i) Define regular surface. Compute the matrix of the differential map $dx_q : I\!R^2 \longrightarrow I\!R^3$ and hence express the regularity condition in Jacobian form. (5)
   (ii) Define a surface of revolution and hence prove that it is a smooth surface. (5)

   (b) (i) Let S be a regular surface. Show that there exist a vector subspace of dimension two which coincides with the set of tangent vectors $T_p(S)$ for $p \in S$. (5)
   (ii) Let $f(x, y, z) = (x + y + z - 1)^2$. Find the values of c for which the set $f(x, y, z) = c$ is a regular surface. (5)

4. (a) (i) Show that the value of second fundamental form for unit vector $v \in T_p(S)$ is equal to the normal curvature of a regular curve passing through p and tangent to v. (5)
   (ii) Show that any normal section of a surface is a geodesic. (5)

[TURN OVER

(b) Consider parametrized equation $\sigma(u,v) = (u+v, u-v, uv)$. Calculate

(i) The coefficients of the first fundamental form (2)

(ii) The coefficients of the Second fundamental form (2)

(iii) The Gaussian curvature (2)

(iv) The Principal curvatures (2)

(v) The Mean curvature (2)

Section B

5. (a) (i) If X is a compact metric then show that $\mathcal{C}(X)$ is complete. (5)

(ii) Is the space l^∞ complete? Justify your answer. (5)

(b) (i) Prove or disprove: An open subset of a complete metric space is of second category. (5)

(ii) If X_1 and X_2 are isometric and X_1 is complete then show that X_2 is also complete. (5)

6. (a) (i) State and prove the Minkowski inequality for l^p space. (5)

(ii) Show that on a finite dimensional vector space, any norm is equivalent to any other norm. (5)

(b) Prove or disprove

(i) Every finite dimensional subspace Y of a normed space X is closed in X. (5)

(ii) Every infinite dimensional subspace Y of a normed space X is closed in X. (5)

7. (a) (i) An integral operator $T : \mathcal{C}[0,1] \to \mathcal{C}[0,1]$ defined by $T(x) = \int_0^1 k(t,\tau)x(\tau)d\tau$ where k is continuous on the closed square $[0,1]\times[0,1]$ in the $t\tau$ plane. Is the integral operator T a bounded linear operator? Justify your answer. (5)

(ii) Let T be a bounded linear operator then show that the null space $N(T)$ is closed. (5)

(b) Define dual space of a normed space. Show that the dual space of $\mathbb{R}^n$ is $\mathbb{R}^n$. (10)

8. (a) State and prove uniform bounded principle. (10)

(b) State and prove Hahn Banach theorem for extension of linear functionals. (10)

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M.SC (MATHS) PART-II (JUNE-2018)
DIFFERENTIAL GEOMETRY (OLD)
(PAPER -III) (JUNE- 2018)

Q.P. Code :09905

[Time: 3 Hours] **[Marks:100]**

Please check whether you have got the right question paper.

N.B:
1. Attempt any five questions
2. All questions carry equal marks.
3. parts (a) (b) in a question carry **10** marks each.

Q.1 Define an inner product on the n-dimensional vector spce $\mathbb{R}^n$ and prove that the inner prodcuct produces a metric topology on $\mathbb{R}^n$.

Prove : i) any two inner products on $\mathbb{R}^n$ give rise to the same topology.

ii) any linear $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is continous with respect to the topology.

a) i) Explain the notion of an orientation of $\mathbb{R}^n$ prove that each $\mathbb{R}^n$ has exactly orientations.

ii) Define orthohonality of a linear $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$. Prove that if T is orthogonal then det $(T) = \pm 1$.

Q.2 a) i) When a subset $\{ u_1, u_2, \ldots, u_k \}$ of $\mathbb{R}^n$ orthonormal ?

Prove that an orthogonal set $\{ u_1, u_2, \ldots, u_n \} \subseteq \mathbb{R}^n$ gives rise to an orthogonal linear

$T: \mathbb{R}^n \longrightarrow \mathbb{R}^n$.

ii) Prove that $T, S : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are orthogonal implies that $T \circ S$ also is orthogoal and deduce that set $O(n)$ of all orthogonal $T: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a group with respect to composition as the group operation.

a) Describe with justfification the group $O(2)$ in items of its matrices

Q.3 a) Let $c : I \longrightarrow \mathbb{R}^3$ be a regular curve. Define

i) cuvature $k(p)$ of c at a point p

ii) The osculating circle of c at p.

Prove that

$\frac{1}{k(p)}$ = Radius of the osculating circle provided $k(p) \neq 0$

b) Obtain the curvature $k(c(t))$ and the torsion $\tau(c(t))$ of the curve

$c(t) = \left(\cos\left(\frac{t}{\sqrt{2}}\right), \sin\left(\frac{t}{\sqrt{2}}\right), \frac{t}{\sqrt{2}} \right),\ t \in \mathbb{R}.$

[P.T.O]

Q.4 State and prove the fundamental theorem space curves.

Q.5 Let *M* be a regular surface.

a) i) Define a smooth tangent field *X* on *M*.

ii) Prove that a smooth tangent field X on M and a smooth function $f : M \longrightarrow \mathbb{R}$ combine to produce a smooth function $X(f) : M \longrightarrow \mathbb{R}$.

b) i) Let *X* be a smooth tangent field on *M*. Define an integral curve of *X* and prove that given any $p \in M$ there exists a unique maximal integral curve of *X* passing through the point *p*.

ii) Descrie a smooth atlas on the ellipsoide $\frac{x^2}{2} + \frac{y^2}{3} + \frac{z^2}{4} = 1$.

Q.6 Let M be a smooth surface.

a) When is a function $f : M \longrightarrow \mathbb{R}$ smooth?

Prove i) If $F : \mathbb{R}^3 \longrightarrow \mathbb{R}$ is smooth then its restriction $F/_M = f : M \longrightarrow \mathbb{R}$ is smooth on *M*.

ii) If $f, g : M \longrightarrow \mathbb{R}$ are smooth on *M* and *a & b* are any real numbers, prove that $af + bg$ is smooth on *M*.

b) Let *p* be a point of *M* and let $f : M \longrightarrow \mathbb{R}$ be a smooth function.

Prove that f determines a linear form $df(p)$ on the tangent space $T_p(M)$ and verify the identity $d(f.g)(p) = f(p)\, dg(p) + g(p).df(p)$ for all smooth functions $f,g : M \longrightarrow \mathbb{R}$.

Q.7 Let *M* be a regular surface and let *p* be a point of it.

a) i) Define the second fundamental form of *M* at *p.*

ii) Taking *M* as the graph of the functions $f(x, y) = 2x^2 + y^3$, $(x,y) \in \mathbb{R}^2$ and $p=(0,0,0)$, describe a vector basis of $T_p(M)$ and obtain the matrix of the second fundamental form at *p* with respect to the vector basis of our choice.

Q.8 a) Let M be a regular surface and p a point of it.

i) Define the Weingarten map $Wp : Tp(M) \longrightarrow Tp(M)$ and prove that it is a self adjoint operator with respect to the inner product on $Tp(M)$ induced by that of $\mathbb{R}^3$.

ii) Prove that either W_p is a real multiple of the identity operator on $T_p(M)$ or it has two mutually perpendicular eigen vectors.

b) Obtain the principal directions on the circular cylinder $M = \{(x, y, z) \in \mathbb{R}^2 : x^2+y^2=4, z\in \mathbb{R})\}$ at the point $(2, 0, 1)$.

M.SC (MATHS) PART-II (JUNE-2018)
GRAPH THEORY (REV)
(PAPER -IV) (JUNE- 2018)

Q. P. Code : 40404

Duration: 3 hrs Marks: 80

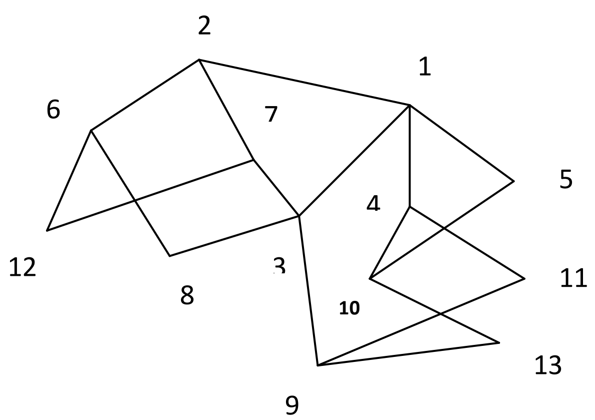
N.B. 1) Both the sections are compulsory.
2) Attempt **ANY TWO** questions from each section.

1. a) Let G(V,E) be an even graph. Then prove that edge set E of G can be partitioned into cycles such that no two cycles will share an edge. 10

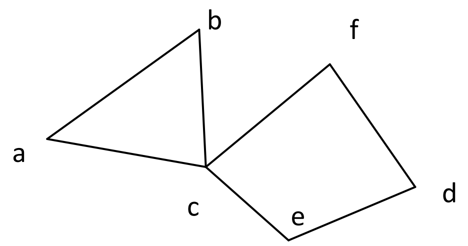
   b) For any graph G , if K is vertex connectivity, K' is edge connectivity and δ is minimum degree of the graph then prove that $K \le K' \le \delta$. 10

2. a) Write an algorithm of Prüfer decoding and decode the Prüfer sequence {2,4,4,4,5,5}. 10

   b) Write the steps of Breadth first search (BFS) algorithm and Implement it on the given graph. 10



3. a) Write an Flueury's algorithm and use it to find Eulerian circuit for the given graph. 10



   b) If a connected graph G has $n \ge 3$ vertices and for every pair u ,v of non adjacent vertices deg(u) +deg(v)$\ge$n then prove that G is Hamiltonian. 10

4. a) State and prove Hall's Theorem . 10

   b)Prove that i) R(2, k) = k for all k ≥ 2 , ii) R(3, 3) = 6 , where R(s, t) is Ramsey number . 10

Section II

5. a) Let G be a graph and let u, v be non adjacent vertices in G then prove that 10
$\chi(G) = \min\{\chi(G+uv), \chi(G.uv)\}$.

   b) Prove that there exist K- colouring of a graph G if and only if V(G) can be partitioned into k-subsets $V_1, V_2 -----, V_K$ Such that no two vertices in V_i $i = 1,2,3,----,k$ are adjacent. 10

TURN OVER

6. a) Prove that there are exactly five regular polyhedron. 10

b) Prove that edges in a plane graph G form a cycle in G if and only if the corresponding dual edges form a bound in G*(Dual graph G). 10

7. a) Prove that a Digraph D is unilaterally connected if and only if there is a directed walk not necessarily closed, containing all its vertices. 10

b) In a directed network the maximum value of an (s, t)- flow equals the minimum capacity of (s, t) cut. 10

8. (a) For every graph G, Prove that $\delta(G) \le \lambda_{\max}(G) \le \Delta(G)$. 10

Where $\delta(G)$ is minimum degree of G, $\Delta(G)$ is maximum degree of G and $\lambda_{\max}(G)$ is maximum eigen value of G.

(b)Define spectrum of graph and find the spectra of path K_3. 10

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M.SC (MATHS) PART-II (JUNE-2018)

GRAPH THEORY (OLD)

(PAPER -IV) (JUNE- 2018)

Q. P. Code: 21629

Marks: 100

N.B. 1) Attempt any **five** questions out of Eight.
2) All questions carry **equal** marks.

1. (a) Write Fleury's algorithm for finding Eulerian trail with one example.
(b) Show that a simple (p,q) graph G with $q > p^2/4$ contains a triangle. State clearly the theorem used.

2. (a) Prove that the center of a tree consists of a single vertex or two vertices joined by an edge. Illustrate the proof.
(b) State and prove Kruskal's algorithm for finding a minimum weight spanning tree.

3. (a) Which is the Hall's matching condition for bipartite graph? Prove it.
(b) Prove that the matching in a graph G is maximum if and only if G contains no M augmenting path.

4. (a) State Menger's theorem and give one of its application.
(b) Prove that in a connected graph G, there do not exist two vertex disjoint longest paths.

5.(a) Prove that graph is Eulerian if and only if it is connected and has no odd vertex.
(b) Prove that a connected graph is isomorphic to its line graph if and only if it is a cycle.

6. (a) If G is a (p,q)- graph($p \geq 3$) such that $\deg(u) + \deg(v) \geq p$, for every non-adjacent pair u,v of vertices in G,then prove that G is a Hamiltonian graph.
(b) If G is a (p,q) graph with $p \geq 3$ and $q \geq (p^2 - 3p + 6) / 2$ then show that G is Hamiltonian.

7. (a) Prove that the graphs K_5 and $K_{3,3}$ are nonplanar.
(b) Prove that there are exactly five regular polyhedral.

8. (a) Prove that flow f in a network N is a maximum flow if and only if N contains no f incrementing path.
(b) Define Ramsey Number R(p,q) for $p,q \geq 2$.Show that $R(p,q) \leq R(p-1,q) + R(p,q-1)$ if $p,q \geq 3$.

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M.SC (MATHS) PART-II (JUNE-2018)

NUMERICAL ANALYSIS. (REV-2017)

(PAPER -V) (JUNE- 2018)

Q. P. Code: 50441

[Total Marks:80

Instructions:

(1) Attempt any two questions from each section.

(2) All questions carry equal marks. Scientific calculator can be used.

(3) Answer to Section-I and Section-II should be written in the same answer book

Section-I

Que. 1 (a) Explain the terms: Inherent error, Round-off error and Truncation error. Find the number of terms of the exponential series such that their sum gives the value of e^x correct to nine decimal places at $x = 1$.

(b) Convert the decimal fraction $(2977.34375)_{10}$ to the binary form and then convert to the hexadecimal form.

Que. 2 (a) Define the term rate of convergence of iterative method and also find the rate of convergence of the Chebyshev method.

(b) Perform two iterations of the Birge-Vieta method to find a root of the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$. Use initial approximation $p_0 = 3.5$.

Que. 3 (a) Let $A = [a_{ij}]$ be a real matrix of order $m \times n$ with $m \geq n$. Derive a formula giving Singular Value Decomposition of a matrix A.

(b) Solve the following system of linear equations using Gauss-Seidel iteration method. $3x + 7y + 13z = 76$; $x + 5y + 3z = 28$; $12x + 3y - 5z = 1$. (Take 5 iterations.)

Que. 4 (a) Define interpolating polynomial and estimate the error in the interpolating polynomial.

(b) Given that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$, find the unique polynomial of degree two or less, which fits the given data. Find the bound on the error.

Section-II

Que. 5 (a) Derive Newton-Cotes quadrature formula and use it to derive Simpson's one third rule for numerical integration.

(b) Evaluate $\int_0^2 \frac{dx}{4 + x^2}$ by Trapezoidal rule taking $h = 2, 1, 0.5$ and then use Romberg's method to get more accurate result correct to four decimal places.

Que. 6 (a) Using the least-squares method, obtain the normal equations to find the values of a, b, c and d when the curve $y = d + cx^2 + bx^4 + ax^6$ is to be fitted for the data points $(x_i, y_i), i = 1, 2, 3......., n$.

(b) Using the Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = x^4 - 5x^3 + 32x + 55$ on $[-1, 1]$ with respect to the weight function $w(x) = \frac{1}{\sqrt{1 - x^2}}$.

[Turn over

Que. 7 (a) Derive the Milne's predictor-corrector formula to solve the differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(b) Use Adams-Bashforth predictor-corrector method to compute $y(1.4)$ correct upto four decimal places, given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ with $y(1) = 1, y(1.1) = 0.996, y(1.2) = 0.986$, $y(1.3) = 0.972$.

Que. 8 (a) Write the difference scheme for Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$.

(b) The one dimensional heat equation $5\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ satisfies the conditions $u(0, t) = 0, u(5, t) = 60$ for $t > 0$ and

$$u(x, 0) = \begin{cases} 20x, & \text{for } 0 < x \leq 3, \\ 60, & \text{for } 3 < x \leq 5. \end{cases}$$

Take $h = 1, k = 0.1$ and use Schmidt's method to compute the values of $u(x, t)$ for three time steps.

M.SC (MATHS) PART-II (JUNE-2018)
NUMERICAL ANALYSIS. (OLD)
(PAPER- V) (JUNE- 2018)

Q.P.Code: 011483

External (Scheme A) (3 Hours) [Total Marks: 100]
Internal (Scheme B) (2 Hours) [Total Marks: 40]

Note:
(1) External (Scheme A) students answer any five questions.
(2) Internal (Scheme B) students answer any three questions.
(3) All questions carry equal marks. Scientific calculator can be used.
(4) Write on top of your answer book the scheme under which your appearing

Q1 a) Define: absolute error, relative error and percentage error. Find absolute error, relative error and percentage error in calculation of $Z = 3x^2 + 2x$ by taking approximate value of x as 3.45, and true value of x as 3.4568.

b) i) Convert decimal number $(0.859375)_{10}$ to corresponding binary number.
ii) Convert binary number $(10110101.110011100)_2$ to Octal number.

Q2 a) Explain Ramanujan Method. Using Ramanujan's method, obtain the first four convergence of $x + x^2 = 1$

b) Derive the Muller's formula to find a root of the algebraic or transcendental equation $f(x) = 0$. Perform one iteration with muller method for
$f(x) = x^2 + x - 1$ & $x_0 = 0, x_1 = 0.5, x_2 = 1$.

Q3 a) Solve the following system by using the Crout's triangularization method.

$$x_1 + x_2 + x_3 = 9$$
$$2x_1 - 3x_2 + x_3 = 13$$
$$3x_1 + 4x_2 + 5x_3 = 40$$

b) Determine the largest eigenvalues and the corresponding eigen vector of the matrix $\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ correct to three decimal places using power method. Take the initial approximate vector as $v^{(0)} = [1 \quad 1]^t$.

Q4 a) Derive Newton's Divided difference formula of interpolation.

b) From the following table, find x for which y is minimum and find this value of y.

x	3	4	5	6	7
y	2.7	6.4	12.5	21.6	34.3

Q5 a) Derive Newton-Cotes Quadrature formula and use it to derive Trapezoidal rule for numerical integration.

b) Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5 + 4\cos x} dx$ by taking 5 ordinates by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.

Q6 a) Using the least –squares method , obtain the normal equation to find the values of a, b and c when the curve $y = c + bx + ax^2$ is to be fitted for the data points (x_i, y_i) $i = 1,2,3, \dots \dots \dots n$.

b) Explain the term Discrete Fourier Transform(D.F.T) and compute the (4-point) D.F.T of the sequence x = (1,2,3,4)

Q7 a) i) Given $\frac{dy}{dx} - 1 = xy$ with $y(0) = 1$, obtain the Taylor series for $y(x)$.
ii) Using Picard's method, obtain the solution of $\frac{dy}{dx} = x(1 + x^3 y)$, $y(4) = 4$.

b) Solve

$$\frac{dx}{dt} = y - t, \qquad \frac{dy}{dt} = x + t$$

TURN OVER

With $x(0) = 1, y(0) = 1$ for $x(0.1)$and $y(0.1)$ by Runge –Kutta Method.

Q8 a) Derive a Jacobi Iteration formula to obtain the numerical solution of one dimensional heat equation

b) Solve $u_t = u_{xx}$ subject to the initial condition $u(x, 0) = sin\pi x \; \forall \; x \in [0,1]$ and $u(0, t) = 0, u(0, t) = 1 \; \forall \; t > 0$ by the Gauss-Seidel Method.

M.SC (MATHS) PART-II (JUNE-2018)
FUNCTIONAL ANALYSIS.
(PAPER- V) *(JUNE- 2018)*

QP Code : 21948

(3 Hours) [Total Marks:100
(2 Hours) [Total Marks: 40

Instructions:

- Write on **top** of your answer book the **scheme** under which you are appearing
- Students of **Scheme B** answer **any three** questions, students of **Scheme A** answer **any five** questions
- All questions carry equal marks

1. (a) Let $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$ be non-negative real numbers. Let $p > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then prove that $\sum a_i b_i \leq (\sum a_i^p)^{1/p} (\sum b_i^q)^{1/q}$

   (b) Let X be a normed linear space and let $f : X \to \mathbb{R}$ be a linear map. Prove that f is continuous iff $\mathrm{Ker}(f)$ is closed in X.

2. (a) Show that the normed linear space l^p $(1 \leq p \leq \infty)$ of all complex sequences $x = (x_n)$ satisfying $\sum_{n=1}^{\infty} |x_n|^p < \infty$ is complete with the norm defined by $\| x \|_p = \left(\sum_{n=1}^{\infty} |x_n|^p \right)^{1/p}$.

   (b) State and prove Riesz's lemma.

3. (a) Let H be a Hilbert space. Show that H is separable iff H admits a countable orthonormal basis.

   (b) Let Y be any closed subspace of a Hilbert space H. Show that $H = Y \oplus Y^{\perp}$ where $Y^{\perp} = \{z \in H \mid \langle z, y \rangle = 0, \forall y \in Y\}$

4. (a) Define an inner product space. Show that if x, y are vectors in an inner product space, then $|\langle x, y \rangle| \leq \|x\| \|y\|$ and equality holds iff $x = cy$ or $y = cx$, for some scalar c.

   (b) Show that every finite dimensional subspace Y of a normed linear space X is complete.

5. (a) Show that the space l^{∞} is not separable.

   (b) State and prove Hahn-Banach Theorem.

6. (a) Let X and Y be Banach spaces and $T : X \to Y$ be a closed linear map. Prove that T is continuous.

   (b) State and prove Closed Graph Theorem.

7. (a) Show that every Hilbert space is reflexive.

   (b) Suppose $T : X \to Y$ is a compact operator with image(T) being closed in Y. Prove that image(T) is a finite dimensional vector space.

8. (a) Show that a linear map $T : X \to Y$ is bounded iff it takes bounded sets of X to bounded sets of Y.

   (b) State and prove Uniform Boundedness Theorem.