Q.P. Code: 28881

(3 hours) **Total marks: 60**

N.B.

1. **Attempt any two questions from question numbers 1, 2, 3 and any two questions from question numbers 4, 5, 6.**
2. **Figures to the right indicate full marks**
3. **Simple non-programmable calculator is allowed.**

1 a. Let X and Y be two independent rvs following $U(0,\theta)$. Suppose we want to test the hypothesis $H_0 : \theta = 2$ against $H_0 : \theta = 1$. Calculate the probability of type one error and power of the test based on the following critical regions. (08)

(i) $W = \{(x, y);\ xy > 0.75\}$ (ii) $W = \left\{(x, y);\ \frac{x}{y} > 0.75\right\}$.

b. Define Uniformly Most Powerful (UMP) test. Let $X_1, X_2, .., X_n$ be iid rvs having pdf $f(x/\theta),\ \theta > 0$ where (07)

$$f(x/\theta) = \begin{cases} \theta x^{\theta-1} & ;\quad 0 < x < 1 \\ 0 & ;\ otherwise \end{cases}$$

Find the UMP test for

(i) $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$

(ii) $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$

2 a. The probability density functions of random variable X under H_0 and H_1 are as follows : (08)

$H_0 : X \sim f_0(x)$ *where*

$$f_0(x) = \frac{1}{\sqrt{2\pi}} \exp[\frac{-x^2}{2}] \qquad ; -\infty < x < \infty$$

against $H_1 : X \sim f_1(x)$, where

$$f_1(x) = \frac{1}{2} \exp(-|x|) \quad ; -\infty < x < \infty.$$

Find MP test of size α based on a single observation for testing H_0 against H_1.

b. Let X be a rv following $C(1, \theta)$. Obtain a most powerful test of level of significance α to test $H_0 : \theta = 0$ against $H_1 : \theta = 1$. State clearly the critical regions for (i) k>1 (ii) k=1. (07)

3 a. Let the rv X has pdf (pmf) $f\left(x/\theta\right)$, where $f\left(x/\theta\right)$ has a MLR in T(x). Consider the one-sided testing problem, $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$; $\theta_0 \in \Theta$. Show that any test of the form (15)

$$\phi(x) = \begin{cases} 1 & ; \quad T(x) > t_0 \\ \gamma & ; \quad T(x) = t_0 \\ 0 & ; \quad T(x) < t_0 \end{cases} \qquad (1)$$

has non-decreasing power function and is UMP of its size α provided that α >0. Moreover show that for every $0 \le \alpha \le 1$ and every $\theta_0 \in \Theta$ there exists a $t_0, -\infty < t_0 < \infty$ and $0 \le \gamma \le 1$ such that the test described in (1) is UMP of its size α for testing H_0 against H_1.

4 a. If R denotes total number of runs when there are n_1 elements of type I and n_2 elements of type II, derive expression for $E(R)$ and $var(R)$. If n_1= 12, n_2 = 10 and observed value of R is 18, using normality, conclude about randomness of the sequence. (10)

b. Show that for large n, $V = 4nD_n^{+\,2}$ follows χ^2 distribution with 2 d.f. If n = 30 and $\chi^2_{2,0.05} = 5.99$ find value of D_n^+. (05)

5 a. State assumptions of Wilicoxon's Signed rank test. Describe test procedure to test $H_0: M = M_0$ against $H_1: M > M_0$. (08)

b. Find the distribution of median for sample of size 4 from uniform distribution U(0, 1). Find mean and variance of the distribution. (07)

6 a. How is Mann-Whitney test different from the Wald-Wolfowitz test? For two samples of size m and n each from continuous populations show that, with standard notations Mann-Whitney statistic satisfies the recurrence relation, (08)

$$r_{m,n}(u) = r_{m,n-1}(u) + r_{m-1,n}(u-n)$$

b. Describe p^th quantile (K_p) of a continuous distribution f(x). How would you find point estimate of K_p from sample? Prove or disprove, (07)

$$P\left[X_{(r)} < K_p\right] = \sum_{i=r}^{n} p^i\,(1-p)^{n-i}$$

Where $X_{(r)}$ denotes r^th order statistics. If n =4, show that

$$P\left[X_{(1)} < K_{0.5} < X_{(4)}\right] = 0.875$$

Hence find 87.5% confidence interval for the median if sample values are 3.2, 2.8, 4.9, 3.7.

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