QP Code : 28089

TOTAL TIME: 3 HRS.
TOTAL MARKS: 80

INSTRUCTIONS: 1) All questions carry equal marks.
2) This paper has TWO sections. Each section carries FOUR questions.
3) Attempt ANY TWO subquestions out of FOUR from each section.

SECTION A

1) a) State and prove Jordan-Holder theorem.
b) Define solvable group. Prove that every group of order eight is solvable.

2) a) With correct justification, write down the charater table for the alternating group on three symbols. Prove that for an abelian group all the characters are one dimensional.
b) State and prove Maschke's theorem.

3) a) State and prove any one of the Noether isomorphism theorems. Explain clearly all the notations used.
b) Let M be an R-module. Prove that a subset N of a M is a submodule of M if and only if N is non-empty and closed under addition as well as scalar multiplication from R.

4) a) Prove that every submodule of a finitely generated module over a principal ideal domain is free.
b) Define rational canonical form. Let V be a finite dimensional vector space over a field F and let T be a linear transformation of V. Prove that there is a basis of V with respect to which the matrix of T is in rational canonical form.

SECTION B

1 a) Define the term: algebraic closure. Prove the existence of an algebraic closure of a field.
b) Prove that if K/F is a finite extension, then it is algebraic. Is the converse always true?

2 a) Prove that separable extensions form a distinguished class.
b) Construct a field of order 8 with correct justification.

3 a) Using Sylow theory prove that the field of complex numbers is algebraically closed.
b) Define the terms: Galois group, fixed field. Determine with correct justification the Galois group of Q(i) over Q. (Here Q denotes the set of rational numbers.)

4 a) Define the term constructible number. Prove that a sum of constructible numbers is constructible.
b) With correct justification, give an example of a polynomial with real coefficients over Q, which is not solvable by radicals.

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