Duration: 3hrs **[Total Marks: 80]**

1) Question **no.1** is **compulsory**.
2) Solve any **three** questions out of remaining **five** questions.
3) All questions carry equal marks as indicated by figures to the right.
4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Prove by induction that n^2+n is an even number, for every natural number n. (**05M**)

b) Find the generating function for the following finite sequences (**05M**)

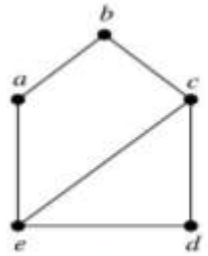
i) 2,2,2,2,2,2 ii) 1,1,1,1,1,1

c) Let A={a,b,c,d,e} and R={(a,a),(a,b),(b,c),(c,e),(c,d),(d,e)} (**05M**)

Compute R^2 and R^{∞}

d) Define Lattice. Check if the following diagram is a lattice or not. (**05M**)

Q.2 a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic. If yes, find the one-to-one correspondence between the vertices. If not justify your answer. (**08M**)



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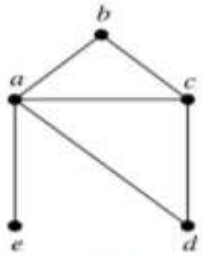
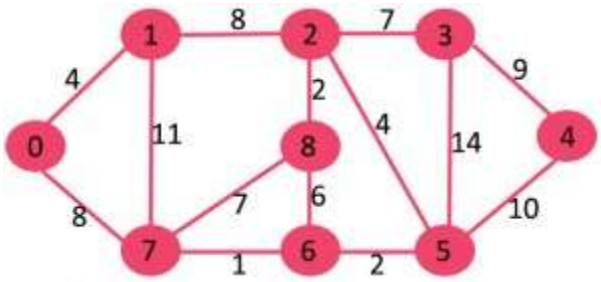


H

b) Find Minimum spanning tree for the following graph using Kruskal's Algorithm. (**08M**)



c) Prove $(p \vee q) \wedge (p \wedge \neg q) \vee q] \leftrightarrow p \vee q$

Q. 3 a) Prove that set G = {0,1,2,3,4,5} is a finite abelian group of order 6 with respect to addition modulo 6. (**08M**)

b) Let A={1,2,3,4}, let R={(1,2),(2,3),(3,4),(2,1)} Find Transitive closure of R using Warshall's Algorithm. (**08M**)

c) Test whether the following function is one-to-one, onto or both. (**04M**)

f: Z→Z, $f(x)=x^2+x+1$

Q.4 a) Show that the (2,6) encoding function $e:B^2 \rightarrow B^5$ defined by (**08M**)

e(00)=000000 e(01)=011110

e(10)=101010 e(11)=111000 is a group code.

Find Minimum distance.How many errors will it detect and correct?

b) Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function associated with e_H. 1) 01111 2)01110 3)11001 (**08M**)

c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month? (**04M**)

Q.5 a) Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a*b=a+b-ab for all a,b €G. Prove that (G,*) is a group.

b) Solve the recurrence relation $a_r-a_{r-1}-6a_{r-2}=-30$ given $a_0=20$, $a_1=-5$ (**08M**)

c) Let A={a,b,c,d,e,f,g,h}. Consider the following subsets of A (**04M**)
A1={a,b,c,d} A2={a,c,e,g,h}
A3={a,c,e,g} A4={b,d} A5={f,h}
Determine whether following is partition of A or not. Justify your answer.
i) {A1, A2} ii) {A3, A4, A5}

Q.6 a) Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers. (**08M**)

I. A={2,4,,12,24}
II. A={1,3,5,15,30}

b) Let the functions f,g, and h defined as follows: (**08M**)
f:R→R, f(x)=2x+3
g: R→R, g(x)=3x+4
h: R→R, h(x)=4x
Find gof, fog, foh, , gofoh

c) Determine Euler Cycle and path in graph shown below (**04M**)