Q. P. Code: 23725

(3 Hours) **Total Marks: 80**

N. B. 1) Question No. **1** is **compulsory**.
2) Answer any **3** questions from the remaining **5** questions.
3) Assume suitable data wherever necessary.

Q1 (a) Determine if the following functions are linear or nonlinear **20**

i) $f(x_1, x_2, x_3) = \sum_{j=1}^{3} C_j x_j$

ii) $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 - x_1 x_3$

iii) $f(x_1, x_2, x_3) = \ln(\beta) x_1 + 9^{\alpha} x_3$

iv) $f(x_1, x_2, x_3) = \sqrt{x_1^2 + x_2^2}$

v) $f(x_1, x_2, x_3) = 2x_1 + x_2 - 5x_3$

(b) Write short note on unimodal and multimodal function.

(c) What is the need of modeling?

(d) Determine whether the given direction at the point is direction of descent for the following function:
$f(X) = 3x_1^2 + 2x_1 + 2x_2^2 + 7; \mathrm{d} = (-1,1)$ at $X = (2,1)$

Q2 (a) Solve by Simplex method **20**

Minimize $Z = x_1 - 3x_2 + 3x_3$

subject to $3x_1 - x_2 + 2x_3 \le 7$

$2x_1 + 4x_2 \ge -12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

Maximize $Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$

(b) *subject to* $x_1 + x_2 = 2,$

$x_1, x_2 \ge 0$

Q3 (a) Find the minimum of the function $f(\alpha) = 7\alpha^2 - 20\alpha + 22$. Using golden section method within an accuracy of 0.01. **20**

(b) Write component continuity equation for a perfectly mixed batch reactor with simultaneous reactions (first – order, isothermal)

$\mathrm{A} \xrightarrow{k_1} \mathrm{B}$ $\mathrm{A} \xrightarrow{k_2} \mathrm{C}$

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Q4 (a) State and prove the properties of Gradient vector. **20**

(b) Fit a quadratic model $y = \beta_0 + \beta_1 x + \beta_2 x^2$ to the following data where, $\beta_0, \beta_1, \beta_2$ are constants and x is variable and y is the output.

x	Y
20	73
20	78
30	85
40	90
40	91
50	87
50	86
50	91
60	75
70	65

Q5 (a) The profit per acre of a farm is given by **20**
$f(x_1, x_2) = 20x_1 + 26x_2 + 4x_1x_2 - 4x_1^2 - 3x_2^2$ where x_1 & x_2 denote, respectively, the labor cost and the fertilizer cost. Find the values of x_1 & x_2 to maximize the profit.

(b) Solve by Two phase method

Maximize $Z = x_1 + 2x_2$

subject to $3x_1 + 2x_2 \le 12$

$2x_1 + 3x_2 \ge 6$

$x_1 \ge 0, x_2$ is unrestricted in sign

Q6 (a) Minimize $f(x_1, x_2) = 25x_1^2 + 20x_2^2 - 2x_1 - x_2$ using Steepest Descent Method starting at point (3,1). **20**

(b) A company that operates 10 hrs a day manufactures each of two products on three sequential processes. The following table summarizes the data of the problem:

Product	Minutes per unit			Unit Profit ($)
	Process 1	Process 2	Process 3	
1	10	6	8	2
2	5	20	10	3

Determine the optimal mix of the two products graphically.