QP CODE : 00019418

(**2:30** Hours) Marks : **75**

NB : 1) **All** Questions are **Compulsory.**
2) **Figures** to the **right** indicate full **marks.**
3) Draw neat diagrams wherever necessary.
4) Symbols have usual meaning unless otherwise stated.
5) Use of non-programmable calculator is allowed.

1. (a) Attempt any one :- **10**

(i) State second order homogeneous linear ordinary differential equation with constant coefficients. Get its auxiliary equation, hence describe the method for solving the same.

(ii) What is method of separation of variables for solving partial differential equation ? Explain the same by solving Helmholtz equation.

(b) Attempt any one :- **05**

(i) Derive equation for integrating factor of general first order linear homogeneous differential equation.

(ii) Test the exactness of the following differential equation. Solve if it is exact

$$(4x^3 + 6xy + y^2)\, dx + 3\,(x^2 + 2xy + 2)\, dy = 0$$

2. (a) Attempt any one :- **10**

(i) Define Fourier's series in the interval $[-\pi, \pi]$. Evaluate the Fourier's coefficient's and a_0, a_n and b_n.

(ii) Find the cosine and sine transforms of $f(x) = e^{-bx}$, where b is a positive integer.

(b) Attempt any one :- **05**

(i) Find the Fourier series expansion for $f(x) = x$ for $0 < x < 2\pi$.

(ii) Obtain the Fourier transforms of first order and second order derivatives of a function $f(x)$.

3. (a) Attempt any one :- **10**

(i) Explain the phase transition in detail and show that at triple point all the three phases co-exit.

(ii) What do you mean by particle state and system state? If four $\frac{1}{2}$ spin particles are placed in magnetic field B. Find number of states allowed to the system.

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(b) Attempt any one :- **05**

(i) Write the short notes on Phase space.

(ii) For an isolated system at temperature T, Prove that $U = k\,T^2\left(\frac{\partial \ln Z}{\partial T}\right)$.

4. (a) Attempt any one :- **10**

(i) Explain the term a priori probability and thermodynamic probability. Hence get the probability of distribution of N particles at random in k cells. Show that in the most probable distribution, the number of particles in a cell is proportional to the area of that cell.

(ii) Derive Plank's law for the black body radiation. Discuss the Plank's law for low frequencies.

(b) Attempt any one :- **05**

(i) If the r.m.s. velocity of the molecules of hydrogen at N.T.P. is 1.84 km/s, calculate the r.m.s. velocity of oxygen molecules at N.T.P. Molecular weight of hydrogen and oxygen are 2 and 32 respectively.

(ii) Show that at high temperature, the average energy of a quantum mechanical oscillator is given by kT, the classical value.

5. (a) Attempt any one :- **04**

(i) Solve :- $\frac{\partial^2 u}{\partial x \partial y} = 0$

by the method of successive integration.

(ii) Solve the equation of radioactive decay ,

$$\frac{dN(t)}{dt} = -\lambda\, N(t)$$

Where N (t) is number of radio active nuclei at time 't'.

(b) Attempt any one :- **04**

(i) Find the Fourier transform of the function

$$f(x) = \begin{cases} \frac{1}{2\varepsilon} & \text{for } |x| \le \varepsilon \\ 0 & \text{for } x > \varepsilon \end{cases}$$

(ii) The function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the range $(0, 2\pi)$ is given as

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2}$$

Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ------ = \frac{\pi^2}{6}$

(c) Attempt any one :- **04**

(i) Calculate the number of particle states for a particle having energy between ε and $\varepsilon + d\varepsilon$.

(ii) For the adiabatic interaction between System A and system A' prove that enthelpy is constant.

(d) Attempt any one :- **03**

(i) A large box of area $1m^2$ is divided into small square cells. If one thousand balls are thrown at random in the box, calculate the most probable number of balls will fall in the square cell of side 10 cm.

(ii) Find out the number of possible arrangements of three particles in four cells, assuming they obey : (a) M-B Statistics, (b) B-E Statistics and (c) F-D Statistics.
