Q.P. Code: 13190

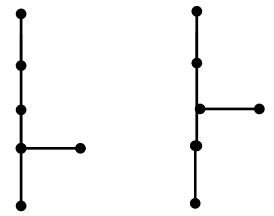
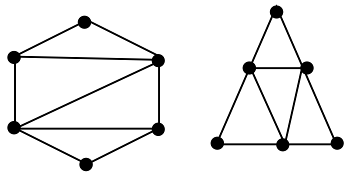
Duration: 3 hrs **Marks: 80**

N.B. 1) Both the sections are compulsory.

2) Attempt **ANY TWO** questions from each section.

Section I

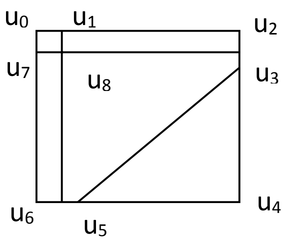
1 (a) Define isomorphic graphs and decide which of the following pairs of graphs are isomorphic. Justify your answer. 4

i.  ii. 

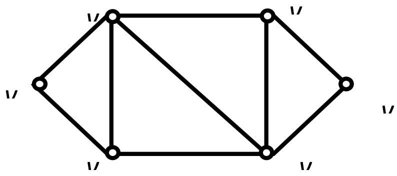
(b) State and prove Menger's theorem – Vertex form. 8

(c) i. Prove that a finite graph is bipartite if and only if it has no odd cycle. 4

ii. Draw line graphs of the following graphs. 4



G_1

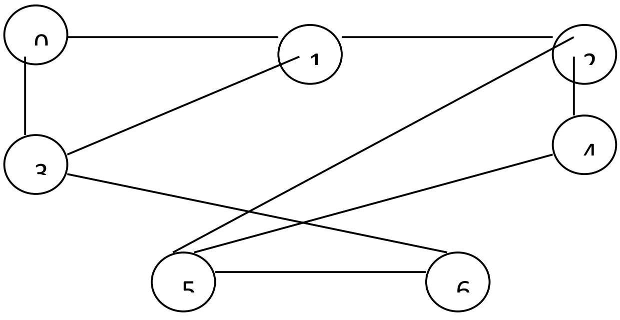


G_2

2 (a) Prove the following results. 6
i. Every tree is bipartite.
ii. A graph is connected if and only if it has a spanning tree.

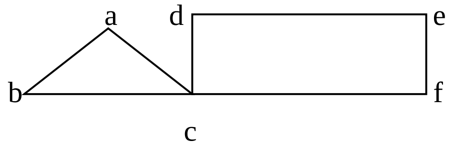
(b) Use Huffman coding to encode these symbols with given frequencies: *a*: 0.20, *b*: 0.10, *c*: 0.15, *d*: 0.25, *e*: 0.30, *f*: 0.12. What is the average number of bits required to encode a character? 6

(c) i. Draw Breadth First Search (BFS) tree for the following graph. 4



ii. Decode the Prufer sequence (2, 3, 3, 4, 4, 5).and draw corresponding tree. 4

3 (a). Discuss Fleury's algorithm for the following graph. 6



(b) Prove that "If a connected graph G has $n \geq 3$ vertices and for every pair u, v of non-adjacent vertices, $\deg(u) + \deg(v) \geq n$, then G is Hamiltonian". 6

(c) i. If the degree sequence of the simple graph G is (2, 2, 3, 3, 4), find degree sequence of G^C. Hence if the degree sequence of the simple graph G is $(d_1, d_2, ..., d_p)$, find degree sequence of G^C. 4

ii. Prove that a graph is Hamiltonian if and only if its closure is Hamiltonian. 4

4 (a) Prove that if G is a bipartite graph with bipartition X and Y , then G has a matching of X into Y if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. 10

(b) Prove that R(3, 4) = 9. Where R(s, t) is Ramsey number. 10

Section II

5. (a) For any graph G prove that, $\chi(G) \leq \Delta(G)+1$. 10

(b) Prove that every k - chromatic graph with n-vertices has a least ${}^kC_2$ edges. 10

.6. (a) Prove that a graph G is planar if and only if it contains no contraction of K_5 or $K_{3,3}$ 10

(b) If G is a (connected) simple, finite planar graph with n vertices, $(n \geq 3)$, then prove that G has at most $3n - 6$ edges. Also if G contains no triangles, then prove that G has at most $2n - 4$ edges. 10

7 (a) Define the following. 10

i) Diagraph

ii) Weakly connected

iii) Strongly connected

iv)Tournament

v) Hamiltonian Path.

(b) Every tournament D contains a vertex from which every other vertex is reachable by a path of length at most two. 10

8. (a) Define Eigen value of a graph G and prove that if G be a connected graph with k distinct eigen values and let d be the diameter of G then k > d. 10

(b) If G is bipartite and λ is an eigenvalue of G with multiplicity m, then $-\lambda$ is also an eigenvalue with multiplicity m. 10

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