Q. P. Code: 11750

External (Revised) (3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

Q.1(A) i) Using principle of mathematical induction, prove that $2^n > n$,for all positive integers n. **(8)**

ii) Explain equivalence relation with example. Also prove that if R and S are equivalence relations in a set then $R \cap S$ is also an equivalence relation. **(6)**

(B) Determine whether each of the following is a tautologies:

a) $(P \wedge Q) \rightarrow (P \vee Q)$ **(3)**

b) $(P \vee Q) \wedge (\neg P \wedge \neg Q)$ **(3)**

Q.2(A) i) If A_m is a countable set for each $m \in N$, then prove that union of all countable sets is countable. **(8)**

ii) If $f : R \rightarrow R$ and $g : R \rightarrow R$ are two functions such that $f(x) = 2x$ and $g(x) = x^2 + 2$. Then **(6)**

a) Prove that fog $\neq$ gof .

b) Find (fog) (3) and (gog) (1).

(B) Let $f : A \rightarrow B$, then prove that **(6)**

a) For each subset X of B, $f(f^{-1}(X)) \subseteq X$.

b) If f is onto then, $f(f^{-1}(X)) = X$.

Q.3(A) i) Let $P(n) = 1+5+9+\ldots\ldots\ldots+(4n-3) = (2n+1)(n-1)$.Then **(8)**

a) Use P(k) to show that P(k+1) is true. **(6)**

b) Is P(n) is true for all $n \geq 1$?

ii) Let a relation R defined on Z^+ as aRb iff a | b then prove that $(Z^+, |)$ is a partially ordered set.

(B) By using Zorn's lemma, prove that a nonzero unit ring contains a maximal proper ideal. **(6)**

Turn Over

Q.4(A) i)Verify whether following permutations commute to each other **(8)**

$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

ii) Prove that every permutation in S_n is a product of disjoint cycles.

(B) i) Define order of a permutation, transposition of a permutation and disjoint cycles with examples. **(6)** **(3)**

ii) Let A = {1,2,3,4,5,6}

Compute (4,1,3,5) o (5,6,3) and (5,6,3) o (4,1,3,5). **(3)**

SECTION-II (Attempt any two questions)

Q.5 i) Give any two definitions of probability. State the limitations if any. **(5)**

ii) Prove that convex combination of probability measures is also a probability measure. **(10)** **(5)**

iii) Define Borel sigma field. Show that set of natural numbers is a Borel sigma field.

Q.6(A) i) State and prove continuity property of probability **(5)**

ii) Explain the concept of following with suitable illustration for each **(5)**

a) Conditional probability of an event A given B.

b) Pairwise Independence

c) Mutual independence (for three events)

(B) i) A secretary goes to work following one of the three routes A, B, C .Her choice for the route is independent of weather. If it rains the probability of arriving late following A, B, C are 0.06, 0.15, 0.12. Corresponding probability if it does not rain (sunny) are 0.05, 0.1, 0.15.One in every four days is rainy. Given a sunny day she arrives late Find the probability that she took route C. **(5)**

ii) Define *P(A)* as $P(A) = \frac{1}{4}\delta_1(A) + \frac{3}{4}P_2(A)$ .Then obtain P(0 ,0.8] if P_2 has a density of $f(x) = 4x^3 \quad 0 < x < 1$. **(5)**

Turn Over

Q.7(A) i) X has exponential distribution with parameter 2. Find it mean and variance. **(5)**

ii) For any r.v.s X, Y show that $E[X+Y]^2 \leq [\sqrt{E(X^2)} + \sqrt{E(Y^2)}]^2$. **(5)**

iii) State properties of Characteristic function. **(5)**

(B) Two balls are drawn from an urn containing one yellow, two red and three blue balls. If X is no. of red balls drawn and Y is no. of blue balls drawn. Obtain joint distribution of X, Y. Hence find P(X=1/Y=2].Also find E[XY]. **(5)**

Q.8(A) i) State and prove Chebyshev's inequality. **(5)**

(B) i) A large lot contains 10% defective. A sample of 100 is taken from this lot. Find the probability that no. of defectives is 13 or more. Given $P[Z< 1] =0.8413$ where Z has N(0,1). **(5)**

ii) The joint p.d.f of X,Y is $f(x,y) = 8xy$ for $0 < x < y <1$;Find find conditional p.d.f of X given y. Hence conditional mean of X given y. **(5)**

iii) Examine whether the the Weak law of large numbers holds for sequence of independent r.v.s $\{ X_k \}$. **(5)**

$X_k = k$ with prob 0.5.and $X_k = -k$ with prob 0.5.
