External (Scheme A)	(3 Hours)	Total Marks: 100
Internal (Scheme B)	(2 Hours)	Total Marks: 40

N.B. : Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q1. (a) State and prove Nested Interval Theorem. 10

(b) Show that every convergent sequence on $\mathbb{R}$ is bounded. Give an example to show that the converse is not true. Justify your answer. 10

Q2. (a) State and prove Root test for the convergence of a positive term series $\sum a_n$. 10

(b) State Leibnitz's test for convergence of an alternating series. Hence or otherwise discuss the convergence of $\sum \frac{(-1)^n x^n}{\sqrt{n}}$ for $|x| < 1$. 5

(c) Show that the series $\sum \frac{\sin nx}{n^2 + 2}$ converges for all real x. 5

Q3. (a) Find the total derivative of $f(x, y) = xy^2$ at (2, 1) as a linear transformation. 10

(b) If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by 10

$$f(x, y) = \frac{x^2 y^2}{x^4 + y^4} \quad for \ (x, y) \neq (0,0)$$
$$= \quad 0 \qquad otherwise$$

Show that f is discontinuous at (0,0) but both the first order partial derivative of f exist at (0,0).

Q4. (a) State and prove Taylor's theorem for n-times continuously differentiable, real valued function of two variables. 10

(b) If $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ are given by $f(x,y)=(xy,x^3)$ and $g(u,v)= (v^3,-u^2)$. Find the Jacobians of f, g at (1,1) and $f(1,1)$ respectively . Hence or otherwise, find the Jacobiian of $g \circ f$ at (1,1). 10

Q5. (a) Prove that a monotonic function is Riemann Integrable. 10

(b) When is a function $f : [0,1] \to \mathbb{R}$ is said to be bounded variation? Show by giving an example that a continuous function need not be bounded variation. 10

Q6. (a) If f is a continuous on $[a,b]$ and if $F(x) = \int_a^x f(t)\,dt$, prove that F is differentiable on $[a,b]$ and $F'(x) = f(x) \forall x \in [a,b]$. 10

Turn Over

(b) Find the extreme values of . $f(x, y) = x^3 + y^3 - 3x - 12y + 10$ 10

Q7. (a) State and prove Fubini's theorem for a double integral over a rectangle in xy plane . 10

(b) Sketch the region of integration and evaluate $\iint_S x^2 y\,dx\,dy$ where S is the region bounded by the lines $y = x$, $y = -x$ and $y = 2$ in the first quadrant. 10

Q8. (a) State only 06

(i) Inverse function theorem.
(ii) Implicit function theorem.
(iii) Mean Value theorem.

for real valued functions of two variables.

(b) Show that the improper integral $\int_1^\infty \frac{dx}{x^2}$ exists but $\int_1^\infty x^2\,dx$ does not. 08

(c) Discuss the convergence of $\int_0^2 \frac{dx}{\sqrt{2-x}}$. 06
