Q.P. Code : 02745

(2½ Hours) **(Total Marks : 75**

N.B :
(1) All questions are compulsory
(2) Figures to the right indicate full marks
(3) Use of calculator is allowed

1. (a) If the joint distribution of (X,Y) is Bivariate Normal distribution with the parameters $(0, 0, 1, 1, \rho)$ then **6**
   (i) Obtain the marginal distribution of Y
   (ii) Obtain conditional distribution of X given $Y = y$. Hence state the conditional mean of X given $Y=y$

   (b) Define Fisher's Z transformation and explain its use in testing $H_0{:}\rho_1 = \rho_2$ for large samples from two bivariate normal populations. **6**

   (c) If (X,Y) have joint p.d.f.: **3**
   $f(x, y) = C\exp(-x^2+xy-y^2), \quad -\infty < x, y < \infty, c > 0$
   Find the parameters of this bivariate normal distribution.

OR

1. (p) What is an orthogonal linear transformation? Using the orthogonal linear transformation obtain the distribution of the sample correlation coefficient r, for a sample from a bivariate normal distribution, with $\rho = 0$. Hence obtain the test statistic for testing the significance of the sample correlation coefficient. **10**

   (q) State the joint moment generating function for a bivariate normal distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Prove that X and Y are independently distributed if $\rho = 0$. **5**

2. (a) If $P_x(s)$ is the probability generating function (p.g.f) of a discrete random variable X, find the p.g.f. of **7**
   (i) $Y = aX + b$, where a, b are constants
   (ii) $P(x = 2n)$, $n = 0, 1, 2.....$
   (iii) $P(x < n)$, $n = 0, 1 ,2....$

   (b) Define convolution of two sequences. Prove that the binomial distribution is an n-fold convolution of Bernoulli distribution. Hence use the p.g.f. of the binomial distribution to find its mean. **8**

OR

2. (p) Define the probability generating function (p.g.f) of a discrete random variable X. **10**
   Obtain the p.g.f. of the truncated Poisson distribuiton, truncated at zero. Hence find its mean and variance.

Turn Over

(q) A uniform die is rolled n times. Let X_i = Number on uppermost face of the i^{th} die, i = 1,2,....,n. **5**
(i) Obtain the probability generating function (p.g.f) of X_i
(ii) Obtain p.g.f. of $Y = X_1+X_2+....+X_n$
Hence obtain $P(Y = n)$

3. Stating the postulates of the Pure Birth process with initially 'a' members in the system, at time t = 0: **15**
(i) Derive the difference-differential equations.
(ii) State the difference differential equations for Yule's process and obtain the expression for $P_n(t)$, the probability of n members in the system at any time t.
(iii) For Yule's process obtain the mean number of members in the system at any time t.

OR

3. (a) What are stochastic processes? What are transient probabilities? **4**
(b) Stating clearly the postulates for the Pure death process, with initially 'a' members in the system, at time t = 0:- **11**
(i) Derive the difference differential equations
(ii) State the difference differential equations for the Poisson death process with $\mu_n = \mu$, and obtain the expression for $P_n(t)$, the probability of n units in the system at any time t. Also state the expression for $P_0(t)$, the probability of ultimate extinction.

4. (a) Write a short note on: **7**
(i) Queue discipline
(ii) Customer behaviour
in a queuing system.
(b) For the (M/M/1):(GD/∞/∞) queuing model, **8**
(i) obtain the steady state probabilities P_n
(ii) obtain the probability distribution of w = waiting time for a customer in the queue.

OR

4. (p) Explain the following terms used in queuing theory: **4**
(i) Calling source
(ii) Capacity of the system
(q) For the (M/M/C):(GD/N/∞), $N \geq C$ queuing model, obtain the steady state probabilities P_n, for n customers in the system. What is the effective arrival rate? **11**

5. (a) If (X, Y) have a bivariate normal distribution with parameters $(0, 0, 1, 1, \rho)$ show that $U = X + Y$ and $V = X - Y$ are in dependently distributed. State the probability distribution functions (p.d.f.s) of U and V. **6**

(b) For the Birth and Death process: **9**
   (i) State the postulates
   (ii) Obtain the difference- differential equations for $P_n(t)$,
   (iii) Obtain the expressions for the steady state probabilities P_n.

OR

5. (p) Let X be a discrete random variable with $p_k = P(X = k)$ and $q_k = P(X > k)$ If $P_x(s)$ and $Q_x(s)$ denote the generating functions of $\{p_k\}$ and $\{q_k\}$, obtain the relationship between $Q_x(s)$ and $P_x(s)$ **6**

(q) For the $(M/M/1)$: $(GD/N/\infty)$ queuing model:- **9**
   (i) State the steady state equations and derive the steady state probabilities P_n, for n customers in the system.
   (ii) Find expected number of customers in the system, Ls,

   when $\rho \neq 1$, where $\rho = \frac{\lambda}{\mu}$.
