

(3 Hours)

[Total Marks : 80

- N.B. :** (1) All questions carry 20 marks.
 (2) Question No. 1 is compulsory and attempt any three questions from Q. Nos. 2 to 5.
 (3) Scientific non-programmable handheld calculator is allowed.
 (4) Graph Paper will be provided on request.
 (5) Use of mathematical and statistical tables is permitted.
 (6) Assume relevant data wherever necessary.

1. (a) Multiple Choice Questions :

10

Use following data show to answer the questions 1 and 2. The number of hours worked by 200 statistics students are shown below :

Number of Hours	Frequency
0-9	40
10 - 19	50
20 - 29	70
30 - 39	40

- (1) The relative frequency of students working 9 hours or less
 (a) is .2
 (b) is .45
 (c) is 40
 (d) cannot be determined from the information given
 (e) None of the above answers is correct.
- (2) The cumulative relative frequency for the class of 10 - 19
 (a) is 90
 (b) is .25
 (c) is .45
 (d) cannot be determined from the information given
 (e) None of the above answers is correct.
- (3) The difference between the largest and the smallest data values is the
 (a) variance
 (b) interquartile range
 (c) range
 (d) coefficient of variation
 (e) None of the above answers is correct.
- (4) Which of the following is not a measure of central location ?
 (a) mean
 (b) median
 (c) variance
 (d) mode
 (e) None of the above answers is correct.
- (5) If a data set has an even number of observations, the median
 (a) cannot be determined
 (b) is the average value of the two middle items
 (c) must be equal to the mean
 (d) is the average value of the two middle items when all items are arranged in ascending order
 (e) None of the above answers is correct.

[TURN OVER

Con. 430-MR-5955-17.

2

- (6) The sum of deviations of the individual data elements from their mean is
- always greater than zero
 - always less than zero
 - sometimes greater than and sometimes less than zero, depending on the data elements
 - always equal to zero
 - None of the above answers is correct.
- (7) The value that has half of the observations above it and half the observations below it is called the
- range
 - median
 - mean
 - mode
 - None of the above answers is correct.
- (8) The most frequently occurring value of a data set is called the
- range
 - mode
 - mean
 - median
 - None of the above answers is correct.
- (9) In a sample of 800 students in a university, 160, or 20%, are Business majors. Based on the above information, the school's paper reported that "20% of all the students at the university are Business majors". This report is an example of
- a sample
 - a population
 - statistical inference
 - descriptive statistics
 - None of the above answers is correct.
- (10) A statistics professor asked students in a class their ages. On the basis of this information, the professor states that the average age of all the students in the university is 21 years. This is an example of
- census
 - descriptive statistics
 - an experiment
 - statistical inference
 - None of the above answers is correct.
- (b) Answer **True** or **False** :
- A store asks shoppers for their zip code to identify market areas. Zip codes are an example of ratio data.
 - An ordinal level of measurement implies some sort of ranking.
 - Data measured on a nominal scale can only be classified into categories.
 - The terms descriptive statistics and inferential statistics can be used interchangeably.
 - A marketing research agency was hired to test a new DVD player. Consumers rated it outstanding, very good, fair, or poor. The level of measurement for this experiment is ordinal.
 - The Union of Electrical Workers of America with 9,128 members polled 362 members about a new wage package that will be submitted to management. The population is the 362 members.

10

2

Con. 430-MR-5955-17.

7. The CIA World Fact-book cited these numbers for the United States; the birthrate is 14.14 births per 1,000 population, the average life expectancy for females is 80 years and Approximately 290 million persons reside in the United States. Each of these numbers is referred to as a statistic.
8. If we select 100 persons from 25,000 registered voters and question them about candidates and issues, the 100 persons are referred to as the population.
9. Statistics is defined as a body of techniques used to facilitate the collection, organization, presentation, analysis, and interpretation of information for the purpose of making better decisions.
10. Categorizing voters as Democrats, Republicans, and Independents is an example of interval level measurement.

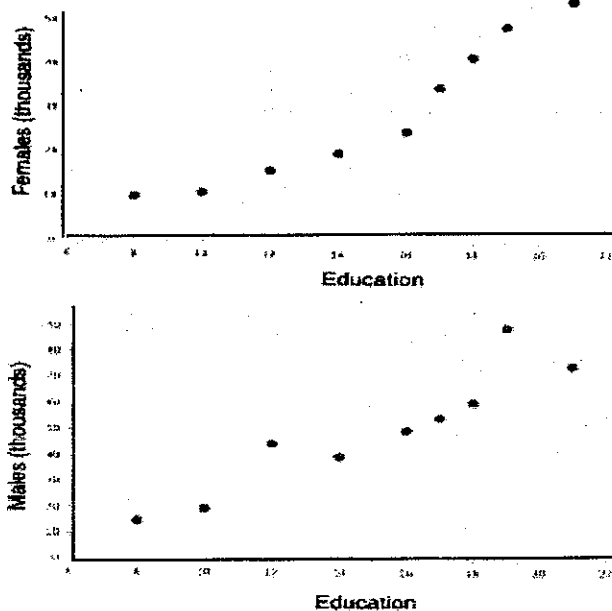
2. (a) Find the covariance for the following data series :

5

x	1	-2	3	0	3
y	3	2	4	6	0

(b) These two scatter plots show the average income for adults based on the number of years of education completed. 16 years of education means graduating from college. 21 years means landing a Ph.D.

5



What type of correlation does this plot represent ?

(3)

[TURN OVER

Use the following data table to solve sub-questions 2. (c) and 2. (d)

Sample	Obs 1	Obs 2	Obs3	Obs 4	Obs5
1	10.68	10.689	10.776	10.798	10.714
2	10.79	10.86	10.601	10.746	10.779
3	10.78	10.667	10.838	10.785	10.723
4	10.59	10.727	10.812	10.775	10.73
5	10.69	10.708	10.79	10.758	10.671
6	10.75	10.714	10.738	10.719	10.606
7	10.79	10.713	10.689	10.877	10.603
8	10.74	10.779	10.11	10.737	10.75
9	10.77	10.773	10.641	10.644	10.725
10	10.72	10.671	10.708	10.85	10.712
11	10.79	10.821	10.764	10.658	10.708
12	10.62	10.802	10.818	10.872	10.727
13	10.66	10.822	10.893	10.544	10.75
14	10.81	10.749	10.859	10.801	10.701
15	10.66	10.681	10.644	10.747	10.728

n	A2	D3	D4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74

- (c) Plot the R-chart. 5
- (d) Draw the x-bar chart. 5

3. (a) The time x in years that an employee spent at a company and the employee's hourly pay, y , for 5 employees are listed in the table below. Calculate and interpret the correlation coefficient r . Include a plot of the data in your discussion. 5

x	y
5	25
3	20
4	21
10	35
15	38

- (b) The estimated slope coefficient is 0.78, the sample size is 26, the standard error of the coefficient is 0.32, and the level of significance is 5%. Is the slope different than zero? 5
- (c) Imagine that you are dealing with samples of size 10, and that you rank the data items from 1 to 10 according to the value of the X variable; then the corresponding ranks for the Y variable will be the numbers 1 to 10 in some order. One possible order is shown in the table below: 5

X-rank	1	2	3	4	5	6	7	8	9	10
Y-rank	5	8	2	1	10	4	9	3	6	7

Find the coefficient of rank correlation.

(4)

- (d) The National Safety Board (NSB) wants to examine the safety of compact cars, midsize cars, and full-size cars. It collects a sample of three for each of the treatments (cars types). For the data provided below, test whether the mean pressure applied to the driver's head during a crash test is equal for each types of car. Use $\alpha = 5\%$. 5

Compact Cars	Mid-size cars	Full-size cars
643	469	484
655	427	456
702	525	402

4. (a) The demand for a product in each of the last five months is shown below : 5

Month	1	2	3	4	5
Demand ('00s)	13	17	19	23	24

- Use a two month moving average to generate a forecast for demand in month 6.
- Apply exponential smoothing with a smoothing constant of 0.9 to generate a forecast for demand for demand in month 6.

- (b) The Department of Environment has theorized that pollution levels are higher in winter (I & IV Quarter) than summer (II & III Quarter) and that they are increasing over the years. The following data was collected : 5

Quarter	I	II	III	IV
1996	193	246	231	282
1997	301	252	227	291
1998	304	259	239	296
1999	306	265	240	300

Calculate the regression line that is described by this data.

- (c) For the data in Table below draw the np chart and comment. 5

Lot Identification	Sample Size (Constant)	Number Defective (In the Sample)
1	100	9
2	100	6
3	100	7
4	100	3
5	100	4
6	100	6
7	100	3
8	100	3
9	100	3
10	100	4
11	100	4
12	100	6
13	100	4
14	100	5
15	100	2
16	100	3
17	100	2
18	100	8
19	100	4
20	100	3
21	100	2
22	100	3
23	100	4
24	100	8
25	100	6

[TURN OVER

Con. 430-MR-5955-17.

6

- (d) A new casino game involves rolling 3 dice. The winnings are directly proportional to the total number of sixes rolled. Suppose a gambler plays the game 100 times, with the following observed counts : 5

Number of Sixes	Number of Rolls
0	48
1	35
2	15
3	3

The casino becomes suspicious of the gambler and wishes to determine whether the dice are fair. What do they conclude at 5% level of significance ?

5. (a) Explain different qualitative methods of forecasting. 20
(b) What do you understand by Chi-Square Statistics ?
(c) What is six sigma ?
(d) What is time series analysis ?

- N.B. :** (1) All questions carry 20 marks.
(2) Q.1 is compulsory and attempt any 3 questions from Q.2 to Q.5.
(3) Figures to the right indicate full marks.
(4) Use of Non-Programmable calculator only is allowed.
(5) Use of Mobile Phones in the Exam Hall is prohibited.
(6) Support your answers with diagram / illustration wherever is required.
(7) Refer Statistical Table if required.

1. (a) Select the appropriate alternative and write the completed statements. 10
- (1) A Matrix in AP is matrix of any order.
(a) Row (b) Column
(c) Square (d) Zero.
- (2) Σ and Σ determines if given TP is balanced or not.
(a) x and y (b) m and n
(c) Supply and Demand (d) None
- (3) Only Method gives optimum solution in TP.
(a) Stepping Stone or Loop (b) Hungarian
(c) VAM (d) MODI
- (4) If $m+n-1$ is not satisfied in TP for IBFS then is introduced as allocated quantity in the unoccupied cell with the least cost in the matrix.
(a) Epsilon = 0
(b) Delta < 0
(c) Pie \neq 0
(d) Alpha \neq 0.
- (5) If given AP is unbalanced and is for maximization the first step to get the optimum solution is-----.
(a) Hungarian method directly applied
(b) Check if MODI method is applicable
(c) Treat the problem as GPP
(d) Use dummy row/ dummy column and convert the problem into balanced AP.
- (6) The principle applied to get OS in AP is called
(a) VAM (c) LCM
(b) MODI method (d) Hungarian Method.

[TURN OVER

- (7) Crashing Cost exists only in
- (a) PERT-NWA (b) CPM- NWA
 (c) PERT- MODI (d) CPM- VAM
- (8) $EST + t = \dots\dots\dots$ [CPM/PERT/NWA]
- (a) EFG (b) GST
 (c) EFT (d) TEST
- (9) 'Saddle Point' may exist in a problem of
- (a) DPP (b) Game Theory
 (c) NWA (d) AP and TP.
- (10) The correct formula out of the following is

(a) $Cost\ Slope = \frac{CC - CD}{ND - NC}$ (b) $Cost\ Slope = \frac{CC - NC}{ND - CD}$

(c) $Cost\ Slope = \frac{NC - CD}{ND - CC}$ (d) $Cost\ Slope = \frac{CC - ND}{CC - NC}$

- (b) State if the following statements are **true** or **false** —
- (1) LCM/ MMM in TP is a method which is solely based on Penalty Calculations.
 (2) $TF - HS = FF$ and $FF - TS = IndF$
 (3) CP sometimes may not have highest duration in Network Analysis.
 (4) The term IBFS in OR means Indian Border Force for Security.
 (5) MODI method is a checkpoint only to find if IBFS is OS.

(c) Match the column (i) and (ii)

10

Column A

Column B

- | | |
|---|------------------|
| 1) $m \neq n$ | a) CPM Problem |
| 2) Penalty | b) PERT Problem |
| 3) $\Sigma\ Supply \neq \Sigma\ Demand$ | c) Balanced AP |
| 4) Various Floats | d) Unbalanced TP |
| 5) Probability distribution | e) Balanced TP |
| | f) VAM. |

2. (a) Consider a firm having 2 factories. The firm is to ship its products from the factories to 3- retail stores. The number of units available at factories X and Y are 200 and 300, respectively, while those demanded at retail stores A, B, and C are 100, 150 and 250 respectively. Rather than shipping directly from factories to retail stores, it is asked to investigate the possibility of trans-shipment. The transportation cost in Rs. per unit is given in the table. 10

	Factory		Retail Store			
	X	Y	A	B	C	
Factory	X	0	18	17	18	19
	Y	16	0	15	14	13
Retail Store	A	17	12	0	15	11
	B	11	15	11	0	14
	C	18	19	17	18	0

Find the Optimum Shipping schedule using VAM for IBFS.

- (b) Four girls A, B, C, D are always seen with four boys P, Q, R, S, in your class. 10
 In fact they do all assignments, projects, submissions together after consulting each other and after discussions within their group of eight. They are now in the fourth semester and during the internship it has struck them that they could consider settling in life with members from within their group. Since OR was their favorite subject they have formulated a happiness matrix given later and wish to decide their ideal life partners using Hungarian principle as applied to the happiness matrix. One constraint in this regard is that 'A' and 'S' are cousins of each other and **marriage between them is not permitted**. What would be your advice for ideal pairing keeping the 'cousins' constraint in mind? What is the value of the Happiness index for such pairing? Can you recommend any alternative pairing ?

	P	Q	R	S
A	100	75	85	110
B	80	60	75	70
C	95	98	95	100
D	90	88	80	95

[TURN OVER

3. (a) Explain the Difference between CPM and PERT. 5
 (b) Write an Algorithm on "Drawing the minimum number of lines to cover all Zeroes" as a part of optimization technique of AP. 5
 (c) Three time estimates in days are given for each activity of the following project : 10
 1. Tabulate expected duration and variance for each of the activities.
 2. Draw Network Diagram complete in all respects.
 3. What will be the project completion time for 90% confidence level?
 4. Find the probability for the project completion if duration is reduced by 2 days.

Activity	Optimistic	Most Likely (m)	Pessimistic (b)
1-2	8	12	24
1-3	9	12	18
1-4	12	16	20
2-5	4	6	8
3-5	16	30	44
4-6	12	30	42
5-6	14	28	56

4. (a) 'LIBERTY' Distributers has 5 distribution outlets A, B, C, D and E and it distributes material to its Clients M, T, N and L. Transportation cost in Rs. '00 is mentioned per cell of the Matrix given below. To find IBFS use VAM and Check further if IBFS you obtained is Optimum Solution or not. 10

	B	E	S	T	SUPPLY
K	3	7	9	4	530
L	8	5	2	6	535
M	5	4	11	7	450
N	6	9	8	12	265
P	11	6	4	9	420
DEMAND	640	465	535	560	

- (b) In an Election Campaign, the strategies adopted by the Ruling and Opposition party along with pay-offs i.e. Ruling party's % share in votes polled, are given below: 10

Ruling Party's Strategies	Opposition Party's Strategies		
	Campaign one day in each city	Campaign two days in large towns	Spending two days in large Rural sectors
Campaign one day in each city	55	40	35
Campaign two days in large towns	60	60	75
Spending two days in large Rural sectors	80	75	55

Assume a Zero sum Game. Find Optimum strategies for both the parties and Expected pay-off to the Ruling Party.

5. (a) There are 3 dairies A, B and C in a small town which supplies all the milk consumed in the town. Assume that initial consumer sample is composed of 1000 respondents distributed over 3 dairies A, B and C. It is known by all the dairies that consumers switch from one dairy to another due to advertising, price and dissatisfaction. All these dairies maintain records of the number of their consumers and the dairy from which they obtained each new consumer. The following table shows the flow of consumers over an observation period of one month. Assume that the Matrix of transition Probabilities remains fairly stable and at the beginning of period one Market shares are A=25%, B=45% and C=35%. Construct the state transition probability matrix to analyze the problem. 10

Dairy	Period 1 Consumers	Change during Period		Period 2 Consumers
		Gain	Loss	
A	250	62	50	262
B	450	53	60	443
C	300	50	55	295
Total	1000	165	165	1000

- (b) Explain the use of Slack and Surplus Variables you are using in solving LPP by Simplex Method. 5
- (c) Write on Various Floats in Network Analysis. 5

N.B. :(1) Q. No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.5.

Q. 1. (a) Fill in the blanks :-

10

- a. _____ Programming is a mathematical technique dealing with the optimisation of multi-stage decision problems.
- b. The technique was originated in 1952 by _____ and G.B. Dantzig.
- c. In some situations of integer programming problems, each variable can take on the values of either 'zero' or 'one', such problems are referred to as _____ programming problems.
- d. To apply goal programming, the first step is to set a _____.
- e. _____ linear programming investigates the effect of predetermined continuous variations of the coefficients on the optimal solution.
- f. A non linear-programming problem which does not fall satisfy the assumptions of convex programming, falls into the category of _____ linear programming.
- g. The object of the Assignment algorithm is to assign _____ objects to the same number of jobs in such a way that will minimise the total cost.
- h. Given a directed network defined by node, arcs and flow capacities, maximum flow can occur between a _____ node and sink node.
- i. A systematic procedure for solving pure integer programming problems was first suggested by _____ in 1958.
- j. _____ algorithm is the most widely used algorithm to solve pure and mixed integer problems in use.

Q.1. (b) Match 'Column A' with 'Column B':-

10

Column A	Column B
1. Dynamic Programming	a. Stage
2. Sub-problems in dynamic programming	b. Non-linear
3. Variables are non-negative integers	c. Cutting plane algorithm
4. R. E. Gomory	d. Branch and bound algorithm
5. A. H. Land and A. G. Doig	e. Stochastic linear programming
6. Multi-objective situations	f. Goal-programming
7. Extension of sensitivity analysis	g. Parametric linear programming
8. No constraints	h. Unconstrained optimisation
9. Minimum cost capacitated flow	i. Shortest path, transportation, trans shipment are examples of
10. IBM	j. Optimisation

[TURN OVER

- Q.2. (a) A food processing firm has compiled the following data for future monthly production requirements and production costs in regular and overtime periods. 10

Month	Quantity	Cost per unit	
		Regular (Rs.)	Overtime (Rs.)
September	4,000	20	30
October	5,200	25	35
November	5,000	24	34
December	3,700	26	36
January	4,200	20	30
February	3,000	20	30

The production capacity of the firm is 6,000 units in regular time and 3,000 units in overtime. The cost of carrying storage is Rs. 7.50 unit per month. If at the end of August, there are 3,500 units in stock at a cost of Rs. 25 each, what is optimal production schedule and the total associated cost? Note that no inventory is required at the end of six months.

- Q2. (b) A company has three media A, B and C available for advertising its product. The data collected over the past years about the relationship between the sales and frequency of advertisement in the different media is as follows :- 10

Frequency/month	A	B	C
1	125	180	300
2	225	290	350
3	260	340	450
4	300	370	500

The cost of advertisement is Rs. 5,000 in medium A, Rs. 10,000 in medium B and Rs. 20,000 in medium C. The total budget allocated for advertising the product is Rs. 40,000. Determine the optimal combination of advertising media and frequency.

OR

- Q2. (c) Solve the problem by Gomory's algorithm: 10

Maximise $Z = 3x_1 + 4x_2$,

subject to $x_1 + x_2 \leq 4$,

$3/5 x_1 + x_2 \leq 3$,

$x_1, x_2 \geq 0$ and integer.

Q2. (d) A refrigeration and air-conditioning company has been awarded a contract for the air-conditioning of a new computer installation. The company has to make a choice between two alternatives :

- hire one or more refrigeration technicians for six hours a day or
- hire one or more part-time refrigeration apprentice technicians for four hours a day.

The rate of wages of a refrigeration technician is Rs. 20 per hour, while the corresponding rate of apprentice technician is Rs. 8 per hour. The company wants to engage the technicians on work for not more than 25 man hours per day and also limit the charges to technicians to Rs. 440. The company estimates, that the productivity of a refrigeration technician is eight units and that of a part-time apprentice technician is three units. Formulate the integer programming problem to enable the company to select the optimum number of technicians and apprentices.

Q3. (a) A rural clinic hires its staff from nearby cities and towns on a part-time basis. The clinic attempts to have a general practitioner (GP), a nurse and an internist on duty during at least a portion of each week. The clinic has a weekly budget of Rs. 1,200. A GP charges the clinic Rs. 40 per hour, a nurse charges Rs. 20 per hour, and an internist charges Rs. 150. The clinic has established the following goals in order of priority.

- A nurse should be available for at least 30 hours per week.
- The weekly budget of Rs. 1,200 should not be exceeded.
- A GP or internist should be available at least 20 hours per week.
- An internist should be available at least 6 hours per week.

Formulate a goal programming model for determining the number of hours to hire each staff member in order to satisfy the various goals.

Q3. (b) A manufacturer of toys makes two types of toys, A and B, processing of these two toys is done on two machines X and Y. Toy A requires 4 hours on machine X and six hours on machine Y, while Toy B requires eight hours on machine X and five hours on machine Y. There are thirty two hours of time per day available on machine X and thirty hours on machine Y. The profit obtained on each of the toys is Rs. 30 per unit. What should be the daily production of each of the toys for maximum profit? A non-integr solution to the problem is not acceptable.

OR

Q3. (c) Consider the parametric problem :-

$$\text{Maximise } Z = (3 - 6\lambda)x_1 + (2 - 2\lambda)x_2 + (5 + 5\lambda)x_3$$

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

$$x_1, x_2, x_3 \geq 0.$$

Perform a complete parametric analysis and identify all the critical values of the parameter λ .

[TURN OVER

Q3. (d) Consider the parametric problem :-

10

$$\text{Maximise } Z = (3 + 3\theta)x_1 + 2x_2 + (5 - 6\theta)x_3$$

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

$$x_1, x_2, x_3 \geq 0.$$

Where θ is a non-negative parameter. Perform a complete parametric analysis.

Q4. (a) Max $Z = 2x_1 + 3x_2 - 2x_1^2$

10

$$\text{subject to } x_1 + 4x_2 \leq 4$$

$$x_1 + x_2 \leq 2$$

$$\text{and } x_1, x_2 \geq 0$$

Q4. (b) Max $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$

10

$$\text{subject to } 3x_1 + 2x_2 \leq 6$$

$$\text{and } x_1, x_2 \geq 0$$

OR

Q4. (c) What are the different softwares available for OR ? Explain any two of them.

10

Q4. (d) Explain SPSS and its working style.

10

Q5. (a) A firm has divided its marketing area into three zones. The amount of sales depends upon the number of salesmen in each zone. The firm has been collecting the data regarding sales and salesmen in each area over a number of past years. The information is summarized in table 1. For the next year firm has only 9 salesmen and the problem is to allocate these salesmen to three different zones so that the total sales are maximum.

Table 1

Profits in thousands of rupees

No. of Salesmen	Zone 1	Zone 2	Zone 3
0	30	35	42
1	45	45	54
2	60	52	60
3	70	64	70
4	79	72	82
5	90	82	95
6	98	93	102
7	105	98	110
8	100	100	110
9	99	100	110

Q5. (b) Solve the following mixed integer problem by the branch and bound technique:- **10**

Maximise $Z = x_1 + x_2$
 subject to $2x_1 + 5x_2 \leq 16,$
 $6x_1 + 5x_2 \leq 30,$
 $x_2 \geq 0,$
 $x_1 \geq 0$ and integer

OR

Q5. (c) Determine the values of x_1 and x_2 graphically so as to - **10**

Minimise $Z = x_1^2 + x_2^2$
 subject to $x_1 + x_2 \geq 8,$
 $x_1 + 2x_2 \geq 10,$
 $2x_1 + x_2 \geq 10$
 $x_1, x_2 \geq 0.$

Q5. (d) Consider the linear programming problem - **10**

Maximise $Z = 4x_1 + 6x_2 + 2x_3,$
 subject to $x_1 + x_2 + x_3 \leq 3,$
 $x_1 + 4x_2 + 7x_3 \leq 9$
 $x_1, x_2, x_3 \geq 0.$

The optimal solution to this problem is given by -

Table 1

	C_j	4	6	2	0	0	
C_B	Basis	X_1	X_2	X_3	X_4	X_5	b
4	X_1	1	0	-1	4/3	-1/3	1
6	X_2	0	1	2	-1/3	1/3	2
$Z_j = \sum C_B a_{ij}$	4	6	8	10/3	2/3		
$C_j = c_j - Z_j$	0	0	-6	-10/3	-2/3		

Solve the problem if the variation right-hand side vector $b' = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$. Perform complete parametric analysis and identify all critical values of parameter λ .