**M.SC.(MATHEMATICS) PART –I**
**Algebra – I (REV -2016)**
**(PAPER – I) (DEC - 2017)**

**Q. P. Code: 11732**

**Time: 3 Hours** **Total Marks: 80**

**Instructions:**

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

**SECTION I (Attempt any two Questions)**

Q1. (a) Prove that every n-dimensional vector space V(F) is isomorphic to $V_n$ (F).

(b) Let F be a field of complex numbers and let T be the function from $F^3$ into $F^3$ defined by

$$T(X_1, X_2, X_3) = (X_1 - X_2 + 2X_3,\ 2X_1 + X_2 - X_3,\ -X_1 - 2X_2)$$

Verify that T is a linear transformation. Describe the null space of T.

Q2. (a) Let A is a linear transformation on a vector space V such that $A^2 - A + I = 0$. Then show that A is invertible.

(b) Find the Rank of the matrix.

$$\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$$

Q3. (a) Show that similar matrices have the same minimal polynomial.

(b) Find all (complex) characteristic values and characteristic vectors of the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q4. (a) If α and β are vectors in an inner product space V(F) and a,b ϵ F, then prove that

$$\mathrm{Re}(\alpha, \beta) = \tfrac{1}{4}\|\alpha + \beta\|^2 - \tfrac{1}{4}\|\alpha - \beta\|^2$$

(b) To every self-adjoint operator T on a finite dimensional inner product space V there corresponding distinct real numbers $C_1, C_2, C_3 \ldots C_k$ and perpendicular projections $E_1, E_2, E_3 \ldots E_k$. (where k is strictly positive integer, and greater than the dimension of the space )

**Turn Over**

Show that

a) The $E_i$ are pairwise orthogonal and different from 0.
b) $E_1 + E_2 + E_3 + \dots + E_k = I$
c) $T = C_1E_1 + C_2E_2 + C_3 E_3 + \dots + C_kE_k$

**SECTION II (Attempt any two Questions)**

Q5. (a) Show that every quotient group of a group is a homomorphic image of the group.

(b) If N & M are normal sub groups of a group G, then show that N ∩ M is a normal sub group of G.

Q6. (a) Prove if a group G of order 28 has a normal subgroup of order 4, then G is abelian.

(b) If G is any finite group such that p / o(G) , where p is a prime number then show that G has an element of order p.

Q7. (a) Prove that a finite integral domain is a field.

(b) Show that the intersection of two ideals of a ring R is an ideal of R. Give one example to show that the union of two ideals of R need not to be an ideal of R.

Q8. (a) Show that in a unique factorization domain every pair of non-zero elements have a g.c.d and l.c.m.

(b) Show that every ring R can be embedded in the polynomial ring R[x].

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# M.SC.(MATHEMATICS) PART –I
## Algebra – I (Old)
**(PAPER – I) *(DEC - 2017)***

**Q. P. Code: 11729**

External (Scheme A) (3 Hours) Total Marks: 100
Internal (Scheme B) (2 Hours) Total Marks: 40

N.B.:Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q1. (a) If H and K are two subgroups of a group G, Prove that H U K is a subgroup of G if and only if either $H \subset K$ or $K \subset H$.

(b) If N & M are normal sub groups of a group G, then show that $N \cap M$ is a normal sub group of G.

Q2. (a) Prove that if a group G of order 28 has a normal subgroup of order 4, then G is abelian.

(b) Let G be a finite abelian group such that p divides o( G ), p being a prime number then show that there exists an element $a \neq e \in G$ such that $a^p = e$ .

Q3. (a) Prove that every group of prime order is abelian.

(b) Show that every quotient group of a cyclic group is cyclic. Give an example to show that the converse need not to be true.

Q4. (a) Let $f: R \to R'$ be a homomorphism of a ring R onto a ring R'. Then show that

$$\frac{R}{Ker\ f} \approx R'$$

(b) Find the maximal ideals of $Z_{12}$ , the ring of integers modulo 12.

Q5. (a) Prove that a finite integral domain is a field.

(b) Show that in a unique factorization domain every pair of non zero elements have a g.c.d and l.c.m. .

Q6. (a) Let U and V be the vector spaces over the field F and let T be a linear transformation from U into V. Suppose that U is finite dimensional then prove that
Rank (T) + Nullity (T) = dim (U).

(b) Show that the mapping T: $V_2(R) \to V_3(R)$ defined as T (a, b) = ( a + b , a - b , b ) is a linear transformation from $V_2(R)$ into $V_3(R)$ . Find the range, rank, null space and nullity of T.

**Turn Over**

Q7. (a) Let A and B be linear transformations on a finite dimensional vector space V and let AB=I. Then show that A and B are both invertible and $A^{-1} = B$.

Give an example to show that this is false when V is not finite dimensional.

(b) Find the Rank of the matrix.

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

Q8. (a) Suppose that α and β are vectors in an inner product space. Then show that

$$\| \alpha + \beta \|^2 + \| \alpha - \beta \|^2 = 2 \| \alpha \|^2 + 2 \| \beta \|^2$$

(b) Find the minimal polynomial for the real matrix.

$$\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$

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**M.SC.(MATHEMATICS) PART –I**
**Analysis & Topology (REV -2016)**
**(PAPER – II) (DEC - 2017)**

**Q. P. Code: 11743**

**REVISED** **Time: 3 Hours** **Total Marks: 80**

**Instructions:**

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

Q.1. (a) State and prove Lebesgue covering lemma.
(b) State and prove Heine Borel theorem.

Q.2. (a) Define a compact set. Show that the continuous image of a compact set is compact.
(b) Define a connected set. If A and B are connected sets is $A\cup B$ and $A\cap B$ are connected? Justify your answer.

Q.3. (a) Let $S$ be an open subset of $\mathbb{R}^2$. If $a \in S$, and the partial derivatives $D_1f,\ D_2f$ exist in some open ball $B(a,r)$ and are continuous at $a$, then show that $f$ is differentiable at $a$.
(b) Use chain rule and find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ where $z = x^2 - 3x^2y^3, x(u,v) = ve^u, y(u,v) = ve^{-u}$ .

Q.4. (a) State and prove Inverse function theorem.
(b) State and prove mean value theorem for scalar fields.

**SECTION II (Attempt any two Questions)**

Q.5. (a) Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
(b) Let X and Y be topological spaces. Then show that a mapping f: X →Y is continuous if and only if the inverse image under f of every closed set in Y is also closed in X.

Q.6. (a) Show that a topological space X is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed in X.
(b) Show that continuous image of a connected space is connected.

Q.7. (a) Show that closed subsets of a compact sets are compact.
(b) Show that a Hausdorff space X is locally compact if and only if each of its points is an interface point of some compact subspace of X.

Q.8. (a) Show that a compact subset of a metric space is closed and bounded.
(b) Show that all completions of a metric space are isometric.

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**M.SC.(MATHEMATICS) PART –I**
**Analysis (Old)**
**(PAPER – II) (*DEC - 2017*)**

**Q. P. Code: 11735**

| | | |
|---|---|---|
| External (Scheme A) | (3 Hours) | Total Marks: 100 |
| Internal (Scheme B) | (2 Hours) | Total Marks: 40 |

N.B. : Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q1. (a) State and prove Nested Interval Theorem. 10

(b) Show that every convergent sequence on $\mathbb{R}$ is bounded. Give an example to show that the converse is not true. Justify your answer. 10

Q2. (a) State and prove Root test for the convergence of a positive term series $\sum a_n$. 10

(b) State Leibnitz's test for convergence of an alternating series. Hence or otherwise discuss the convergence of $\sum \frac{(-1)^n x^n}{\sqrt{n}}$ for $|x| < 1$. 5

(c) Show that the series $\sum \frac{\sin nx}{n^2 + 2}$ converges for all real $x$. 5

Q3. (a) Find the total derivative of $f(x, y) = xy^2$ at (2, 1) as a linear transformation. 10

(b) If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by 10

$$f(x, y) = \frac{x^2 y^2}{x^4 + y^4} \quad for \ (x, y) \neq (0,0)$$
$$= 0 \qquad otherwise$$

Show that $f$ is discontinuous at (0,0) but both the first order partial derivative of $f$ exist at (0,0).

Q4. (a) State and prove Taylor's theorem for $n$-times continuously differentiable, real valued function of two variables. 10

(b) If $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ are given by $f(x,y)=(xy,x^3)$ and $g(u,v)= (v^3,-u^2)$. Find the Jacobians of $f$, $g$ at (1,1) and $f(1,1)$ respectively . Hence or otherwise, find the Jacobiian of $g \circ f$ at (1,1). 10

Q5. (a) Prove that a monotonic function is Riemann Integrable. 10

(b) When is a function $f : [0,1] \to \mathbb{R}$ is said to be bounded variation? Show by giving an example that a continuous function need not be bounded variation. 10

Q6. (a) If f is a continuous on $[a,b]$ and if $F(x) = \int_a^x f(t)\,dt$, prove that F is differentiable on $[a,b]$ and $F'(x) = f(x) \, \forall x \in [a,b]$. 10

**Turn Over**

(b) Find the extreme values of . $f(x, y) = x^3 + y^3 - 3x - 12y + 10$ 10

Q7. (a) State and prove Fubini's theorem for a double integral over a rectangle in xy plane . 10

(b) Sketch the region of integration and evaluate $\iint_S x^2 y\,dx\,dy$ where S is the region bounded by the lines $y = x$, $y = -x$ and $y = 2$ in the first quadrant. 10

Q8. (a) State only 06

(i) Inverse function theorem.

(ii) Implicit function theorem.

(iii) Mean Value theorem.

for real valued functions of two variables.

(b) Show that the improper integral $\int_1^\infty \frac{dx}{x^2}$ exists but $\int_1^\infty x^2 dx$ does not. 08

(c) Discuss the convergence of $\int_0^2 \frac{dx}{\sqrt{2-x}}$ . 06

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**M.SC.(MATHEMATICS) PART –I**

**Topology**

**(PAPER – III) (DEC - 2017)**

**QP Code : 21070**

**Scheme A (External)]** (3 Hours) [Total Marks:100

**Scheme B (Internal)]** (2 Hours) [Total Marks: 40

**Instructions:**

- **Scheme A** students should attempt **any five** questions.
- **Scheme B** students should attempt **any three** questions.
- **All** questions carry **equal marks**.
- **Mention** clearly the **Scheme** under which you are appearing.

1. (a) Show that the set consisting of all finite subsets of $\mathbb{N}$ is a countable set.

   (b) Show that for any non-empty set $A$, the cardinality of the power set of $A$ is strictly greater than that of $A$.

2. (a) Define a topological space. Define the closure $\overline{A}$ of a subset $A$ of a topological space $X$. Show that $\overline{A}$ is a closed subset of $X$.

   (b) Let $X$ and $Y$ be two topological spaces. Let $f : X \to Y$. Prove that the following statements are equivalent:
   (i) $f$ is continuous
   (ii) For every subset $A$ of $X$, $f(\overline{A}) = \overline{f(A)}$
   iii) For every closed subset $B$ of $Y$, the set $f^{-1}(B)$ is a closed subset of $X$.

3. (a) Define countable and uncountable sets and show that the set $X$ of all sequences taking values 0 or 1 is an uncountable set.

   (b) Let $f : X \to Y$ be a function where $X$ is a metric space. Show that the function $f$ is continuous iff for every convergent sequence $x_n \to x$ in $X$ the sequence $f(x_n)$ converges to $f(x)$ in $Y$.

4. (a) If $X$ and $Y$ are connected topological spaces, prove that $X \times Y$ is connected.

   (b) Find two connected subsets $A, B$ of $\mathbb{R}^2$ such that $A \cap B$ is not connected.

5. (a) Prove that a complete and totally bounded metric space is compact.

   (b) Show that every continuous bijective map from a compact topological space to a Hausdorff topological space is a homeomorphism.

6. (a) State and prove the Tube Lemma.

   (b) Prove that a complete metric space is a Baire space.

7. (a) Define the terms : Second Countable space, Separable space. Show that a Second Countable space is Separable.

   (b) Define terms: Complete metric space, totally bounded metric space. Prove that a complete, totally bounded metric space is compact.

8. (a) Let $k$ be a fixed positive integer and let $p : E \to B$ be a covering map. Let $K$ denote the subset of $B$ such that $b \in K$ iff $p^{-1}(b)$ has exactly $k$ elements. Show that both $K$ and its complement are open subsets of $B$.

   (b) If $X$ is a path connected topological space then show that for any two points $x_0$ and $x_1$ of $X$, $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.

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**Con. 74-17.**

# M.SC.(MATHEMATICS) PART –I
## Complex Analysis (REV -2016)
### (PAPER – IV) (DEC - 2017)

**Q. P. Code: 28233**

**Time: 3 Hours** **Total Marks: 80**

**Instructions:**

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

**SECTION I (Attempt any two Questions)**

1) (a) If $a_n \neq 0$ for all but finitely many values of n then the radius of convergence R of $\sum_{n=0}^{\infty} a_n z^n$, then prove that $\liminf \left|\frac{a_{n+1}}{a_n}\right| \leq \frac{1}{R} \leq \limsup \left|\frac{a_{n+1}}{a_n}\right|$. In particular, if $\lim \left|\frac{a_{n+1}}{a_n}\right|$ exist, then $\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |a_n|^{1/n}$.

(b) Given a series $\sum_{n=1}^{\infty} z^n (1-z)$. Prove that

(i) The series converges for $|z| < 1$ and find its sum.

(ii) The series uniformly converges to the sum $z$ for $|z| \leq 1/2$.

2) (a) Prove that if $G$ is an open connected set and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ $\forall z \in G$, then $f$ is constant.

(b) Find the Bilinear Transformation which maps the points $z = 1, i, -1$ onto points $i, 0, -i$. Also find the fixed points of the transformation.

3) (a) Let $0 \notin G$ be an open connected set in $\mathbb{C}$ and suppose that $f : G \to \mathbb{C}$ is analytic. Then prove that $f$ is a branch of logarithm if and only if $f'(z) = \frac{1}{z}$, $\forall z \in G$ and $e^{f(a)} = a$ for atleast one $a \in G$.

(b) If $f : G \to \mathbb{C}$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |\operatorname{Re} f(z)|^2 = 2|f'(z)|^2$.

4) (a) State and prove Cauchy's Deformation Theorem.

(b) Evaluate $\int_0^{1+i} z^2 \, dz$ along

(i) The line $y = x$ (ii) Along the parabola $y = x^2$. Is the integral independent of path?

**SECTION II (Attempt any two Questions)**

5) (a) State and prove Cauchy's estimate.

(b) Evaluate $\int_C \frac{z+6}{z^2-4} dz$, using Cauchy's Integral Formula where C is the circle

(i) $|z|=1$ (ii) $|z-2|=1$ (iii) $|z+2|=1$.

6) (a) State and prove Schwartz Lemma

(b) Suppose $f$ is non-constant and analytic in a domain of $G$. if $|f|$ attains minimum in G at $\alpha$, then $f(\alpha)=0$.

7) (a) State and prove Casorti Weiestrass theorem.

(b) Find all the possible Laurent Series expansions of $f(z)=\frac{4-3z}{z(1-z)(2-z)}$.

8) (a) State and prove Residue theorem.

(b) Use the residue theorem to evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$.

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**Q. P. Code: 28235**

| | | |
|---|---|---|
| External (Scheme A) | (3 Hours) | Total Marks: 100 |
| Internal (Scheme B) | (2 Hours) | Total Marks: 40 |

N.B.:
1. Scheme A students should attempt any five questions.
2. Scheme B students should attempt any three questions.
3. Write the scheme under which you are appearing, on the top of the answer book.

1) (a) If $a_n \neq 0$ for all but finitely many values of n then the radius of convergence R of $\sum_{n=0}^{\infty} a_n z^n$, then prove that $\liminf \left|\frac{a_{n+1}}{a_n}\right| \leq \frac{1}{R} \leq \limsup \left|\frac{a_{n+1}}{a_n}\right|$. In particular, if $\lim \left|\frac{a_{n+1}}{a_n}\right|$ exist, then $\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |a_n|^{1/n}$.

(b) Given a series $\sum_{n=1}^{\infty} z^n (1-z)$. Prove that

(i) The series converges for $|z| < 1$ and find its sum.

(ii) The series uniformly converges to the sum $z$ for $|z| \leq 1/2$.

2) (a) Prove that if $G$ is an open connected set and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ $\forall z \in G$, then $f$ is constant.

(b) Find the Bilinear Transformation which maps the points $z = 1, i, -1$ onto points $i, 0, -i$. Also find the fixed points of the transformation.

3) (a) Let $0 \notin G$ be an open connected set in $\mathbb{C}$ and suppose that $f : G \to \mathbb{C}$ is analytic. Then prove that $f$ is a branch of logarithm if and only if $f'(z) = \frac{1}{z}$, $\forall z \in G$ and $e^{f(a)} = a$ for atleast one $a \in G$.

(b) If $f : G \to \mathbb{C}$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |\operatorname{Re} f(z)|^2 = 2 |f'(z)|^2$.

4) (a) State and prove Cauchy's Deformation Theorem.

(b) Evaluate $\int_0^{1+i} z^2 \, dz$ along

(i) The line $y = x$

(ii) Along the parabola $y = x^2$. Is the integral independent of path?

5) (a) State and prove Cauchy's estimate.

(b) Evaluate $\int_C \frac{z+6}{z^2-4} dz$, using Cauchy's Integral Formula where C is the circle

(i) $|z|=1$ (ii) $|z-2|=1$ (iii) $|z+2|=1$.

6) (a) State and prove Schwartz Lemma

(b) Suppose $f$ is non-constant and analytic in a domain of $G$. if $|f|$ attains minimum in G at $\alpha$, then $f(\alpha)=0$.

7) (a) State and prove Casorti Weiestrass theorem.

(b) Find all the possible Laurent Series expansions of $f(z)=\frac{4-3z}{z(1-z)(2-z)}$.

8) (a) State and prove Residue theorem.

(b) Use the residue theorem to evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$.

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**Q. P. Code: 11746**

| | | |
|---|---|---|
| Scheme A(External) | (3 Hours) | Total marks: 100 |
| Scheme B(Internal/External) | (2 Hours) | Total marks: 40 |

N.B: 1) Scheme A students answer **any five** questions.
2) Scheme B students answer **any three** questions.
3) All questions carry equal marks.
4) Write on the top of your answer book the scheme under which you are appearing.

1. (a) How many integers strictly between0 and 10,000 have exactly one digit equal to 5?
(b) Determine number of 8-permutations of the multiset T = {3.a, 2.b, 4.c}

2. (a) Prove the recurrence relation using combinatorial argument for $D_n$, derangement of n objects; $D_n = (n-1)(D_{n-1} + D_{n-2})$ , $n \geq 3$.
(b) If S(n,k) denotes Stirling numbers of second kind then show that
(i) S(n,1) = 1 = S(n,n), (ii) $S(n,2) = 2^{n-1} - 1$,
(iii) $S(n,n\text{-}1) = \binom{n}{2}$, for $n \geq 2$.

3. (a) What is circular permutation?
Ten people including two who do not wish to sit next to one another are to be seated at a round table. How many circular sitting arrangements are there?
(b) State and prove strong form of Pigeon hole principle. Give example.

4. (a) Find the sum of all coefficients in $(4x - 3y + z)^5$.
(b) State and prove Baye's theorem.

5. (a) Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$ subject to the initial condition $a_0 = 2$; $n \geq 1$.
(b) Compute the Möbius function of linearly ordered set $(X_n, \leq)$ where $X_n = \{1, 2, 3,\ldots\ldots., n\}$.

6. (a) Define SDR, system of distinct representatives. Find the number of SDR's for the family {1},{1,2},…………..,{1,2,3,…,n}.
(b) There are three groups of children containing 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected at random from each group. Show that the chance that the three selected consists of 1 girl and 2 boys is 13/32.

7. (a) Show that every sequence of $n^2+1$ distinct real numbers contains a subsequence of length n+1 that is either strictly increasing or strictly decreasing.
(b) Determine number of regions that are created by n mutually overlapping circles in general position in the plane

**Turn Over**

8. (a) Show that $\sum_{k=1}^{n} k \binom{n}{k}^2 = n \binom{2n-1}{n-1}$.

   (b) What is variance of discrete random variable? Compute variance of random variable with normal distribution.

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**Q. P. Code: 11750**

External (Revised) (3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I ( Attempt any two questions)

**Q.1(A)** i) Using principle of mathematical induction, prove that $2^n > n$ ,for all positive integers n. **(8)**

ii) Explain equivalence relation with example. Also prove that if R and S are equivalence relations in a set then $R \cap S$ is also an equivalence relation. **(6)**

**(B)** Determine whether each of the following is a tautologies:

a) $(P \wedge Q) \rightarrow (P \vee Q)$ **(3)**

b) $(P \vee Q) \wedge (\neg P \wedge \neg Q)$ **(3)**

**Q.2(A)** i) If $A_m$ is a countable set for each $m \in N$, then prove that union of all countable sets is countable. **(8)**

ii) If $f : R \rightarrow R$ and $g : R \rightarrow R$ are two functions such that $f(x) = 2x$ and $g(x) = x^2 + 2$. Then **(6)**

a) Prove that $fog \neq gof$ .

b) Find (fog) (3) and (gog) (1).

**(B)** Let $f : A \rightarrow B$, then prove that **(6)**

a) For each subset X of B, $f(f^{-1}(X)) \subseteq X$.

b) If f is onto then, $f(f^{-1}(X)) = X$.

**Q.3(A)** i) Let $P(n) = 1+5+9+\ldots\ldots\ldots+(4n-3) = (2n+1)(n-1)$.Then **(8)**

a) Use P(k) to show that P(k+1) is true. **(6)**

b) Is P(n) is true for all $n \geq 1$ ?

ii) Let a relation R defined on $Z^+$ as aRb iff $a \mid b$ then prove that $(Z^+, \mid)$ is a partially ordered set.

**(B)** By using Zorn's lemma, prove that a nonzero unit ring contains a maximal proper ideal. **(6)**

**Turn Over**

**Q.4( A)** i)Verify whether following permutations commute to each other **(8)**

$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

ii) Prove that every permutation in $S_n$ is a product of disjoint cycles.

i) Define order of a permutation, transposition of a permutation and disjoint cycles **(6)**

**(B)** with examples. **(3)**

ii) Let A = {1,2,3,4,5,6}

Compute (4,1,3,5) o (5,6,3) and (5,6,3) o (4,1,3,5). **(3)**

SECTION-II ( Attempt any two questions)

**Q.5** i) Give any two definitions of probability. State the limitations if any. **(5)**

ii) Prove that convex combination of probability measures is also a probability **(10)**

measure. **(5)**

iii) Define Borel sigma field. Show that set of natural numbers is a Borel sigma field.

**Q.6(A)** i) State and prove continuity property of probability **(5)**

ii) Explain the concept of following with suitable illustration for each **(5)**

a) Conditional probability of an event A given B.

b) Pairwise Independence

c) Mutual independence (for three events)

**(B)** i) A secretary goes to work following one of the three routes A, B, C .Her choice for the route is independent of weather. If it rains the probability of arriving late following A, B, C are 0.06, 0.15, 0.12. Corresponding probability if it does not rain (sunny) are 0.05, 0.1, 0.15.One in every four days is rainy. Given a sunny day she arrives late Find the probability that she took route C. **(5)**

ii) Define *P(A)* as $P(A) = \frac{1}{4}\delta_1(A) + \frac{3}{4}P_2(A)$ .Then obtain P(0 ,0.8] if $P_2$ has a density of $f(x) = 4x^3 \quad 0 < x < 1$. **(5)**

**Turn Over**

**Q.7(A)** i) X has exponential distribution with parameter 2. Find it mean and variance. **(5)**

ii) For any r.v.s X, Y show that $E[X+Y]^2 \leq [\sqrt{E(X^2)} + \sqrt{E(Y^2)}]^2$. **(5)**

iii) State properties of Characteristic function. **(5)**

**(B)** Two balls are drawn from an urn containing one yellow, two red and three blue balls. If X is no. of red balls drawn and Y is no. of blue balls drawn. Obtain joint distribution of X, Y. Hence find P(X=1/Y=2].Also find E[XY]. **(5)**

**Q.8(A)** i) State and prove Chebyshev's inequality. **(5)**

**(B)** i) A large lot contains 10% defective. A sample of 100 is taken from this lot. Find the probability that no. of defectives is 13 or more. **(5)**

Given P[Z< 1] =0.8413 where Z has N(0,1).

ii) The joint p.d.f of X,Y is f(x,y) = 8xy for $0 < x < y < 1$ ;Find find conditional p.d.f of X given y. Hence conditional mean of X given y. **(5)**

iii) Examine whether the the Weak law of large numbers holds for sequence of independent r.v.s $\{ X_k \}$ . **(5)**

$X_k = k$ with prob 0.5.and $X_k = -k$ with prob 0.5.

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**Q. P. Code: 28231**

(3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I ( Attempt **any two** questions)

1. A) Solve the linear Diophantine equations $247x + 91y = 39$.. [10]
   B) State and prove Euler's criterion for quadratic residue of p. [10]

2. A) Let $r, n \in N$ and $l = \min\{r, n\}$. then show that $n^r = \sum_{k=1}^{l} C(n,k)k!\,S(r,k)$. [10]

   B) For $n \in N$, How many square free integers do not exceed $n$? [10]

3. A) Give any sequence of $mn + 1$ distinct real numbers then prove that there exist either an increasing sequence of length $m + 1$ or decreasing sequence of length $n + 1$ or both. [10]
   B) During a month with 30 days a baseball team plays at least one game a day, but on more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. [10]

4. A) Let G be a graph and e be an edge. Then show that e is a cut edge if and only if e is not on a cycle. [10]
   B) Let $d_1 \le \cdots \ldots \le d_n$ be the vertex degrees of G. Suppose that, for each $k < n/2$ with $d_k \le k$, the condition $d_{n-k} \ge n - k$ holds. Then, prove that G is Hamiltonian. [10]

P.T.O….

SECTION-II ( Attempt any two questions)

5. A) State and prove Gronwall's inequality to the uniqueness of the solution of the initial value problem. [10]

   B) Obtain approximate solution to with in $t^5$ of the initial value problem

   $\frac{dx}{dt} = xt + t^2 y, \ \ x(0) = 1.$

   $\frac{dy}{dt} = xy + t, \quad y(0) = 2.$ [10]

6. A) If $\emptyset_1(x)$ is a solution of $L_2(y) = 0$ on an interval $I$ and $\emptyset_1(x) \neq 0$ on $I$ then show that the other linearly independent solution of $L_2(y) = 0$ is $\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \left[\frac{1}{\emptyset_1(t)^2} e^{-\int a_1 t dt}\right] dt.$ [10]

   B)Solve the following IVP.

$$\frac{dx}{dt} = 2x + y + z, \quad x(1) = 1$$

$$\frac{dy}{dt} = 2y + 2z, \quad y(1) = 2$$

$$\frac{dz}{dt} = 2z \quad z(1) = 3.$$ [10]

7. A)Show that the Legendre polynomial $P_n(x)$ of degree $n$ is given by

   $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$ [10]

   B) Obtain solution in the form of power series of the following Differential equation:

   $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + (4t^2 - 2)x = 0.$ [10]

8. A) Solve $\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0, \ u = u(x, y)$ with $u(x, 0) = h(x)$ for a given $h: \mathbb{R} \to \mathbb{R}.$ [10]

   B) Solve $u_x . u_y = u, \ u(x, 0) = x^2.$ [10]

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