Solution Q.P. Code 38392.

Sem-VI (CBSGS) (Rev-2012)/T0853- Control System-I

Q.1(c) $\dot{x} = \begin{bmatrix} 0 & 3 & 1 \\ 2 & 8 & 1 \\ -10 & -5 & -2 \end{bmatrix} x + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$

Characteristics equation

$\det(sI - A) = 0$

$s^3 - 6s^2 - 7s - 52 = 0$

Routh Harwitz array to be arranged

1 pole on RHS of s plane

2 poles on LHS " " "

No pole on imaginary axis

Q.1(e) Draw free body diagram in s-domain & obtain simultaneous equations.

$(s^2+s+1)X_1(s) - (s+1)X_2(s) = F(s)$

$-(s+1)X_1(s) + (s^2+s+1)X_2(s) = 0$

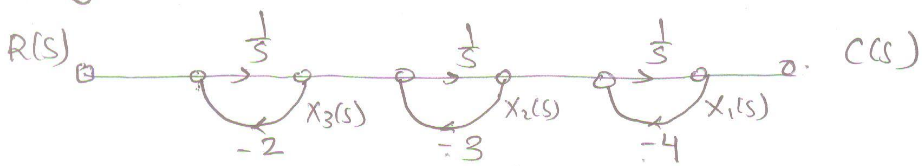
$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^2(s^2+2s+2)}$$

Q.2(a) Cascade representation.

$$G(s) = 24 \times \frac{1}{s+1} \times \frac{1}{(s+2)} \times \frac{1}{s+3}$$

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

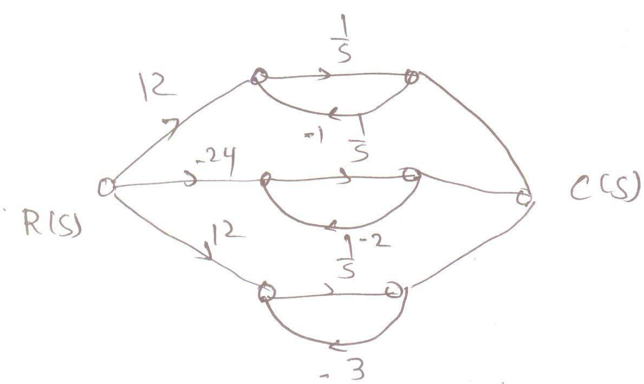


Parallel representation.

$$G(s) = \frac{12}{s+1} - \frac{24}{s+2} + \frac{12}{s+3}$$

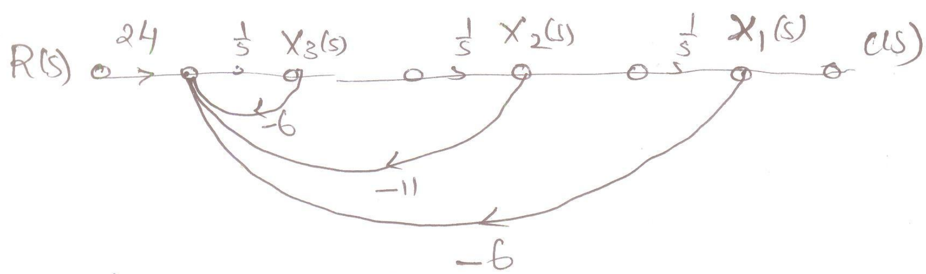
$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 12 \\ -24 \\ 12 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x$$



Phase variable form

$$G(s) = \frac{24}{s^3 + 6s^2 + 11s + 6}$$



$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

2(b) $G(s) = \dfrac{K(s+3)}{s(s+1)(s+2)(s+4)}$; $H(s) = 1$

Ch. equation

$1 + G(s)H(s) = 0$

$s^4 + 7s^3 + 14s^2 + (8+K)s + 3K = 0$

s^4	1	14	$3K$
s^3	7	$(8+K)$	0
s^2	$\frac{90-K}{7}$	$3K$	0
s^1	$\frac{720-65K-K^2}{90-K}$	0	0
s^0	$3K$	0	0

For system response to oscillate

$$\frac{720-65K-K^2}{90-K} = 0$$

$K = 9.64$

$A(s) = \left(\frac{90-K}{7}\right)s^2 + 3K = 0$

Freq. of oscillation ; $\omega_n = 1.59$ rad/sec.

3(a) $T(s) = \dfrac{T_1 \Delta_1}{\Delta}$

$T_1 = G_1(s)G_2(s)G_3(s)$; $\Delta_1 = 1$

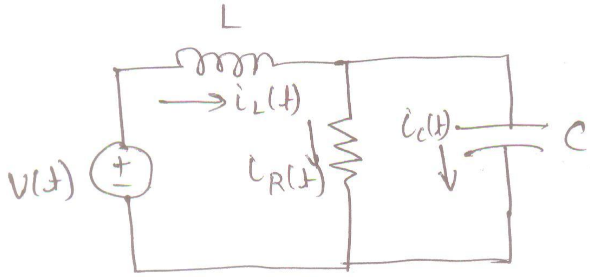
$\Delta = 2 + G_2(s)G_3(s)G_4(s) + 2G_3(s)G_4(s) + 2G_4(s)$

$$T(s) = \frac{G_1(s)G_2(s)G_3(s)G_4(s)}{2 + G_2(s)G_3(s)G_4(s) + 2G_3(s)G_4(s) + 2G_4(s)}$$

3b)

$$T(s) = \frac{G_6(s)\left[G_4(s) + G_5(s)G_3(s) + G_5(s)G_2(s)\right]}{G_6(s)\left[G_7(s)G_4(s) + G_7(s)G_5(s)G_3(s) + G_7(s)G_5(s)G_2(s) + G_3(s)G_1(s) + G_2(s)G_1(s) + 1\right] + G_1(s)\left[G_3(s) + G_2(s) + 1\right]}$$

4(b)



$$C\frac{dv_c}{dt} = i_c$$

$$L\frac{di_L}{dt} = v_L$$

Let v_L & i_L be the state variables

$$i_c = -i_R + i_L$$

$$= -\frac{1}{R}v_c + i_L$$

$$v_L = v(t) - v_c$$

$$= -v_c + v(t)$$

$$C\frac{dv_c}{dt} = -\frac{1}{R}v_c + i_L$$

$$\dot{v}_c = -\frac{1}{CR}v_c + \frac{1}{C}i_L$$

$$L\frac{di_L}{dt} = v_L$$

$$\frac{di_L}{dt} = \frac{1}{L}\left[-v_c + v(t)\right]$$

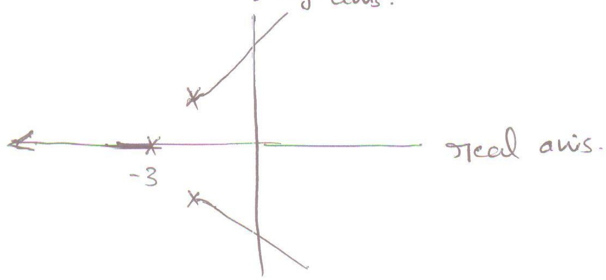
$$\dot{i}_L = -\frac{1}{L}v_c + \frac{1}{L}v(t) \quad ; \quad y = v_c$$

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{CR} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{Bmatrix} v_c \\ I_L \end{Bmatrix}$$

Q.6(a) $G(s) = \dfrac{k}{(s+3)(s^2+4s+5)}$; $H(s) = 1$

Poles at $s=-3$; $s=-2\pm j1$



angles of asymptotes

$\theta_1 = 60°$; $\theta_2 = 180°$; $\theta_3 = 300°$.

$\sigma = -2.33$.

ξ for 15% O.S.

$\xi = 0.517$; $\theta = 121.131°$

Dominant poles $s_{1,2} = -1.243 \pm j\,2.058$

K for 15% OS. = 11.09.