Q.1. A) Given : $Q = 10\,C$; $\vec{E} = 10x\,\hat{a}_x + 10y\,\hat{a}_y$ (V/m) ; path : $y = 2-2x$, $z=0$.

Initial co-ordinates : $B(1,0,0)$; Final co-ordinates : $A(0,2,0)$.

To find : Energy (W).

Solⁿ : Here, $y = 2-2x \Rightarrow dy = -2dx$.

Now, $W = -Q\int_B^A \vec{E}\cdot\vec{dl}$

$$\therefore \quad W = -10\int_B^A (10x\,\hat{a}_x + 10y\,\hat{a}_y)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= -100\int_B^A [(x\,dx) + (2-2x)(-2dx)]$$

$$= -100\int_{x=1}^{x=0} (5x-4)\,dx = -100\left(\frac{3}{2}\right) = -150\,J.$$

$$\therefore \quad \boxed{W = -150\,J}$$

B) Comparison of MOM, FEM & FDM should have following points.

i) Type of equation the method handles.
ii) Working domain
iii) Simplicity.
iv) Discretization.
v) Resultant matrix.
vi) Open boundary problems solving capability.
vii) Radiation condition.

c) With neat labeled diagram & appropriate theory, the explanation of Super refraction & Sub refraction should be done.

D) Only write Maxwell's eqⁿ. in "Point form" and "Integral form". No derivation required. Give significance of each equation. "Modified Ampere's Law" eqⁿ. tells the propagation of electromagnetic wave in air.

	Time	Maximum attenuation

Q.2. A) Given : $\epsilon_{r_1} = 1$; $\epsilon_{r_2} = 50$; $\vec{E_1} = 10\hat{a}_x + 40\hat{a}_y + 50\hat{a}_z$ (V/m)

Interfacing boundary plane : $z = 0$

To find : $\vec{E_2}$

Sol^n : $\vec{E_1} = \vec{E_{1t}} + \vec{E_{1n}}$

Since $z=0$ plane is interface, $\vec{E_{1n}} = 50\hat{a}_z$.

$\therefore \quad \vec{E_{1t}} = 10\hat{a}_x + 40\hat{a}_y$

Boundary cond^n. for Tangential E-Field.

$$\vec{E_{2t}} = \vec{E_{1t}} = 10\hat{a}_x + 40\hat{a}_y \quad \text{— (1)}$$

Boundary cond^n for Normal E field.

$$\vec{D_{2n}} = \vec{D_{1n}} \rightarrow \vec{E_{2n}} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} \vec{E_{1n}}$$

$$\therefore \vec{E_{2n}} = \frac{1}{50}(50\hat{a}_z) = \hat{a}_z \quad \text{— (2)}$$

$\therefore$ from (1) & (2)

$$\boxed{\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}} = 10\hat{a}_x + 40\hat{a}_y + \hat{a}_z \quad (V/m)}$$

B) Derive the following eq^n. and state significance of each term in it.

$$\int_v (\vec{E}\cdot\vec{J})\, dv = -\frac{\partial}{\partial t}\int_v \left(\frac{\mu}{2}H^2 + \frac{\epsilon}{2}E^2\right) dv - \oint_s (\vec{E}\times\vec{H})\cdot d\vec{s}$$

Q.3. A) Given : $\vec{D} = 4xy\,\hat{a}_x + 2(x^2+z^2)\,\hat{a}_y + 4yz\,\hat{a}_z$ (C/m^2)

To find : i) ρ_v at $(-1, 0, 3)$

ii) Ψ through cube defined by $0 \le x \le 2$; $0 \le y \le 3$; $0 \le z \le 5$.

iii) $Q_{enclosed}$ by cube.

Sol^n :

i) $$\rho_v = \nabla\cdot\vec{D}\Big|_{at\,(-1,0,3)}$$

$$= \nabla\cdot(4xy\,\hat{a}_x + 2(x^2+z^2)\,\hat{a}_y + 4yz\,\hat{a}_z)$$

$$= 4y + 4y = 8y\Big|_{at\,(-1,0,3)} = \underline{0\ (C/m^3)}$$

ii) $$\Psi = \oint_s \vec{D}\cdot d\vec{s} = \int_v \rho_v\, dv = \iiint_{x\,y\,z} \rho_v\, dx\,dy\,dz \quad (\because dv = dx\,dy\,dz)$$

$$= \int_0^2 dx \cdot \int_0^3 (8y)\,dy \cdot \int_0^5 dz = (2)(4\times 9)(5) = \underline{360\ C}$$

iii) By Gauss's law, $Q_{enclosed} = \Psi = \underline{360\ C}$

B) <u>Given</u> : $\frac{\partial^2 V}{\partial x^2} = 0$, $0 \leq x \leq 4$, & $V(0) = 0$; $V(4) = 20$.

<u>To find</u> : $V(1)$ using FDM performing Band matrix method.

<u>Soln</u> : $V_0 = V(0)$, V_1, V_2, V_3, $V_4 = V(4)$, at points 0, 1, 2, 3, 4.

Using FDM we get, $V_1 = \frac{V_0 + V_2}{2} \Rightarrow 2V_1 - V_2 = 0$ — ①

$V_2 = \frac{V_1 + V_3}{2} \Rightarrow -V_1 + 2V_2 - V_3 = 0$ — ②.

$V_3 = \frac{V_2 + V_4}{2} \Rightarrow -V_2 + 2V_3 = 20$ — ③.

$$\therefore \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}$$

Solving above matrix. we get.

$V_1 = 5\ V$

$V_2 = 10\ V$

$V_3 = 15\ V$.

$\therefore$ $\boxed{V(1) = V_1 = 5V}$

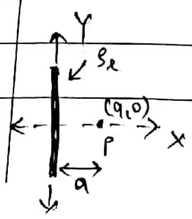
c) With neat diagram & proper explanation of working of Electromagnetic pump, give the application of electromagnetics ~~pump~~. Use of ~~Using~~ Fleming's Left hand rule should be there in explanation.

Q.4. A) Derivation of following expressions is required.

$$\Gamma_{R\perp} = \left|\frac{\vec{E_r}}{\vec{E_i}}\right| = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$\Gamma_{T\perp} = \left|\frac{\vec{E_t}}{\vec{E_i}}\right| = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

B) Derivation of following expression is required.



$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\ \hat{a}_R$ (V/m) ← Cylindrical co-ordinate system.

$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 a}\ \hat{a}_x$ (V/m) ← Cartesian co-ordinate system.

Q.5. A) Obtain following expression for MUF in terms of d, H & f_c.

$$f_{MUF} = f_c \sqrt{1+\left(\frac{d}{2H}\right)^2}$$

Given : $d = 2000$ km, $H = 200$ km, $f_c = 5$ MHz.

$$\therefore\ f_{MUF} = f_c \sqrt{1+\left(\frac{d}{2H}\right)^2} = \underline{25.495 \text{ MHz}}.$$

B) Given : $h_t = 60$ m ; $h_r = 6$ m ;
To find : d.

Soln : $d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right] = \underline{42 \text{ km}}.$

C) E-layer & Sporadic-E layer can be explained in difference form with following points.
i) Behaviour of layers.
ii) ~~Ex.~~ Time of presence (day / night).
iii) Ionization density.
iv) Purpose in communication. (wireless).

Q.6. A) Given : $\mu_r = 1$; $\epsilon_r = 9$; $\vec{E}(z,t) = \cos(10^9 t - \beta z)\hat{a}_x$ (V/m).
To find : i) β & ii) $\vec{H}$.

Soln : i) $\omega = 10^9$ rad/sec ; $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{3\sqrt{\mu_0\epsilon_0}} = 10^8$ (m/s).

$$\therefore\ \boxed{\beta = \frac{\omega}{v} = 10 \text{ (rad/m)}}.$$

ii) $\frac{E_x}{H_y} = \eta = \sqrt{\frac{\mu}{\epsilon}} = 40\pi \Rightarrow H_y = \frac{E_x}{40\pi}$.

$$\therefore\ \boxed{\vec{H}_y = \frac{1}{40\pi}\cos(10^9 t - \beta z)\,\hat{a}_y \text{ (A/m)}}.$$

B) Given : $\epsilon_r = 1$; $\mu_r = 1$; $\sigma = 10^{-4}$ (mho/m). ; $f = 1$ GHz.

Solⁿ :

i) Propagation constant (γ) :

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ (rad/sec)}.$$

$$\therefore \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{[-\omega^2\mu\epsilon + \sigma\omega\mu j]}$$

$$= \sqrt{-438.65 + 0.7896j}$$

$$\therefore \boxed{\gamma = (0.019 + 20.944j)} = \alpha + j\beta.$$

ii) Attenuation constant in dB :

$$\therefore \alpha = 0.019 \text{ (Np/m)}.$$

$$\therefore \alpha_{dB} = 8.686 \times 0.019 = 0.165 \text{ dB}.$$

iii) Wavelength (λ) :

$$\boxed{\lambda = \frac{2\pi}{\beta} = 0.299 \approx 0.3 \text{ m.}}$$

iv) Refractive Index (n) :

$$\boxed{n = \frac{c}{v} = 1} \quad (\because \epsilon_r = 1 \ \& \ \mu_r = 1).$$

v) Loss tangent :

$$\tan\Delta = \frac{\sigma}{\omega\epsilon} = 1.798 \times 10^{-3}$$

$\because \tan\delta << 1 \Rightarrow$ Medium is lossy dielectric.

c) Formation of duct propagation should be described by proper theory & neat diagram.

Condⁿˢ for duct propagation :

i) Transmitting antenna should be inside the duct.

ii) Radio wave should enter the duct at very low angle of incidence.

Q.1. A) Given : $Q_1 = 300\,\mu C$ at $P_1(1,-1,-3)$ m

$Q_2 = Q$ at $P_2(3,-3,-2)$ m.

$\vec{F}_{21} = 8\hat{a}_x - 8\hat{a}_y + 4\hat{a}_z$ (N).

To find : $Q_2 = Q$.

Sol^n : $\vec{F}_{21} = \dfrac{Q_1 Q_2}{4\pi\epsilon_0 (R_{21})^2}\hat{a}_{21}$; $\vec{R}_{21} = -2\hat{a}_x + 2\hat{a}_y - \hat{a}_z$

$\therefore R_{21} = 3$ m.

$\therefore \hat{a}_{21} = \dfrac{\vec{R}_{21}}{R_{21}} = \dfrac{1}{3}(-2\hat{a}_x + 2\hat{a}_y - \hat{a}_z)$

$\therefore$ $\boxed{Q_2 = Q = -40\,\mu C.}$

B) E & F regions are present during day & night time. Maximum attenuation of wave takes place in D-region.

C) With neat labeled diagram & appropriate theory, the explanation of Super refraction & Sub refraction should be done.

D) "Modified Ampere's law" eq^n. tells the propagation of EM wave in air.

Q.2. A) Following boundary cond^{ns}. need to be derived,

For E Field : $\vec{D}_t = \vec{E}_t = 0$ & $\vec{D}_n = \epsilon\vec{E}_n = \rho_s$

For H-Field : $\underbrace{\vec{H}_n = \vec{B}_n = 0}_{\text{Inside conductor.}}$ & $\vec{H}_t = \dfrac{\vec{B}_t}{\mu} = \bar{a}_{N12} \times \bar{K}$ ← Surface current.

B) Derive following eq^n. & state significance of each term in it.

$$\int_V (\vec{E}\cdot\vec{J})\,dv = -\frac{\partial}{\partial t}\int_V \left(\frac{\mu}{2}H^2 + \frac{\epsilon}{2}E^2\right)dv - \oint_S (\vec{E}\times\vec{H})\cdot d\vec{s}$$

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$$V_2 = \frac{V_1 + V_3}{2} \Rightarrow -V_1 + 2V_2 - V_3 = 0 \quad - (2)$$

$$V_3 = \frac{V_2 + V_4}{2} \Rightarrow -V_2 + 2V_3 = 20 \quad - (3).$$

$$\therefore \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}$$

$\therefore V_1 = 5V \; ; \; V_2 = 10V \; ; \; V_3 = 15V.$

$\therefore \boxed{V(1) = V_1 = 5V.}$

c) Use Fleming's left hand rule to describe the theory.

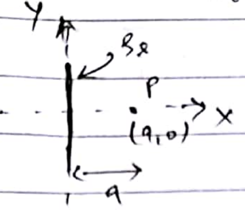
Q. 4. A) Derivation of following expressions is required.

$$\Gamma_{R\perp} = \frac{|E_r|}{|\vec{E_i}|} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$\Gamma_{T\perp} = \frac{|\vec{E_t}|}{|\vec{E_i}|} = \frac{2n_2}{n_2 + n_1}$$

B) Derivation of following expression is required.

$$\vec{E} = \frac{\rho_l}{2\pi \epsilon_0 R} \hat{a}_R \ (V/m)$$ ← Cylindrical Co-ordinate system.



$$\vec{E} = \frac{\rho_l}{2\pi \epsilon_0 a} \hat{a}_x \ (V/m)$$ ← Cartesian Co-ordinate System.

Q.5 A) Obtain following expression for MUF in terms of d, H & f_c.

$$f_{MUF} = f_c \sqrt{1 + \left(\frac{d}{2H}\right)^2}$$

for $d = 2000$ Km ; $H = 200$ Km ; $f_c = 5$ MHz.

$$\boxed{f_{MUF} = 25.495 \text{ MHz.}}$$

B) Given : $h_t = 60$ m ; $h_r = 6$ m.

To find : d.

Soln : $d = 4.12 \left[\sqrt{h_t} + \sqrt{h_r}\right] = 42$ Km.

c) E-layer & Sporadic-E layer can be explained in differential form with following points.

ii) 'α' in dB :

$\alpha = 0.019$ (Np/m).

$\therefore \quad \alpha_{dB} = 8.686\,\alpha = 0.165$ dB.

iii) λ

$\lambda = \frac{2\pi}{\beta} = 0.299 \simeq 0.3$ m.

iv) (n)

$\boxed{n = \frac{c}{v} = 1}$ $(\because \epsilon_r = 1 \text{ \& } \mu_r = 1)$

v) Loss tangent :

$\tan\Delta = \frac{\sigma}{\omega\epsilon} = 1.798 \times 10^{-3}$.

$\because$ $\tan\Delta << 1 \Rightarrow$ Medium is lossy dielectric.

c) Formation of duct propagation should be described by proper theory & neat diagram.

Cond^n. for duct propagation :

i) Transmitting antenna should be inside the duct.

ii) Radio wave should enter the duct at very low angle of incidence.

1 3

HINTS & SOLUTIONS (Only Numericals).

Q.1. b) Given : $\epsilon_{r_1} = 1$; $\epsilon_{r_2} = 5$; $\theta_1 = 58°$.

To Find : θ_2.

Sol^n : $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_{r_1}}{\epsilon_{r_2}}$.

$\therefore \theta_2 = \tan^{-1}\left(\dfrac{\epsilon_{r_2}}{\epsilon_{r_1}} \tan\theta_1\right) = \tan^{-1}\left[5 \cdot \tan(58°)\right]$

$\therefore \boxed{\theta_2 = 82.88°}$

c) True. a. Since, less loss of E field strength.

Q.2. b) $\vec{E} = \dfrac{\rho_l}{2\pi\epsilon_0 R}\hat{a}_R$, Hence, $\vec{E}$ is independent of wire length.

Q.3. a) By FDM, $V_1 = \dfrac{V_2 - 10 + 0 + 0}{4} \Rightarrow 4V_1 - V_2 = -10$ — (1).

$V_2 = \dfrac{V_1 + V_3 + 0 + 0}{4} \Rightarrow -V_1 + 4V_2 - V_3 = 0$ — (2).

$V_3 = \dfrac{V_2 + 10 + 0 + 0}{4} \Rightarrow -V_2 + 4V_3 = 10$ — (3).

$$\therefore \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 10 \end{bmatrix}$$

$\therefore \boxed{V_1 = -2.5V \ ; \ V_2 = 0V \ ; \ V_3 = 2.5V}$

b) Type of polarization : Perpendicular polarization.

Here,

$\vec{E}_i = E_i \cos(\omega t - \vec{\beta}_i \cdot \vec{r})\,\hat{a}_{Ei}$

& $\vec{E} = 8\cos(\omega t - 4x - 3z)\,\hat{a}_y$ (V/m).

$\therefore \beta_{xi} = 4$; $\beta_{yi} = 0$; $\beta_{zi} = 3$

$\therefore \tan\theta_i = \dfrac{\beta_{xi}}{\beta_{zi}} = \dfrac{4}{3} \Rightarrow \boxed{\theta_i = 53.13°}$

Q.4. a) $\tan\Delta = \frac{\sigma}{\omega\epsilon}$; $\tan\Delta = 0$ for lossless dielectric.
$\tan\Delta > 1$ for lossy dielectric.
$\tan\Delta \gg 1$ for good conductor.

b) Given : $Q_1 = 5nC$ at $P(2,3,5)$ m.
$Q_2 = 4nC$ at $R(3,2,3)$ m.

To find : $\vec{E}$ at $A(6,7,8)$.

Sol^n :- $\vec{E_1} = \frac{Q_1}{4\pi\epsilon_0} \frac{(4\hat{i}+4\hat{j}+3\hat{k})}{(41)^{3/2}}$ (V/m)

$\vec{E_2} = \frac{Q_2}{4\pi\epsilon_0} \frac{(3\hat{i}+5\hat{j}+5\hat{k})}{(59)^{3/2}}$ (V/m)

$\therefore \vec{E_{net}} = \vec{E_1} + \vec{E_2} = 45\frac{(4\hat{i}+4\hat{j}+3\hat{k})}{(41)^{3/2}} + 36\frac{(3\hat{i}+5\hat{j}+5\hat{k})}{(59)^{3/2}}$

$$\boxed{\vec{E_{net}} = (0.924\,\hat{i} + 1.083\,\hat{j} + 0.911\,\hat{k}) \ (V/m)}$$

Q.5. b) Given :- $\vec{H}\Big|_{\text{at } P(3,4,0)} = 10(-0.8\hat{a}_x + 0.6\hat{a}_y)$ (A/m).

To find :- I.

Sol^n :- $\vec{R} = 3\hat{a}_x + 4\hat{a}_y \Rightarrow \hat{a}_R = \frac{3\hat{a}_x + 4\hat{a}_y}{5}$

Conductor is along z-axis $\Rightarrow \vec{a_l} = \hat{a}_z$.

$\therefore \hat{a}_\phi = \hat{a}_l \times \hat{a}_R = \frac{-4\hat{a}_x + 3\hat{a}_y}{5}$

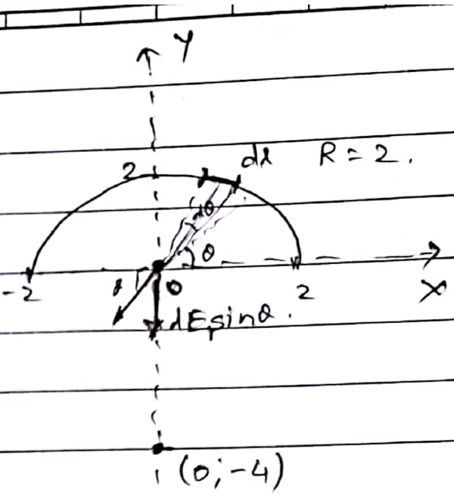
Now, $\vec{H} = \frac{I}{2\pi R}\hat{a}_\phi$

$$\therefore \boxed{I = 100\pi = 314.159 \text{ A}}$$

c)

$dl = 2\, d\theta$

$\rho_l = \dfrac{dQ}{dl} \Rightarrow dQ = \rho_l\, dl$



But $\rho_l = \dfrac{Q_{net}}{2\pi} = \dfrac{5}{\pi}\ (nC/m)$.

$\therefore \quad dQ = \dfrac{10\times10^{-9}}{\pi} = \dfrac{10^{-8}}{\pi}\, d\theta$

$\therefore \quad E_r = \int dE \sin\theta = \int \dfrac{dQ}{4\pi\epsilon_0 R^2} \sin\theta$

$= \dfrac{9\times10^9}{4} \displaystyle\int_0^{\pi} \left(\dfrac{10^{-8}}{\pi}\right) \sin\theta\, d\theta$

$E_r = \dfrac{90\,(2)}{4\pi} = \dfrac{45}{\pi} = 14.324\ (V/m)$.

$\therefore \quad \vec{E}_r = \vec{E}\ (\text{due to ring}) = -14.324\, \hat{a}_y \ (V/m)$

Now, $\vec{E}_q = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{r^2} \hat{a}_r = 9\times10^9 \times \dfrac{Q}{4^2} \times \hat{a}_y \ (V/m)$.

$\therefore$ for $\vec{E}_{net} = 0 \Big|_{\text{at origin}} = \vec{E}_r + \vec{E}_q$

$\therefore \quad E_q = E_r \Rightarrow \dfrac{9\times10^9 \times Q}{16} = 14.324$

$\therefore \quad \boxed{Q = 25.465\ nC}$