**QP Code : 79299**

Duration:[2½Hours] [Marks: 60]

N.B. 1) All questions are compulsory.
2) Attempt any **TWO** subquestions from each question.
3) Each question carry **12** marks.

1. (a) $G$ is a $(p,q)$-graph with $q < \frac{kp}{2}$, where $k$ is a positive integer. Prove that $G$ is not $k$-connected. (6)

   (b) Let $G$ be a graph with $p$ vertices, where $p \geq 2$. Then prove that $G$ has at least two vertices which are non-cut vertices. (6)

   (c) Let $G$ be a graph on $p$ vertices and let $A$ denotes adjacency matrix of $G$. Let $B$ be the matrix where $B = A + A^2 + A^3 + \ldots + A^{(n-1)}$. Then prove that $G$ is connected graph if and only if for every pair of distinct indices $i, j$ we have $b_{ij}$ is non-zero, where $b_{ij}$ is the $(i,j)$-th entry of the matrix $B$. (6)

2. (a) Define a bond in a graph. Show that in a connected graph $G$, and edge cut $F$ is a bond if and only if $G - F$ has exactly two components. (6)

   (b) Show that graph $G$ is tree if and only if every two vertices of $G$ are connected by a unique path. (6)

   (c) Explain Breadth First Search Algorithm and Depth First Search Algorithm. (6)

3. (a) State and prove the necessary and sufficient condition for a graph to be Eulerian. (6)

   (b) Let $G$ be a simple graph vertex degree sequence $(d_1 \leq d_2 \leq \cdots \leq d_p)$, whenever $p \geq 3$. If $i < p/2$ implies that $d_i > i$ or $d_{p-i} \geq p - i$ then then show that G is Hamiltonian. (6)

   (c) Define closure of a graph $C(G)$. Show that if the closure of graph $G$ is complete then $G$ is Hamiltonian. (6)

4. (a) Prove that the matching $M$ in a graph G is maximum if and only if $G$ contains no $M$ augmenting path. (6)

   (b) Define Ramsey number $r(k,l)$. Show that $r(k,l) \leq \begin{pmatrix} k+l-2 \\ k-1 \end{pmatrix}$. (6)

   (c) If $M$ be a matching and $K$ be a covering such that $|M| = |K|$ then prove that $M$ is a maximum matching and $K$ is a minimum covering. (6)

5. (a) Show that a graph is bipartite if and only if for every subgraph $H$ of $G$, $H$ contains an independent set of at least $n(H)/2$ vertices, where $n(H)$ is the number of vertices of $H$. (6)

   (b) Let $G$ be a $(p,q)$ graph with $k$ components. Show that $p - k \leq q \leq \frac{p-k+1}{2}$. (6)

   (c) If $G$ is a graph on $|V(G)|$ vertices with $deg(u) + deg(v) \geq |V(G)| - 1$ for every pair of non adjacent vertices $u$ and $v$ in $G$, then show that $G$ contains a Hamiltonian path. (6)

**TM-Con.1352-17.**