01 Q.P. Code: 38827

Signals & systems
Answer key -

Q.1

a) $L[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

Put $s = \sigma + j\Omega$ in above eq^n gives

$L[x(t)] = X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma + j\Omega)t}\, dt$

By def^n of Fourier transform of $x(t)$ is

$F[x(t)] = X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$

$\therefore X(s) = X(j\Omega)$ when $\sigma = 0$

(b) exponential form of Fourier series:

$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\Omega_0 t}$

Reconstructed signal exhibits oscillations & these are compressed towards discontinuties. At point of discontinuities, fourier series converges to avg. value of signal. This is Gibbs phenomenon.

(c) properties of Z.T. :-

i) $Z[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1[z] + a_2 X_2[z]$

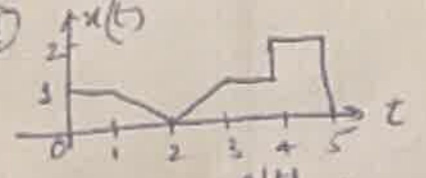
ii) $Z[x(n+m)] = z^{m} X(z)$

iii) $Z[n^m x(n)] = \left(-z \frac{d}{dz}\right)^m X(z)$

iv) $Z[a^n x(n)] = X(a^{-1}z)$

v) $Z[x_1(n) * x_2(n)] = X_1(z)\, X_2[z]$

(d)



(e) $x(t) = A e^{-a|t|}$ for all t

$X(j\Omega) = \int_{-\infty}^{0} A e^{at} e^{-j\Omega t}\, dt + \int_{0}^{\infty} A e^{-at} e^{-j\Omega t}\, dt$

$\therefore F[A e^{-a|t|}] = \frac{2aA}{a^2 + \Omega^2}$

(Q 2)

Q.3 (b)

(i) $E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \frac{1}{4}$ J

$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = 0$

given signal is an energy signal.

(ii) $x(t) = 3\cos 5\Omega_0 t$

$E = \lim_{T \to \infty} \int_{-T}^{T} (3\cos 5\Omega_0 t)^2 dt$

$\therefore E = \infty$.

$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} (3\cos 5\Omega_0 t)^2 dt$

$P = 4.5$ Watt

Given signal is power signal.

(c) (i) $x(t) = 2\cos\left(\frac{t}{4}\right)$

$= A\cos(2\pi f t)$

$\therefore T = \frac{1}{f} = 8\pi$

given signal is periodic with $T = 8\pi$.

(ii) $x(t) = e^{-j2\pi t/7} = A e^{j2\pi f t}$

periodic signal

$F = \frac{1}{7} = \frac{1}{T}$

Given signal is periodic with $T = 7$ sec

Q.4. (a) $X(z) = \frac{-10z}{z-1} + \frac{10z}{z-2}$

$x(n) = -10u(n) + 10(2^n)u(n)$

case (i) $|z| > 2$

$x(n) = -10u(n) + 10(2)^n u(n)$

case (ii) $1 < |z| < 2$

$x(n) = -10u(n) - 10(2)^n u(-n-1)$

case (iii) $|z| < 1$

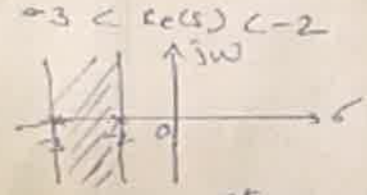
$x(n) = 10u(-n-1) - 10(2)^n u(-n-1)$

O3

(b) $X(s) = \frac{s}{(s+2)(s+3)^2}$ (4)

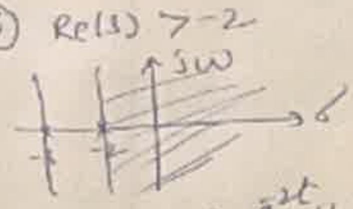
$X(s) = \frac{-2}{s+2} + \frac{2}{s+3} + \frac{3}{(s+3)^2}$

(i) $-3 < Re(s) < -2$



$x(t) = 2e^{-2t}u(-t) + 2e^{-3t}u(t) + 3te^{-3t}u(t)$

(ii) $Re(s) > -2$



$x(t) = -2e^{-2t}u(-t) + 2e^{-3t}u(t) + 3te^{-3t}u(t)$

Q.5 (a) $y(n) - 4y(n-1) + 4y(n-2) = x(n-1)$

Take Z.T.

$Y(z) - 4z^{-1}Y(z) + 4z^{-2}Y(z) = z^{-1}X(z)$

$Y(z)[1 - 4z^{-1} + 4z^{-2}] = z^{-1}X(z)$

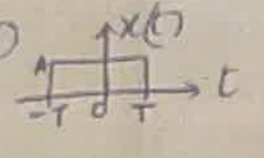
$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-4z^{-1}+4z^{-2}} = H(z)$

Taking inverse Z.T.

$h(n) = z^{-1}\{H(z)\} =$

$h(n) = \frac{1}{2} n 2^n u(n) = n 2^{(n-1)} u(n)$

(b) (i)



$X(j\omega) = \int_{-T}^{T} A e^{-j\omega t} dt$

$X(j\omega) = 2AT \sin\frac{\omega T}{\omega T}$