QP - 38205

Solution 2018 (May) Choice Based. 1

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1) a) Along $y = x^2$, $dy = 2x\,dx$

$$\therefore I = \int_0^3 (x + x^2)\,dx + x^2x^2(2x\,dx)$$

$$= \int_0^3 (x + x^2 + 2x^5)\,dx = \left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^6}{3}\right]_0^3 = \boxed{256\tfrac{1}{2}}$$ #

b) $$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} x\,dx = 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx = -\frac{2}{\pi}$$

$$\therefore x = \pi - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$$ #

c) $$F\{f(x)\} = \int_{-1}^{1} (1 - x^2)\,e^{-isx}\,dx$$

$$= 2\int_0^1 (1 - x^2)\cos sx\,dx$$

$$= 2\left[(1 - x^2)\frac{\sin sx}{s} - 2x\frac{\cos sx}{s^2} + 2\frac{\sin sx}{s^3}\right]_0^1$$

$$= \frac{4}{s^3}(\sin s - s\cos s)$$ #

d) $E[f(x)] = f(x+h)$, $E(e^x) = e^{x+h}$ — 1

$$\Delta e^x = e^{x+h} - e^x = e^x[e^h - 1]$$

$$\Delta^2 e^x = \Delta\cdot\Delta(e^x) = \Delta e^x[e^h - 1]$$

$$= (e^h - 1)\Delta e^x = (e^h - 1)e^x(e^h - 1)$$

$$= (e^h - 1)^2 e^x \quad \text{— 2}$$

$$\left(\frac{\Delta^2}{E}\right)e^x = (\Delta^2E^{-1})e^x = \Delta^2 e^{x-h} = e^{-h}\Delta^2 e^x = e^{-h}(e^h - 1)^2 e^x \quad \text{— 3}$$

$$\therefore LHS = e^{-h}(e^h - 1)^2 e^x \cdot \frac{e^{x+h}}{(e^h - 1)^2 e^x} = e^x = RHS$$

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2) a) $\frac{\ell}{2}$ $f(x) = \frac{l}{2} - x$

$$a_0 = \frac{1}{l/2}\int_0^l f(x)\,dx = \frac{2}{l}\int_0^l \left(\frac{l}{2} - x\right)dx = \frac{2}{l}\left[\frac{lx}{2} - \frac{x^2}{2}\right]_0^l = 0.$$

$$a_n = \frac{1}{l/2}\int_0^l f(x)\cos\frac{n\pi x}{l/2}\,dx = 0.$$

$$b_n = \frac{1}{l/2}\int_0^l f(x)\sin\frac{n\pi x}{l/2}\,dx = \frac{2}{l}\int_0^L\left(\frac{1}{2} - x\right)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{l}{n\pi}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_1^\infty a_n\cos\frac{n\pi x}{l/2} + \sum_1^\infty b_n\sin\frac{n\pi x}{l/2}$$

$$\frac{l}{2} - x = \frac{l}{\pi}\sum\frac{1}{n}\sin\frac{2n\pi x}{l}. \quad \#.$$

b) $L(x_1, x_2, \lambda) = (4x_1 + 8x_2 - x_1^2 - x_2^2) - \lambda(x_1 + x_2 - 4)$

$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - \lambda$, $\frac{\partial L}{\partial x_2} = 8 - 2x_2 - \lambda$, $\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 4)$

solving, $\frac{\partial L}{\partial x_1} = 0$, $\frac{\partial L}{\partial x_2} = 0$, $\frac{\partial L}{\partial \lambda} = 0$, we get

$4 - 2x_1 - \lambda = 0$, $8 - 2x_2 - \lambda = 0$, $x_1 + x_2 = 4$

$\therefore$ solving, $\lambda = 2$, $x_1 = 1$, $x_2 = 3$

$\therefore X_0$ is $(1, 3)$

Now $h(x_1, x_2) = x_1 + x_2 - 4 = 0$

$\frac{\partial h}{\partial x_1} = 1$, $\frac{\partial h}{\partial x_2} = 1$

$f(x_1, x_2) = 4x_1 + 8x_2 - x_1^2 - x_2^2$

$\frac{\partial f}{\partial x_1} = 4 - 2x_1$, $\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$, $\frac{\partial^2 f}{\partial x_1^2} = -2.$

$\frac{\partial f}{\partial x_2} = 8 - 2x_2$, $\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$, $\frac{\partial^2 f}{\partial x_2^2} = -2$

$$\Delta = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda\frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda\frac{\partial^2 h}{\partial x_1 \partial x_2} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda\frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda\frac{\partial^2 h}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{vmatrix} = 4$$

$\because \Delta$ is positive, X_0 is maxima

$\therefore x_1 = 1$, $x_2 = 3$ $\quad Z_{max} = 18.$

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2. c) $f(z) = \frac{a}{z} + \frac{b}{z^2} + \frac{c}{z-1} + \frac{d}{z-2}$

Solving,

$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3(z-1)} - \frac{1}{12(z+2)}$$

Case 1. $0 < |z| < 1 \Rightarrow |z| < 2 \Rightarrow \left|\frac{z}{2}\right| < 1$

$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3}(1-z)^{-1} - \frac{1}{24}\left(1+\frac{z}{2}\right)$$

$$= -\frac{1}{4z} - \frac{1}{2z^2} - \frac{1}{3}\left(1+z+z^2+\cdots\right) - \frac{1}{24}\left(1 - \frac{z}{2} + \frac{z^2}{4} - \cdots\right)$$

Case 2. $1 < |z| < 2$

$1 < |z| \Rightarrow \left|\frac{1}{z}\right| < 1$, $|z| < 2 \Rightarrow \left|\frac{z}{2}\right| < 1$

$$\therefore f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z}\left(1-\frac{1}{z}\right)^{-1} - \frac{1}{12\cdot 2}\left(1+\frac{z}{2}\right)^{-1}$$

$$= -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\cdots\right) - \frac{1}{24}\left(1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right)$$

Case 3 $|z| > 2 \Rightarrow |z| > 1$

$\therefore$ ~~$\left|\frac{z}{2}\right|$~~ $\left|\frac{2}{z}\right| < 1$, $\left|\frac{1}{z}\right| < 1$

$$\therefore f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z}\left(1-\frac{1}{z}\right)^{-1} - \frac{1}{12z}\left(1+\frac{2}{z}\right)^{-1}$$

$$= -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\cdots\right) - \frac{1}{12z}\left(1 - \frac{2}{z} + \frac{4}{z^2} - \cdots\right)$$

3. a) we have $x_0 = 1,\ x_1 = 2,\ x_2 = 7,\ x_3 = 8$

$y_0 = 4,\ y_1 = 4,\ y_2 = 5,\ y_3 = 4$

and $x = 5$.

$$y = y_0 \cdot \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$
$$+ y_2 \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$y = 4 \cdot \frac{(5-2)(5-7)(5-8)}{(1-2)(1-7)(1-8)} + 4 \cdot \frac{(5-1)(5-7)(5-8)}{(2-1)(2-7)(2-8)}$$
$$+ 5 \cdot \frac{(5-1)(5-2)(5-8)}{(7-1)(7-2)(7-8)} + 4 \cdot \frac{(5-1)(5-2)(5-1)}{(8-1)(8-2)(8-7)}$$

$$= \frac{26}{5}$$

b) $$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad \text{—(1)}$$

$\Rightarrow u = (c_1 \cos pt + c_2 \sin pt)(c_3 \cos px + c_4 \sin px)$

$x = 0,\ u = 0 \quad \therefore \quad c_3 = 0$

$\therefore\ u = (c_1 \cos pt + c_2 \sin pt)(c_4 \sin px)$

put $x = \pi,\ u = 0$

$c_4 \sin p\pi = 0 \Rightarrow \sin p\pi = 0 = \sin n\pi$

$p\pi = n\pi \Rightarrow p = n, \quad n = 1, 2, 3, \ldots$

$\therefore\ u = (c_1 \cos nt + c_2 \sin nt)\, c_4 \sin nx$

$\frac{du}{dt} = (-c_1 n \sin nt + c_2 n \cos nt)\, c_4 \sin nx$

$\frac{du}{dt} = 0,\ t = 0 \Rightarrow 0 = (c_2 n)(c_4 \sin nx)$.

$\Rightarrow c_2 = 0$

$\therefore\ u = c_1 c_4 \cos nt \sin nx$

3.b) contd

(08) (05)

$$u(x,0) = 2(\sin x + \sin 3x)$$

$$t=0,\ u(x,0) = 4C_4 \sin nx$$

$$2(\sin x + \sin 3x) = 4C_4 \sin nx$$

$$4\sin 2x \cos x = 4C_4 \sin nx$$

$$4C_4 = 4\cos x, \quad \sin nx = \sin 2x$$
$$n=2$$

$$u(x,t) = 4\cos x \cos 2t \sin 2x \ .\ \#.$$

c) $f(x_1,x_2) = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

$h(x_1,x_2) = x_1 + 3x_2 - 6$

$h(x_1,x_2) = 5x_1 + 2x_2 - 10$

Kuhn Tucker conditions for maxima are

$$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_1} = 0 \quad \therefore 2 - 2x_1 - \lambda_1 - 5\lambda_2 = 0$$

$$\frac{\partial f}{\partial x_2} - \lambda_1 \frac{\partial h}{\partial x_2} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0 \quad \therefore 3 - 4x_2 - 3\lambda_1 - 2\lambda_2 = 0$$

$$\lambda_1 (x_1 + 3x_2 - 6) = 0$$

$$\lambda_2 (5x_1 + 2x_2 - 10) = 0$$

$$x_1 + 3x_2 - 6 \le 0$$

$$5x_1 + 2x_2 - 10 \le 0$$

$$x_1, x_2 \geq 0 \qquad \lambda_1, \lambda_2 \geq 0 .$$

Case I $\quad \lambda_1 = 0 \ \&\ \lambda_2 = 0$

$2 - 2x_1 = 0, \quad x_1 = 1, \quad 3 - 4x_2 = 0 \quad \therefore x_2 = 3/4$

$$H = \begin{bmatrix} \frac{\partial^2 z}{\partial x_1^2} & \frac{\partial^2 z}{\partial x_1 \partial x_2} \\ \frac{\partial^2 z}{\partial x_2 \partial x_1} & \frac{\partial^2 z}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$$

$A_1 = |a_{11}| = -2$ & $A_2 = \begin{vmatrix} -2 & 0 \\ 0 & -4 \end{vmatrix} = 8$ (06)

since the principal minors are alternatively negative positive $x_1 = 1$, $x_2 = 3/4$ gives a maxima.

$$Z_{max} = 2 + 3\left(\frac{3}{4}\right) - 1 - 2\left(\frac{9}{16}\right) = \frac{17}{8}.$$

<u>case 2</u> $\lambda_1 = 0$ & $\lambda_2 \neq 0$

$2 - 2x_1 - 5\lambda_2 = 0$

$3 - 4x_2 - 2\lambda_2 = 0$

Solving $x_1 = \frac{89}{54} = 1.648$, $x_2 = \frac{95}{108} = 0.880$

$5\lambda_2 = -\frac{70}{54}$ $-$ve.

<u>reject the pair.</u>

Case 3. $\lambda_1 \neq 0$ and $\lambda_2 = 0$

$2 - 2x_1 - \lambda_1 = 0$, $3 - 4x_2 - 3\lambda_1 = 0$

$\therefore$ solving, $x_1 = 3/2$, $x_2 = 3/2$

But x_1, x_2 donot satisfy the eqns.

$\therefore$ <u>Reject the pair</u>.

<u>case 4.</u> $\lambda_1 \neq 0$, $\lambda_2 \neq 0$

$x_1 + 3x_2 = 6$ & $5x_1 + 2x_2 = 10$

$x_1 = \frac{20}{13}$, $x_2 = \frac{18}{13}$

$\lambda_1 + 5\lambda_2 = -\frac{14}{13}$, $3\lambda_1 + 2\lambda_2 = -\frac{33}{13}$

$\therefore \lambda_1 = -\frac{137}{169}$ & $\lambda_2 = -\frac{9}{169}$ negative

$\therefore$ Reject the pair.

$$\therefore \boxed{Z_{max} = \frac{17}{8}}$$

07 7

4. a) Half range cosine series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$

Here $l = \pi$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x(\pi - x)\,dx = \frac{\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x(\pi - x)\cos nx\,dx = \frac{2}{\pi}\left(\frac{\pi}{n^2}\right)\left[-(-1)^n - 1\right]$$

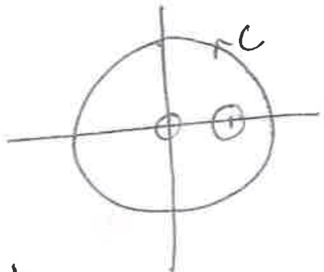
$$= \begin{cases} -\frac{4}{n^2} & \text{when } n \text{ is even} \\ 0 & \text{when } n \text{ is odd} \end{cases}$$

Hence $x(\pi - x) = \frac{\pi^2}{6} - 4\left[\frac{\cos 2x}{2^2} + \frac{\cos 4x}{4^2} + \cdots\right]$ #.

b) $f(z)$ is not analytic at

$z(z-1)(z-2) = 0 \Rightarrow z = 0, z = 1, z = 2$.

$z = 0, 1$ lies inside C but $z = 2$ lies outside C.



$$\int_C = \int \frac{\frac{4-3z}{z-2}}{z(z-1)}\,dz$$

$$= \int_C \frac{\frac{4-3z}{(z-2)(z-1)}}{z}\,dz + \int_C \frac{\frac{4-3z}{z(z-2)}}{z-1}\,dz$$

$$= 2\pi i\left[\frac{4-3z}{(z-1)(z-2)}\right]_{z=0} + 2\pi i\left[\frac{4-3z}{z(z-2)}\right]_{z=1}$$

$$= 2\pi i \frac{4}{(-1)(-2)} + 2\pi i \frac{4-3}{1(1-2)} = 2\pi i(2-1) = 2\pi i$$ #.

c) We have $\frac{\partial u}{\partial t} = \frac{1}{h^2} \frac{\partial u}{\partial x^2}$

08

0

$$u = c_1 e^{-\frac{p^2 t}{h^2}} \left(c_2 \cos px + c_3 \sin px \right)$$

$u = 0, \ x = 0 \quad \therefore c_1 c_2 e^{-p^2 t^2/h^2} = 0$

$c_1 \neq 0 . \quad c_2 = 0$

$\therefore u = c_3 \sin px \, c_1 e^{-p^2 t/h^2}$

$x = l, \ u = 0$

$0 = c_3 \sin pl \, e^{-p^2 t/h^2}$

$c_3 \neq 0 \quad \therefore \sin pl = 0 = \sin n\pi$

or $p = \frac{n\pi}{l}$

$\therefore u = c_3 \sin \frac{n\pi x}{l} e^{-\frac{n^2\pi^2 l}{h^2 t^2}}$

$t = 0, \ u = \sin \frac{\pi x}{l}$

$\therefore \sin \frac{\pi x}{l} = c_3 \sin \frac{n\pi x}{l}$

$\Rightarrow n = 1, \quad c_3 = 1$

$\therefore u = \sin \frac{\pi x}{l} e^{-\pi^2 t/h^2 t^2}$ #

5. a) We have six values so assume sixth dif to be zero.

$D^6 y_0 = 0 \quad \therefore (E-1)^6 y_0 = 0$

$(E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) y_0 = 0$

$\therefore y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$

$\therefore y_6 - 6(1222) + 15(492) - 20(148) + 15(22) - 6(-6) - 8 = 0$

$y_6 - 2554 = 0 \quad \Rightarrow \boxed{y_6 = 2554}$

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b) $f(x) = \sum_{-\infty}^{\infty} C_n e^{in\pi x/l}$

$$C_n = \frac{1}{2l}\int_{-l}^{l} f(x) e^{-in\pi x/l}\,dx$$

$$= \frac{1}{2l}\int_{-l}^{l} e^{ax} e^{-in\pi x/l}\,dx$$

$$= \frac{1}{2l}\int_{-l}^{l} e^{\left(a-\frac{in\pi}{l}\right)x}\,dx$$

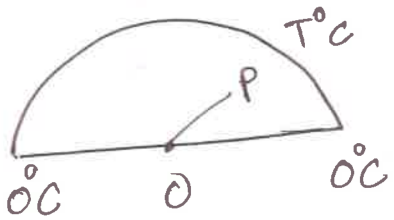
$$= \frac{1}{2l}\left[\frac{e^{(a-in\pi/l)x}}{a-\frac{in\pi}{l}}\right]_{-l}^{l}$$

$$= \frac{(al+in\pi)\cdot l}{2l\,(a^2l^2+n^2\pi^2)}\left[e^{al}\cdot e^{-in\pi} - e^{-al}e^{in\pi}\right]$$

$$= \frac{1}{2}\frac{(al+in\pi)}{a^2l^2+n^2\pi^2}(-1)^n\left[e^{al}-e^{-al}\right]$$

$$= \frac{al+in\pi}{a^2l^2+n^2\pi^2}(-1)^n \sinh al$$

$$\therefore\ e^{ax} = \sum_{-\infty}^{\infty}(-1)^n\frac{\sinh al\,(al+in\pi)}{a^2l^2+n^2\pi^2}e^{in\pi x/l}.$$

c) 

Let the centre O of the semicircular plate be the pole and the bounding diameter be as the initial line. Let $u(r,\theta)$ be the steady state temperature at any point $P(r,\theta)$ and u satisfies the equation

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0.$$

(10)

The boundary conditions are

i) $u(r,0) = 0 \qquad 0 \le r \le a$

ii) $u(r,\pi) = 0 \qquad 0 \le r \le a$

iii) $u(a,\theta) = T$

From (ii) & (iii), we have $u \to 0$ as $r \to 0$.

Hence the appropriate solution of (i) is ~~as~~

$$u = (c_1 r^p + c_2 r^{-p})(c_3 \cos p\theta + c_4 \sin p\theta) \quad (2)$$

$u(r,0) = 0$ in (2)

$$0 = (c_1 r^p + c_2 r^{-p}) c_3 \rightarrow c_3 = 0$$

$$\therefore \quad u = (c_1 r^p + c_2 r^{-p}) c_4 \sin p\theta$$

putting $u(r,\pi) = 0$ in (3),

$$0 = (c_1 r^p + c_2 r^{-p}) c_4 \sin p\pi$$

$$\Rightarrow \sin p\pi = 0 = \sin n\pi$$

$$\Rightarrow p\pi = n\pi \Rightarrow p = n$$

$$\therefore \quad u = (c_1 r^n + c_2 r^{-n}) c_4 \sin n\theta$$

$u = 0$ when $r = 0$, $0 = c_2$

$$u = c_1 c_4 r^n \sin n\theta$$

$$\therefore \quad u(r,\theta) = \sum_{n=1}^{\infty} b_n r^n \sin n\theta$$

$r = a$, $u = T$

$$T = \sum_{n=1}^{\infty} b_n a^n \sin n\theta$$

By Fourier half range series we get

$$b_n a^n = \frac{2}{\pi}\int_0^{\pi} T \sin n\theta \, d\theta$$

$$= \frac{2}{\pi} T \left(-\frac{\cos n\theta}{n}\right)_0^{\pi}$$

$$= \frac{2T}{n\pi}\left[-(-1)^n + 1\right]$$

$$= \begin{cases} 0, & \text{if } n \text{ is even} \\ \dfrac{4T}{n\pi}, & n \text{ is odd.} \end{cases}$$

$$\therefore \; b_n = \frac{4T}{a^n n\pi}$$

$$\therefore \; u(r,\theta) = \frac{4T}{\pi}\left[\frac{r/a}{1}\sin\theta + \frac{(r/a)^3}{3}\sin 3\theta + \frac{(r/a)^5}{5}\sin 5\theta + \cdots\right]$$

6. a) $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$

Let $u = X(x)\,T(t)$ where X is the ~~[illegible]~~ function of x only and T, function of T only.

$$\therefore \; \frac{\partial (X.T)}{\partial x} = 2\frac{\partial}{\partial t}(X.T) + X.T$$

$$T\frac{dX}{dx} = 2X\frac{dT}{dt} + X.T$$

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or $T.X' = 2x\,T' + X.T$

or $\frac{X'}{X} = 2\frac{T'}{T} + 1 = c$ (say)

$\frac{X'}{X} = c$ or $\frac{1}{X}\frac{dX}{dx} = c$

$\frac{dX}{X} = c\,dx$

$\log X = cx + \log a$

$\log \frac{X}{a} = cx$

$\frac{X}{a} = e^{cx}$ or $\boxed{X = ae^{cx}}$

$2\frac{T'}{T} + 1 = c$ or $\frac{T'}{T} = \frac{1}{2}(c-1)$

$\frac{1}{T}\frac{dT}{dt} = \frac{1}{2}(c-1)$

$\frac{dT}{T} = \frac{1}{2}(c-1)\,dt$

$\log T = \frac{1}{2}(c-1)t + \log b$

$\log T/b = \frac{1}{2}(c-1)t$

$T/b = e^{\frac{1}{2}(c-1)t}$

$\boxed{T = b\,e^{\frac{1}{2}(c-1)t}}$

$\therefore u = X.T = ae^{cx}\; be^{\frac{1}{2}(c-1)t}$

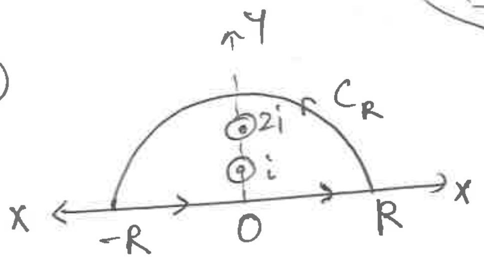
$u = ab\,e^{cx + \frac{1}{2}(c-1)t}$

$u(x,0) = abe^{cx} = 6e^{-3x}$

$\Rightarrow ab = 6,\quad c = -3$

$\therefore u = 6\,e^{-3x + \frac{1}{2}(-4)t} = \underline{\underline{6e^{-3x-2t}}}$.

6. b)



Consider $\int_C \frac{z^2 dz}{(z^2+1)(z^2+4)}$

$= \int_C f(z)\, dz$

where C is the contour consisting of the semi circle C_R of radius R together with the part of the real axis from $-R$ to $+R$.

The integral has simple poles at $z = \pm i$, $z = \pm 2i$ of which $z = i$, $2i$ only lie inside C.

The residue (at $z = i$) $= \lim_{z \to i} (z-i) \frac{z^2}{(z-i)(z+i)(z^2+4)}$

$= \lim_{z \to i} \frac{z^2}{(z+i)(z^2+4)} = -\frac{1}{6i}$

The residue (at $z = 2i$) $= \lim_{z \to 2i} (z-2i) \frac{z^2}{(z^2+1)(z+2i)(z-2i)}$

$= \lim_{z \to 2i} \frac{z^2}{(z^2+1)(z+2i)} = \frac{1}{3i}$

By Cauchy's Residue Theorem

$\int_C f(z)\, dz = 2\pi i \left[\text{Res}\, f(i) + \text{Res}\, f(2i) \right]$

$= 2\pi i \left(-\frac{1}{6i} + \frac{1}{3i} \right) = \frac{\pi}{3}$

ie $\int_{-R}^{R} f(x)\, dx + \int_{C_R} f(z)\, dz = \pi/3$

for any pt on C_R as $|z| \to \infty$, $f(z) \to 0$

$\therefore \lim_{|z| \to \infty} \int_{C_R} f(z)\, dz = 0$

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$\therefore R \to \infty,$

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{\pi}{3}$$

$$\therefore \int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3}. \quad \#$$

c) (i) Consider Newton's Backward difference formula.

$x_0 = 90$, $a = 90$, $h = 10$

$\therefore a - rh = 84$ ie $90 - 10r = 84$

$\therefore r = 0.6$

x	u	Δu	$\Delta^2 u$	$\Delta^3 u$
$x_3 = 60$	$u_{-3} = 226$	24	2	0
$x_2 = 70$	$u_{-2} = 250$	26	2	
$x_1 = 80$	$u_{-1} = 276$	28		
$x_0 = 90$	$u_0 = 304$			

$$y_{-r} = y_{-0.6} = (E^{-1})^{0.6} y_0 = (1-\nabla)^{0.6} u_0$$

$$= \left(1 - 0.6\nabla - \frac{(0.6)(0.4)}{2!}\nabla^2 - \dots\right) u_0$$

$$= u_0 - 0.6\nabla u_0 - \frac{(0.6)(0.4)}{2!}\nabla^2 u_0 - \dots$$

$$= 304 - (0.6)(28) - \frac{(0.6)(0.4)}{2!}\cdot 2 = \boxed{286.96}$$

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c) (ii) $f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$

$$= ax(x-1)(x-2)(x-3) + bx(x-1)(x-2) + cx(x-1) + dx + e$$

$$= a x^{(4)} + b x^{(3)} + c x^{(2)} + d x^{(1)} + e$$

When $x = 0$, $9 = e$

$x = 1$, $10 = d + e$ $\therefore d = 1$

$x = 2$, $37 = 2c + 2d + e$ $\therefore c = 13$

$x = 3$, $54 = 6b + 6c + 3d + e$, $b = -6$

Comparing the coeff of x^4, $a = 1$

$$\therefore f(x) = x^{(4)} - 6x^{(3)} + 13x^{(2)} + x^{(1)} + 9$$