Sub: BDSP sem V

Biomedical Engineering

Question paper code no: 00050565

3 Hrs

80 Marks

Answer key

Q1)

(a) $x(n) = \cos\left[\frac{3\pi}{4} n\right]$ $\quad \Omega = \frac{3\pi}{4} \quad \frac{\Omega}{2\pi} = \frac{3}{8} = \frac{m}{N}$

$N = 8$; periodic signal, it can't be energy signal

$P = \frac{1}{8}\left[1 + 0.5 + 0 + 0.5 + 1 + 0.5 + 0 + 0.5\right]$

$= \underline{0.5}$ unit power signal, energy is infinite

(b) $x(n) = (1/3)^n u(n)$

$$E = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{1 - \left(\frac{1}{9}\right)^{\infty}}{1 - 1/9}$$

$$= \frac{1}{8/9} = \underline{9/8} \text{ unit}$$

Energy is finite hence energy signal. power is zero

(c) $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right) e^{-j\omega}\right]^n$$

$$= \frac{1}{1 - \frac{1}{2} e^{-j\omega}} = \cancel{\frac{1}{\frac{1}{2}[\cos\omega - j\sin\omega]}}$$

$$= \frac{e^{j\omega}}{e^{j\omega} - 1/2} = \frac{\cos\omega + j\sin\omega}{\cos\omega - \frac{1}{2} + j\sin\omega}$$

$|H(\omega)| = \cancel{\ }$ 2 $= \dfrac{1}{1-\cos\omega + \frac{1}{4}}$

$= \dfrac{1}{\left[(\cos\omega) - \frac{1}{2}\right]^2 + \sin^2\omega}$

$|X(\omega)| = \dfrac{1}{\frac{5}{4} - \cos\omega}$

$\phi(\omega) = \cancel{\tan^{-1}(\omega)}\ \omega - \tan^{-1}\dfrac{\sin\omega}{\cos\omega - \frac{1}{2}}$

$X(0) = \dfrac{1}{\frac{5}{4} - 1} = 4 \quad \phi(\omega) = 0$

$\phi(\pi/4) = \pi/4 - \cancel{73}$

$28.62 \quad 45 - 73.62$

$X\left(\frac{\pi}{4}\right) = 1.84$

$X\left(\frac{\pi}{2}\right) = 0.8$

$X\left(\frac{3\pi}{4}\right) = 0.51$

$X(\pi) = 0.44$

$\phi(0) = 0$

$\phi\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - \tan^{-1}\left[\dfrac{\sin \pi/4}{(\cos \pi/4 - \frac{1}{2})}\right] = -0.5$

$\phi\left(\frac{\pi}{2}\right) = 2.668$

$\phi(3\pi/4) = 2.886$

$\phi(\pi) = \pi$

1c $\dfrac{s+0.1}{(s+0.1)^2+9}$ 3 ~~$\frac{\pi}{4} = \frac{2}{T}\tan^{-1}\left(\frac{3}{2}\right)$~~

~~$T = \frac{\pi}{4}\ \frac{4}{\pi}\tan^{-1}\left(\frac{3}{2}\right) = 1.59$~~

~~$T = \frac{8}{\pi}\tan^{-1}(3/2) = 0.59$~~

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right) = \frac{2}{T}\tan\left(\frac{\pi}{4}\right)$$

$$3 = \frac{2}{T} \qquad T = \frac{2}{3} = 0.667.$$

$$H(z) = \frac{\frac{2}{T}\frac{z-1}{z+1} + 0.1}{\left(\frac{2}{T}\left(\frac{z-1}{z+1}\right)+0.1\right)^2 + 9}$$

$$= \frac{2.998\left(\frac{z-1}{z+1}\right) + 0.1}{\left[2.998\frac{(z-1)}{(z+1)} + 0.1\right]^2 + 9}$$

$$= \frac{2.998(z^2-1) + 0.1(z+1)^2}{\left[2.998(z-1) + 0.1(z+1)\right]^2 + 9(z+1)^2}$$

$$= \frac{2.998(z^2-1) + 0.1(z+1)^2}{[2.098z + 2.898]^2 + 9(z+1)^2}$$

$$= \frac{2.998(z^2-1) + 0.1(z^2+2z+1)}{[2.098z+2.898]^2 + 9z^2 + 18z + 9}$$

$$= \frac{3.098z^2 + 0.2z - 2.898}{4.4z^2 + 12.16z + 8.398 + 9(z^2+2z+1)}$$

$$= \frac{3.098z^2 + 0.2z - 2.898}{13.4z^2 + 14.16z + 9.398}$$

Q1 d

4

$$X(k) = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$X_1(k) = \sum_{n=0}^{N-1} x(n-l)_N \, e^{-j\left(\frac{2\pi}{N}\right)kn}.$$

$$= \sum_{n=0}^{l-1} x(n-l)_N e^{-j\left(\frac{2\pi}{N}\right)kn} + \sum_{n=l}^{N-1} x(n-l)_N e^{-j\frac{2\pi}{N}kn}$$

put $n-l+N = p$

$n=0 \quad p = N-l$

$n = l-1; \; p = N-1$

$n-l = p$

$n = l; \; p = 0$

$n = N-1 \quad p = N-1-l$

$$X_1(k) = \sum_{p=N-l}^{N-1} x(p) \, e^{-j\left(\frac{2\pi}{N}\right)k(p+l-N)} + \sum_{p=0}^{N-1-l} x(p) \, e^{-j\left(\frac{2\pi}{N}\right)k(p+l)}$$

$$= e^{-j\left(\frac{2\pi}{N}\right)kl} \cdot \sum_{p=0}^{N-1} x(p) \, e^{-j\left(\frac{2\pi}{N}\right)kp}$$

$$= e^{-j\left(\frac{2\pi}{N}\right)kl} X(k)$$

Q2 a

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j3 \\ -3 \\ 2+j3 \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 8 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ -1+j \\ 12 \\ -1-j \end{bmatrix}$$

$Y(k) = X(k)\,H(k) = +$ 05

$Y(0) = 11 \times 18 = 198$

$Y(1) = (2-j3)(-1+j) = [-2+j3+j2+3] = 1+j5$

$Y(2) = -3 \times 12 = -36$

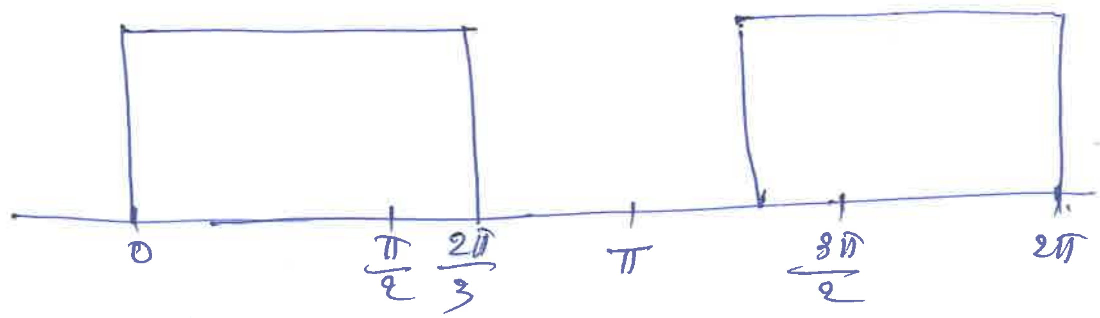
$Y(3) = (2+j3)(-1-j) = -2-j3-j2+3 = 1-j5$

$$y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 198 \\ 1+j5 \\ -36 \\ 1-j5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 198+1+j5-36+1-j5 \\ 198+j-5+36-j+5 \\ 198-1-j5-36-1+j5 \\ 198-j+5+36+j+5 \end{bmatrix} = \begin{bmatrix} 41 \\ 56 \\ 40 \\ 61 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 & 5 \\ 5 & 3 & 2 & 1 \\ 1 & 5 & 3 & 2 \\ 2 & 1 & 5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 1 \\ 8 \\ 2 \end{bmatrix} = \begin{bmatrix} 41 \\ 56 \\ 40 \\ 61 \end{bmatrix}$$ verification

Q2b, $X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1+j4 \\ \cdot \\ \\ \\ \end{bmatrix}$

Q2b



$$H(k) = e^{-j2w} \quad 0 \le |w| \le \frac{2\pi}{3}$$
$$= 0 \quad \frac{2\pi}{3} \le |w| \le \pi$$

The samples of the given frequency response is taken uniformly at $\omega_k = \frac{2\pi}{M}k$, for $0 \le k \le 7$.

so $H(k) = e^{-j\frac{4\pi}{7}k}$ $\quad k = 0, 1, 2.$

$H(k) = e^{-j\frac{4\pi}{7}k}$ $\quad k = 5, 6.$

The filter co-efficients are given by the inverse discrete Fourier transform

$$h_d(n) = \frac{1}{7}\left[\sum_{k=0}^{2} \tilde{H}(k)\, e^{j\frac{2\pi}{7}kn} + \sum_{k=5}^{6} \tilde{H}(k)\, e^{j\frac{2\pi}{7}kn}\right]$$

$$= \frac{1}{7}\left[\sum_{k=0}^{2} e^{-j\frac{4\pi}{7}k} \cdot e^{j\frac{2\pi}{7}kn} + \sum_{k=5}^{6} e^{-j\frac{4\pi}{7}k} \cdot e^{j\frac{2\pi}{7}kn}\right]$$

$$= \frac{1}{7}\left[\sum_{k=0}^{2} e^{j\frac{2\pi}{7}k(n-2)} + \sum_{k=5}^{6} e^{j\frac{2\pi}{7}k(n-2)}\right]$$

$$= \frac{1}{7}\left[1 + e^{j\frac{2\pi}{7}(n-2)} + e^{j\frac{4\pi}{7}(n-2)} + e^{j\frac{2\pi}{7}5(n-2)} + e^{j\frac{2\pi}{7}6(n-2)}\right]$$

$$h_d(n) = \frac{1}{7}\left[1 + e^{j\frac{2\pi}{7}(n-2)} + e^{j\frac{4\pi}{7}(n-2)} + e^{j\frac{10\pi}{7}(n-2)} + e^{j\frac{12\pi}{7}(n-2)}\right]$$

$$= \frac{1}{7}\left[1 + e^{j\frac{2\pi}{7}(n-2)} + e^{j\frac{4\pi}{7}(n-2)} + e^{-j\frac{4\pi}{7}(n-2)} + e^{-j\frac{2\pi}{7}(n-2)}\right]$$

$$= \frac{1}{7}\left[1 + 2\cos\frac{2\pi}{7}(n-2) + 2\cos\frac{4\pi}{7}(n-2)\right]$$

Q3 (a) ~~Linear phase~~ FIR filters (07) IIR filter

1) linear phase
2) always stable
3) design methods are generally linear
4) realised efficiently in Hardware.
5). The start up transmission have finite duration
6) IIR has lower order compared to FIR filters for the same specification.

(b) $y(n) = n\, x^2(n) \rightarrow$ nonlinear

$y_1(n) = n\, x_1^2(n)$; $y_2 = n\, x_2^2(n)$,

$x_3 = x_1(n) + x_2(n)$

$\rightarrow$ non linear

$y_3(n) = n\,(x_1^2(n) + x_2^2(n))$

$= n\, x_1^2(n) + n\, x_2^2(n)$

$y_1(n-n_0) = (n-n_0)\, x_1^2(n-n_0)$

$y(n) = n\, x_1^2(n-n_0)$

$y_2(n) = n\, x_2^2(n-2)$

~~$x_2^2(n-2)$ is the single.~~

replace n by $(n-n_0)$,

$y_2(n) = (n-2)\, x^2(n-2)$ not linear nor

Time invariant

Q3 c

$x(n) = [2, -1, 3, 1, -2, 4, 1, -3, -1, 2, 5, 3]$

$h(n) = [1, 2, -1, 1]$

$x_1(n) = 2, -1, 3, 1$

$x_2(n) = -2, 4, 1, -3$

$x_3(n) = [-1, 2, 5, 3]$

Overlap add method

$y_1(n) = (2, -1, 3, 1) * [1, 2, -1, 1]$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 1 & 3 & -1 \\ -1 & 2 & 0 & 0 & 0 & 1 & 3 \\ 3 & -1 & 2 & 0 & 0 & 0 & 1 \\ 1 & 3 & -1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 3 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 10 \\ -2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 0 & 0 & -3 & 1 & 4 \\ 4 & -2 & 0 & 0 & 0 & -3 & 1 \\ 1 & 4 & -2 & 0 & 0 & 0 & -3 \\ -3 & 1 & 4 & -2 & 0 & 0 & 0 \\ 0 & -3 & 1 & 4 & -2 & 0 & 0 \\ 0 & 0 & -3 & 1 & 4 & -2 & 0 \\ 0 & 0 & 0 & -3 & 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 11 \\ -7 \\ -3 \\ 4 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 3 & 5 & 2 \\ 2 & -1 & 0 & 0 & 0 & 3 & 5 \\ 5 & 2 & -1 & 0 & 0 & 0 & 3 \\ 3 & 5 & 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & 5 & 2 & -1 & 0 & 0 \\ 0 & 0 & 3 & 5 & 2 & -1 & 0 \\ 0 & 0 & 0 & 3 & 5 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 10 \\ 10 \\ 3 \\ 2 \\ 3 \end{bmatrix}$$

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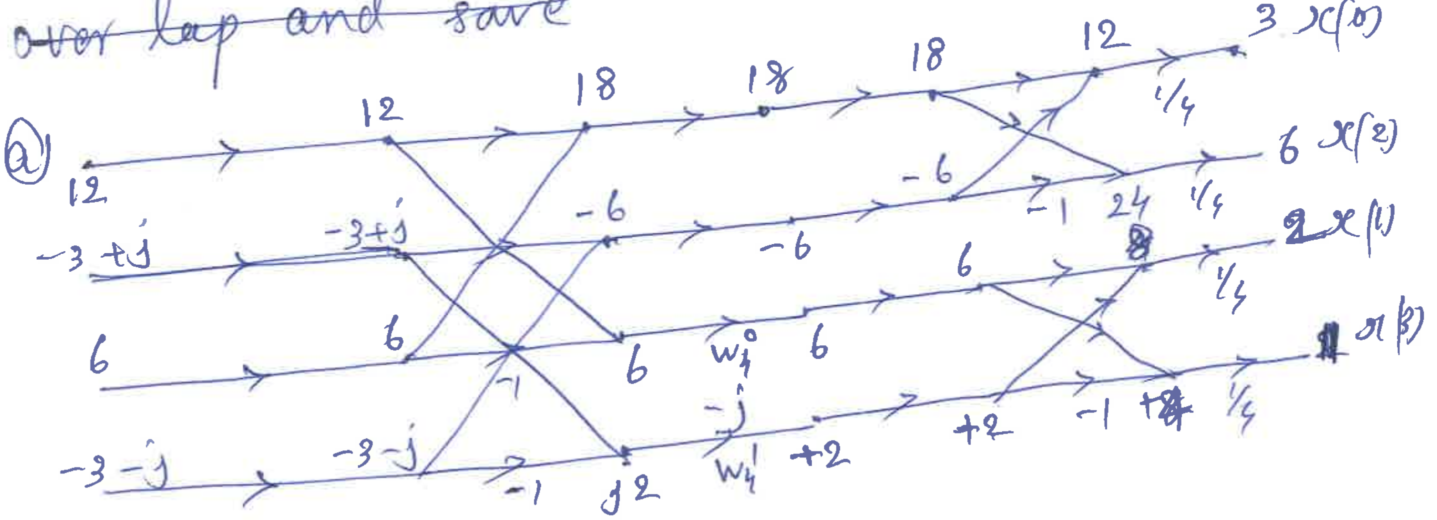
2 3 -1 10 -2 2 1
-2 0 11 -7 -3 4 -3
-1 0 10 10 3 2 3

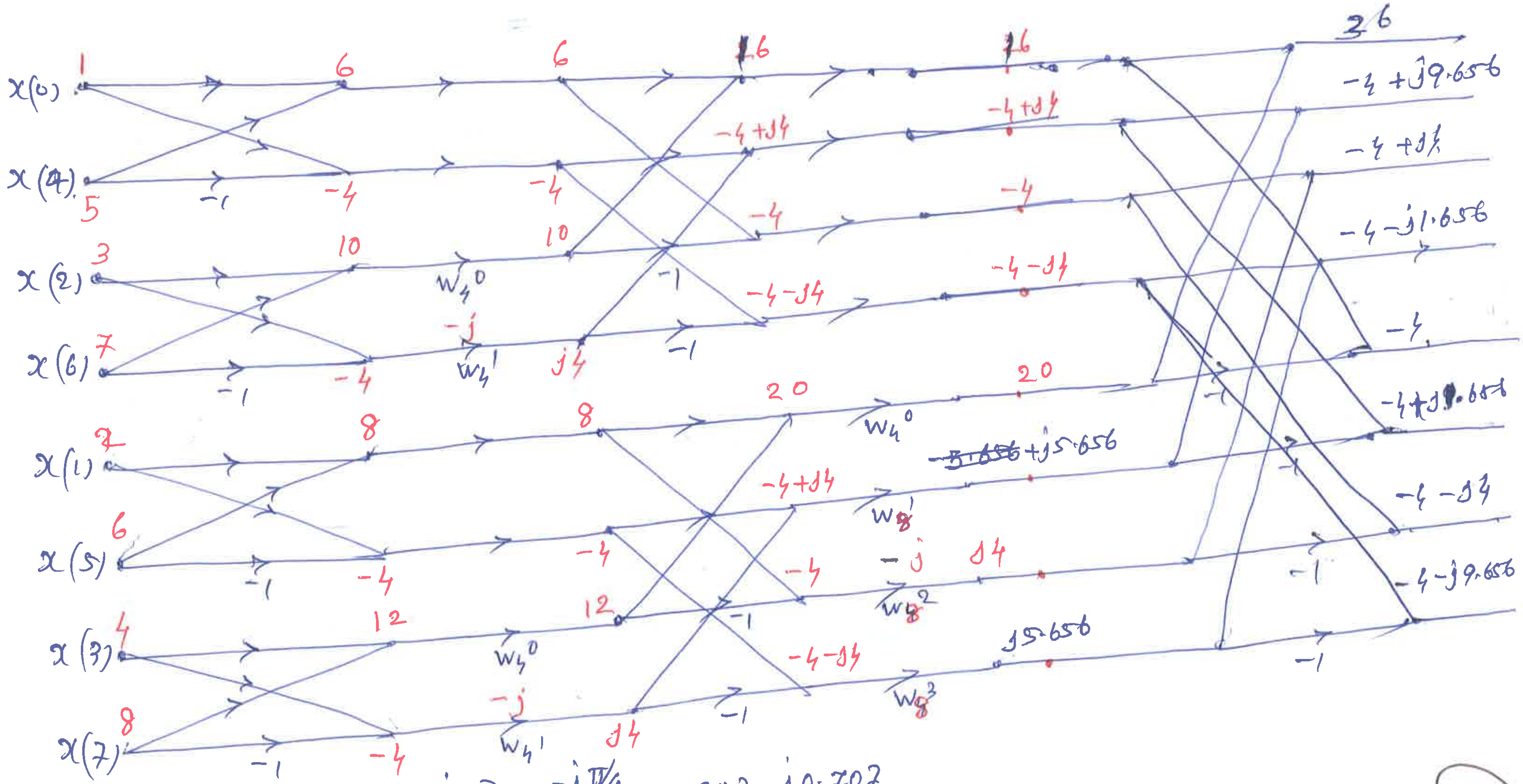
$y(n)$ [2 3 -1 10 -4 2 12 -7 -4 4 7 10 3 -2 3]

~~over lap and save~~

Q4 (a)



$x(n) = [3, 2, 6, 1]$



$e^{-j\frac{2\pi}{8}} = e^{-j\pi/4} = 0.707 - j0.707$

$W_8^2 = e^{-j\pi/2} = -j$

$W_8^3 = e^{-j3\pi/4} = -0.707 - j0.707$

$(-4+j4)[0.707-j0.707] = -2.828 + j2.828 + j2.828 + 2.828 = \cancel{5.656} + j5.656$

$(-4-j4)(-0.707-j0.707) = 2.828 + j2.828 + j2.828 - 2.828 = +j5.656$

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$$H(s) = \frac{(2\cdot 2)}{\left[2\left(\frac{z-1}{z+1}\right) + 2\cdot 2\right]\left[\left[2\left(\frac{z-1}{z+1}\right)\right]^2 + 2\cdot 2\alpha\, 2\left(\frac{z-1}{z+1}\right) + 2\cdot 2^2\right]}$$

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Q4 c, $H(\omega) = e^{-j2\omega} \quad -\frac{\pi}{3} \le \omega \le \pi/3$ 11

$$h[n] = \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{-j2\omega} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{j\omega(n-2)}\, d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega(n-2)}}{j(n-2)}\right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{\pi 2j(n-2)}\left[e^{j\pi/3(n-2)} - e^{-j\pi/3(n-2)}\right]$$

$$h(n) = \frac{\sin \pi/3(n-2)}{3\cdot\frac{\pi}{3}(n-2)} = \frac{1}{3}\,\frac{\sin\frac{\pi}{3}(n-2)}{\pi/3(n-2)}$$

$$h(0) = \frac{\sin(-2\pi/3)}{\pi(-2)} = 0.1378$$

$h(n) = h_d(n) \times w_H(n)$

$$h(1) = \frac{\sin \pi/3(-1)}{\pi(-1)} = 0.27566$$

$$h(2) = 1/3$$

$$h(3) = \frac{\sin(\pi/3)}{\pi} = 0.27566$$

$$h(4) = \frac{\sin(2\pi/3)}{\pi 2} = 0.1378$$

$w_H(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$

$w_H(0) = 0.08$, $w_H(1) = 0.54$, $w_H(2) = 1$

$w_H(3)$ 0.54, 0.08

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Q5 b. $\delta_1 = \cancel{0.707}\ 0.9$ $\quad \omega_1 = \pi/2, \quad T = 1\,sec.$

$\delta_2 = 0.2 \quad \omega_2 = 3\pi/4$

$\Omega_1 = \frac{\Omega}{T} = \pi/2 \quad \Omega_2 = 3\pi/4.$

$$N \geq \frac{1}{2}\frac{\log\left[\dfrac{\left(\frac{1}{0.2^2}-1\right)}{\left(\frac{1}{0.707^2}-1\right)}\right]}{\log\left(\dfrac{3\pi/4}{\pi/2}\right)} = \frac{1}{2}\frac{\log \overset{1.63}{\cancel{2.009}}}{0.1760}$$

$$\geq \frac{1}{2}\frac{\log(24)}{\log(3/2)} = 3.91 = 4$$

~~$\Omega_c =$~~

Q5 (b) $\delta_1 = 0.9 \quad \omega_1 = \pi/2$

$\delta_2 = 0.2 \quad \omega_2 = 3\pi/2$

$\Omega_1 = \frac{\Omega}{T} = \pi/2, \quad \Omega_2 = 3\pi/2$

$$N \geq \frac{1}{2}\frac{\log\left\{\dfrac{\left(\frac{1}{0.2^2}-1\right)}{\left(\frac{1}{0.9^2}-1\right)}\right\}}{\log\left(\dfrac{3\pi/2}{\pi/2}\right)} \geq \frac{1}{2}\frac{\log\left[\dfrac{24}{0.23457}\right]}{\log(3)}$$

$$\geq \frac{1}{2}\frac{\cancel{\log(2)}\ 2.009}{\log(3)}. = 2.106 = \underline{\underline{3}}$$

Q5 $\delta_p = 0.9,\ \delta_s = 0.1$ (5)

b) $\Omega_p = \frac{2}{T}\tan\left(\frac{0.2\pi}{2}\right) = 2\tan(0.1\pi) = 0.6498$

$\Omega_s = 2\tan(0.25\pi) = 2$

$$N \geq \frac{\cancel{\log}\ \cosh^{-1}\left[\frac{\left(\frac{1}{\delta_s^2}-1\right)}{\left(\frac{1}{\delta_p^2}-1\right)}\right]^{1/2}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\cosh^{-1}\left[\frac{\left(\frac{1}{0.1^2}-1\right)}{\left(\frac{1}{0.9^2}-1\right)}\right]^{1/2}}{\cosh^{-1}\left(\frac{2}{0.6498}\right)} = \frac{\cosh^{-1}(20.54)}{\cosh^{-1}(3.0778)} = 2.07 = \underline{\underline{3}}$$

Q5e

$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y[n-2] = x(n) + \frac{1}{3}x[n-1]$

$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right) = X(z)\left[1 + \frac{1}{3}z^{-1}\right]$

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right)} = \frac{A}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$\frac{1 + \frac{1}{3}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - \frac{1}{4}z^{-1}}$$

$1 - \frac{1}{2}z^{-1} = 0$

$1 - \frac{1}{2z} = 0$

$\frac{1}{2z} = 1$

$z = 1/2$

$$A = \left.\frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{4}z^{-1}}\right|_{z^{-1} = 2} = \frac{1 - \frac{1}{3}2}{1 - \frac{1}{4}2} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$B = \left.\frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}\right|_{z^{-1} = 4} = \frac{1 - \frac{1}{3}4}{1 - \frac{4}{2}} = \frac{-1/3}{-1} = \frac{1}{3}$$

$$= \frac{2/3}{1 - 1/2\, z^{-1}} + \frac{1/3}{1 - \frac{1}{4}z^{-1}}$$

$$H(z) = \frac{2/3}{1-\frac{1}{2}z^{-1}} + \frac{1/3}{1-\frac{1}{4}z^{-1}}$$

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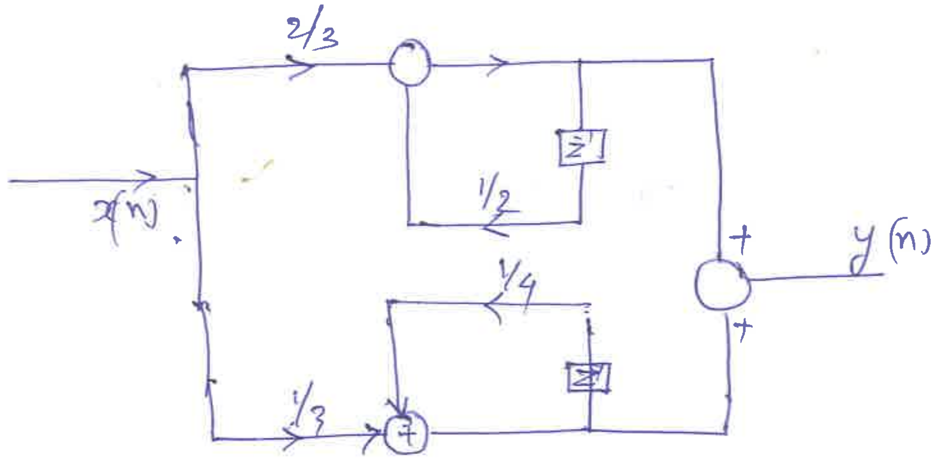


(17) $1.4626\left[\dfrac{s+0.5978}{(s+0.5978)^2+1.46^2} - 0.5702\cdot\dfrac{1.46}{(s+0.5978)^2+1.46^2}\right]$

$$H_1(z) = -1.4626\cdot\frac{1 - e^{-0.5978}z^{-1}\cos(4.517)z^{-1}}{1 - e^{-0.5978}z^{-1}\cos(4.517)z^{-1} + 1.1986\,z^{-2}}$$

$$-0.5702\;\frac{e^{-0.5978}z^{-1}\cos(4.517)}{1 - e^{-0.5978}z^{-1}\;\cancel{2\cos}\;4.517\,z^{-2}}$$

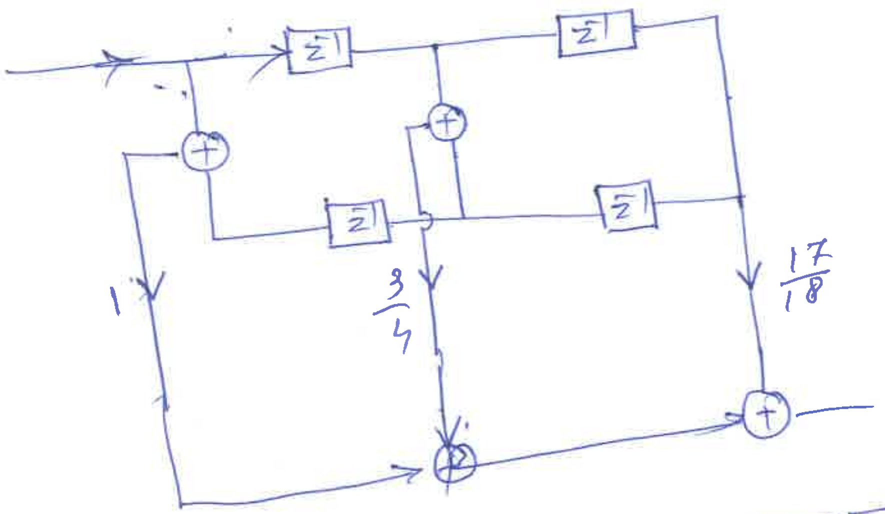
simplyfying

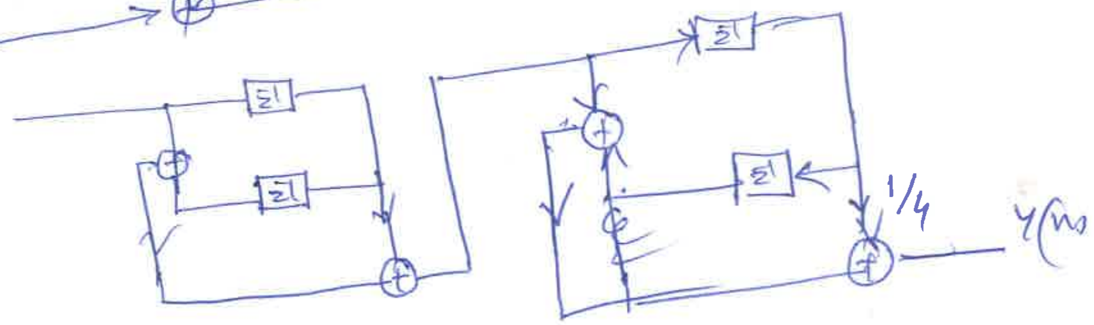
$$H(z) = \frac{-1.4509 - 0.232\,z^{-1}}{1 - 0.131\,z^{-1} + 0.3006\,z^{-2}} + \frac{1.4509 + 0.1848\,z^{-1}}{1 - 0.3862\,z^{-1} + 0.0556}$$

Q6 (a) $H(z) = \left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4}z^{-1} + z^{-2}\right)$

$= 1 + \frac{3}{4}z^{-1} + z^{-2} + \frac{1}{8}z^{-2} + z^{-2} + \frac{3}{4}z^{-3} + z^{-4}$

$= 1 + \frac{3}{4}z^{-1} + \frac{17}{18}z^{-2} + \frac{3}{4}z^{-3} + z^{-4}$





$$= \frac{2.2(z+1)^3}{[2(z-1)+2.2(z+1)][4(z-1)^2 + 4.4(z^2-1) + 4.84(z+1)]}$$

$$= \frac{2.2(z^3+3z^2+3z+1)}{(4.2z+0.2)(4(z^2-2z+1) + 4.4z^2 - 4.4 + 4.84z^2 + 9.68z + 4.84]}$$

$$= \frac{2.2z^3 + 6.6z^2 + 6.6z + 2.2}{(4.2z+0.2)(4z^2 - 8z + 4 + 4.4z^2 - 4.4 + 4.84z^2 + 9.682 + 4.84]}$$

$$= \frac{2.2z^3 + 6.6z^2 + 6.6z + 2.2}{(4.2z+0.2)(13.24z^2 + 1.682z + 4.44)}$$

$$\frac{z^3 + 6.6z^2 + 6.6z + 2.2}{\ldots 2 \ldots 2364z + 18.648}$$

Q4 c, $H(w) = e^{-j2w} \quad -\frac{\pi}{3} \le w \le \pi/3$ (1)

$$h[n] = \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{-j2w} \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{jw(n-2)}\, dw = \frac{1}{2\pi}\left.\frac{e^{jw(n-2)}}{j(n-2)}\right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{\pi 2j(n-2)}\left[e^{j\pi/3(n-2)} - e^{-j\pi/3(n-2)}\right]$$

$$h(n) = \frac{\sin \pi/3(n-2)}{3 \cdot \frac{\pi}{3}(n-2)} = \frac{1}{3}\,\frac{\sin \frac{\pi}{3}(n-2)}{\pi/3(n-2)}$$

$$h(0) = \frac{\sin(-2\pi/3)}{\pi(-2)} = 0.1378$$

$$h(n) = h_d(n) \times w_H(n)$$

$$h(1) = \frac{\sin \pi/3(-1)}{\pi(-1)} = 0.27566$$

$$h(2) = 1/3$$

$$h(3) = \frac{\sin(\pi/3)}{\pi} = 0.27566$$

$$h(4) = \frac{\sin(2\pi/3)}{\pi 2} = 0.1378$$

$w_H(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$

$w_H(0) = 0.08$, $w_H(1) = 0\ \cancel{.3978}\ 0.54$, $w_H(2) = \cancel{0.0121}\ 1$

$w_H(3)\ 0.54, 0.08$

Q5 b. $\delta_1 = \cancel{0.707}$ 0.9 $\omega_1 = \pi/2$, $T = 1 sec$.

$\delta_2 = 0.2$ $\omega_2 = 3\pi/4$

$\Omega_1 = \frac{\Omega}{T} = \pi/2$ $\Omega_2 = 3\pi/4$.

$$N \geq \frac{\frac{1}{2}\log\left[\frac{\left(\frac{1}{0.2^2}-1\right)}{\left(\frac{1}{0.707^2}-1\right)}\right]}{\log\left(\frac{3\pi/4}{\pi/2}\right)} = \cancel{\frac{1}{2}\frac{\log 2.009}{0.1760}} \quad 1.63$$

$$\geq \frac{1}{2}\frac{\log(24)}{\log(3/2)} = 3.91 = 4$$

~~$\Omega_c =$~~

Q5 (b) $\delta_1 = 0.9$ $\omega_1 = \pi/2$

$\delta_2 = 0.2$ $\omega_2 = 3\pi/2$

$\Omega_1 = \frac{\Omega}{T} = \pi/2$, $\Omega_2 = 3\pi/2$

$$N \geq \frac{\frac{1}{2}\log\left[\frac{\left(\frac{1}{0.2^2}-1\right)}{\left(\frac{1}{0.9^2}-1\right)}\right]}{\log\left(\frac{3\pi/2}{\pi/2}\right)} \geq \frac{\frac{1}{2}\log\left[\frac{24}{0.23457}\right]}{\log(3)}$$

$$\geq \frac{1}{2}\frac{\cancel{\log(2)}\ 2.009}{\log(3)} = 2.106 = \underline{\underline{3}}$$