Q1 (a) $x(t) = 7\cos(20\pi t)$

QP-code: 27523
Sem V CBSGS
BDSP-(Biomedical Eng
Exam Date - 8/12/14

$$f_s = \frac{1}{12.5\times10^{-3}}$$

$$x(nT_s) = 7\cos(20\pi\, n \times 12.5\times10^{-3})$$

$$x(n) = 7\cos\left[\frac{1}{4}\pi n\right]$$

$$\Omega = \frac{\pi}{4} \qquad \frac{\Omega}{2\pi} = \frac{1}{8} = \frac{m}{N} \; ; \; N = 8 \text{ periodic}$$

(b) $X(k) = [2,\ 1-j,\ 0,\ 1+j]$

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$2 + j[1-j] - j[1+j] = 2 + j + 1 - j + 1$

$2 - 1(1-j) - 1(1+j) = 2 - 1 + j - 1 - j = 0$

$2 - j(1-j) + 0 + j(1+j) = 2 - j - 1 + 0 + j - 1 = 0$

(c) Linear phase, stability, order of the filter, complexity of the filter,

(d) $x(n) \longleftrightarrow X(k)$

$$x(n-l) = X(k)\cdot e^{-j\left(\frac{2\pi}{N}\right)k.l}.$$

Let $x_1(n) = x(n-l)$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j\left(\frac{2\pi}{N}\right)kl} = \sum_{n=0}^{N-1} x(n-l)\, e^{-j\left(\frac{2\pi}{N}\right)kl}.$$

$$= \sum_{n=0}^{l-1} x(n-l)\, e^{-j\left(\frac{2\pi}{N}\right)kn} + \sum_{n=l}^{N-1} x(n-l)\, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$= \sum_{n=0}^{l-1} x(N+n-l)\, e^{-j\left(\frac{2\pi}{N}\right)kn} + \sum_{n=l}^{N-1} x(n-l)\, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

put $N+n-l = m$.

when $n=0$ $m = N-l$

$n = l-1$ $m = N-1$

put $m = n-l$

$\cancel{m} = l$ $m = 0$

$n = N-1$ $\cancel{m = m+l}$

$m = N-1-l$.

$$X_1(k) = \sum_{m=N-l}^{N-1} x(m)\, e^{-j\left(\frac{2\pi}{N}\right)k.(m+l-N)} + \sum_{m=0}^{N-1-l} x(m)\, e^{-j\left(\frac{2\pi}{N}\right)k(m+l)}$$

$$= \sum_{m=0}^{N-1-l} x(m)\, e^{-j\left(\frac{2\pi}{N}\right)k(m+l)} + \sum_{m=N-l}^{N-1} x(m)\, e^{-j\left(\frac{2\pi}{N}\right)k(m+l)}$$

$$= e^{-j\left(\frac{2\pi}{N}\right)kl} \cdot \sum_{m=0}^{N-1} x(m)\, e^{-j\left(\frac{2\pi}{N}\right)km}.$$

$$\underline{X_1(k) = e^{-j\left(\frac{2\pi}{N}\right)kl} \cdot X(k).}$$

(e) $H(s) = \dfrac{s+0.2}{(s+0.2)^2+9}$ $T_s = 1$. $\dfrac{s+a}{(s+a)^2+b^2} \rightarrow \dfrac{1-e^{-aT}\cos(bT)\, z^{-1}}{1-2e^{-aT}\cos bT\, z^{-1}+e^{-2aT} z^{-2}}$

$$H(z) = \frac{1-e^{-0.2T}\cos(3T)\, z^{-1}}{1-2e^{-0.2T}\cos(bT)\, z^{-1}+e^{-2\times 0.2T} z^{-2}}$$

$$= \frac{1-e^{-0.2}\cos(3)\, z^{-1}}{1-2e^{-0.2}\cos(3)\, z^{-1}+e^{-0.4} z^{-2}}$$

$$= \frac{1-0.8187(-0.9899)\, z^{-1}}{1+1.621\, z^{-1}+0.6703\, z^{-2}}$$

$$= \frac{1+0.8105\, z^{-1}}{1+1.621\, z^{-1}+0.6703\, z^{-2}}$$

Q2 (a) $y(n) = e^{x(n)}$

Let $x_1(n)$ input and $y_1(n)$ is the output, then

$y_1(n) = e^{x_1(n)}$.

if $x_2(n) = c\,x_1(n)$, then

$y_2(n) = e^{x_2(n)} = e^{c x_1(n)} \neq c\,y_1(n)$ hence not homogenous

not linear

Let $x_1(n) \longrightarrow y_1(n)$; $x_2(n) \longrightarrow y_2(n)$

Then $y_1(n) = e^{x_1(n)}$ $\quad y_2(n) = e^{x_2(n)}$,

$x_3(n) \longrightarrow y_3(n)$ where $x_3(n) = x_1(n) + x_2(n)$

$y_3(n) = e^{x_3(n)} = e^{x_1(n)+x_2(n)} = e^{x_1(n)} \cdot e^{x_2(n)} \neq y_1(n) + y_2(n)$

hence not additive hence not linear

Time invariance

$x_2(n) = x_1(n-n_0)$

$y_2(n) = e^{x_1(n-n_0)}$

replace n by $(n-n_0)$ then

$y_1(n-n_0) = e^{x_1(n-n_0)}$

$y_2(n) = y_1(n-n_0)$ hence Time invariance.

$L+M-1 = 3+2-1 = 4$

(b)
$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ 4 \\ 7 \\ 6 \end{bmatrix}}}$$

(c) $H(z) = \dfrac{2 - \frac{5}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 2 z^{-1})} = \dfrac{z(2z - \frac{5}{2})}{(z - \frac{1}{2})(z-2)}$, $\dfrac{H(z)}{z} = \dfrac{2z - \frac{5}{2}}{(z-\frac{1}{2})(z-2)}$

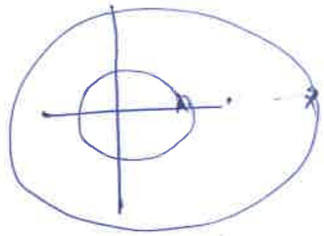
$$\frac{H(z)}{z} = \frac{A}{z - \frac{1}{2}} + \frac{B}{(z-2)}$$

$$A = \left.\frac{2z - 5/2}{(z-2)}\right|_{z=1/2} = \frac{2\cdot\frac{1}{2} - 5/2}{1/2 - 2} = \frac{-3/2}{-3/2} = 1$$

$$B = \left.\frac{2z - 5/2}{z - 1/2}\right|_{z=2} = \frac{4 - 5/2}{2 - 1/2} = \frac{3/2}{3/2} = 1$$

$$\frac{H(z)}{z} = \frac{1}{z - 1/2} + \frac{1}{z-2}$$

$$H(z) = \frac{z}{z - 1/2} + \frac{z}{(z-2)}$$



when it is causal $|z| > 2$

$$h(n) = \left(\frac{1}{2}\right)^n u(n) + 2^n u(n)$$

when it is stable $\frac{1}{2} < |z| < 2$

$$h(n) = \left(\frac{1}{2}\right)^n u(n) - 2^n u(-n-1)$$

(d) $$x(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 16 \\ -4 + j4 \\ -4 \\ -4 - j4 \end{bmatrix}$$

$[1, 7, 5, 3] = x(-n) \longleftrightarrow X(-k))_N = X(N-k))_4$

$X_1(k) = X(N-k))_4 \quad X_1(0) = X(0) = 16$

$X_1(1) = X(3) = -4 - j4$

$X_1(2) = X(2) = -4 \quad X_1(3) = X(1) = \underline{-4 + j4}$

Q3 9. $X(k) = \sum_{n=0}^{N-1} x(n)\, W_8^{kn}$.

$$= \sum_{n=0}^{3} x(n)\, W_8^{kn} + \sum_{n=4}^{7} x(n)\, W_8^{kn}.$$

$$= \sum_{n=0}^{3} x(n)\, W_8^{kn} + \sum_{n=0}^{3} x(n+4)\, W_8^{k(n+4)}$$

$$= \sum_{n=0}^{3} x(n)\, W_8^{kn} + \sum_{n=0}^{3} x(n+4)\, W_8^{kn} \cdot W_8^{4k}.$$

$$X(k) = \sum_{n=0}^{3} x(n)\, W_8^{kn} + W_8^{4k} \sum_{n=0}^{3} x(n+4)\, W_8^{kn}$$

$$X(2r) = \sum_{n=0}^{3} x(n)\, W_8^{2rn} + W_8^{8r} \sum_{n=0}^{3} x(n+4)\, W_8^{2rn}.$$

$$= \sum_{n=0}^{3} x(n)\, W_4^{rn} + \sum_{n=0}^{3} x(n+4)\, W_4^{rn}.$$

$$X(2r) = \sum_{n=0}^{3} \left[x(n) + x(n+4)\right] W_4^{rn}.$$

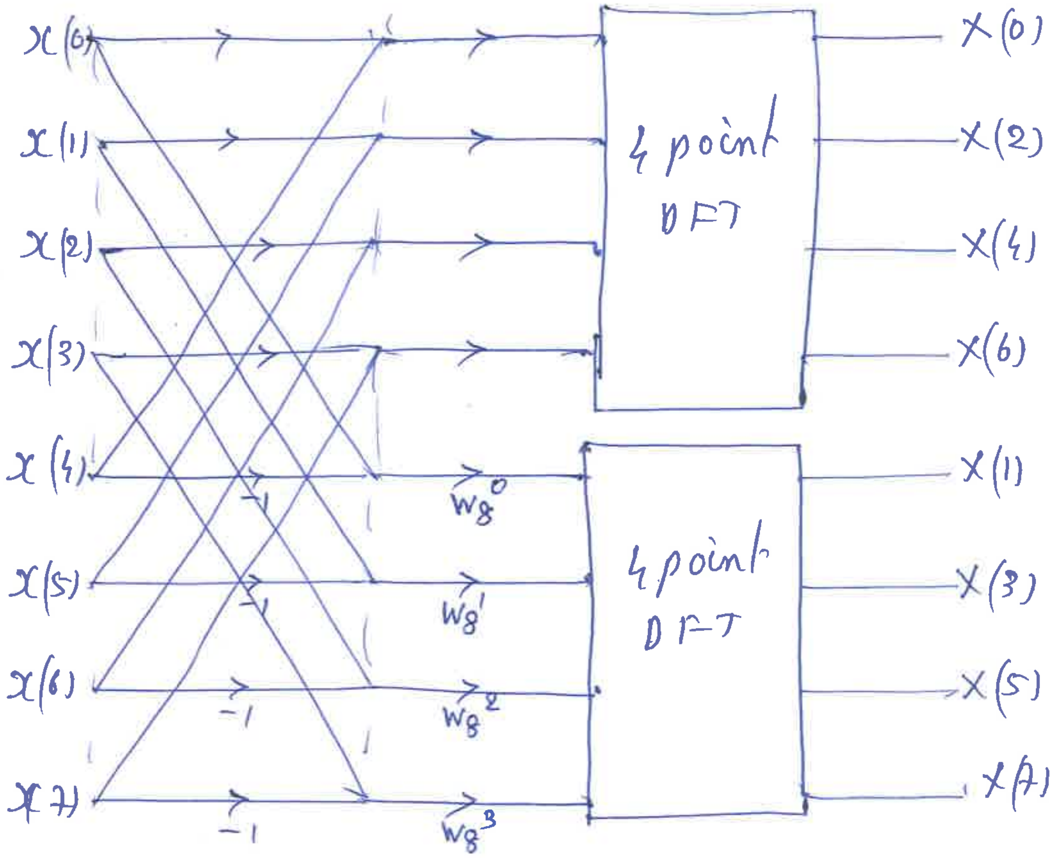
put $k = 2r+1$

$$X(2r+1) = \sum_{n=0}^{3} x(n)\, W_8^{(2r+1)n} + W_8^{4(2r+1)} \sum_{n=0}^{3} x(n+4)\, W_8^{(2r+1)n}$$

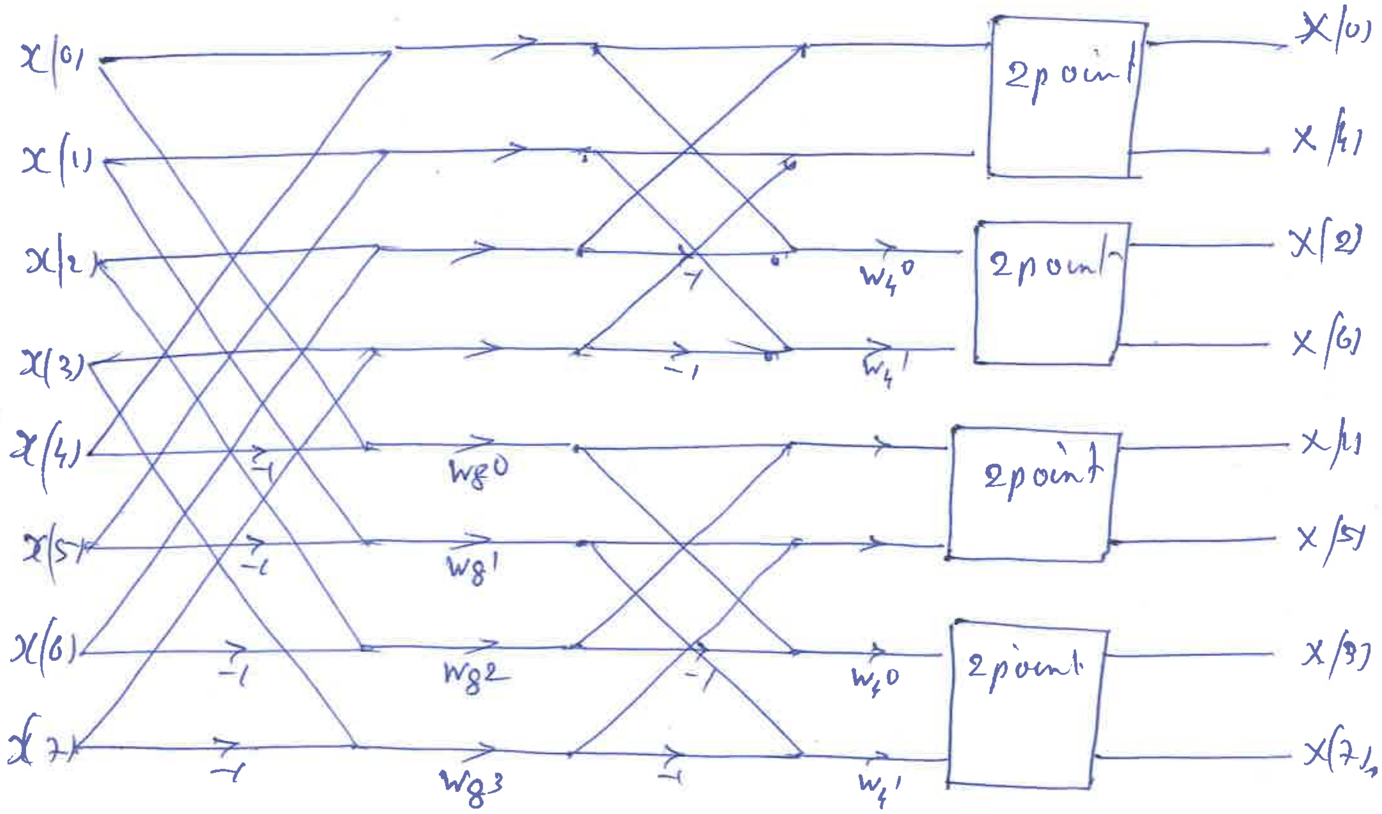
$$= \sum_{n=0}^{3} x(n)\, W_4^{rn} \cdot W_8^{n} + W_8^{4} \sum_{n=0}^{3} x(n+4)\, W_4^{rn}\, W_8^{n}.$$

$$= \sum_{n=0}^{3} x(n)\, W_8^{n}\, W_4^{rn} - \sum_{n=0}^{3} x(n+4)\, W_8^{n}\, W_4^{rn}.$$

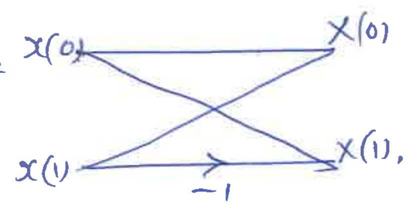
$$= \sum_{n=0}^{3} \left[\left[x(n) - x(n+4)\right] W_8^{n}\right] W_4^{rn}.$$

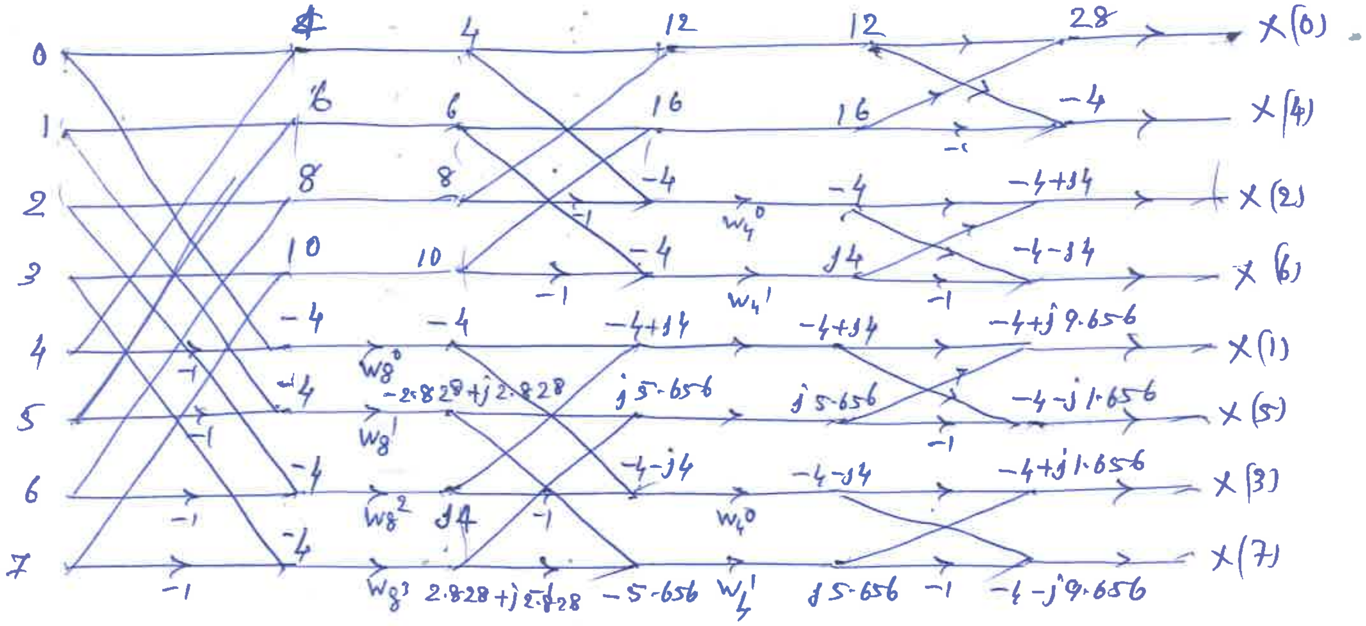


continuing to 4 point DFT



. 2 point DFT =





$$W_8^1 = 0.707 - j0.707$$
$$W_8^2 = -j$$
$$W_8^3 = -0.707 - j0.707$$

3b. $x(n) = [1, -1, 0, 1, -1, -3, 1, -1, 3, 0, 1, \ldots]$

$h(n) = [1, 2, -1, 1]$

$L = 4, \quad M = 4 \quad N = L + M - 1 = 7.$

Number of overlapping 3.

$x_1(n) = [1, -1, 0, 1] = [0, 0, 0, 1, -1, 0, 1]$

$x_2(n) = [-1, -3, 1, -1] = -1, 0, 1, -1, -3, 1, -1$

$x_3(n) = [3, 0, 1, 0] = [3, 1, -1, 3, 0, 1, 0]$

$x_4(n) = [0, 0, 0, 0] = [0, 1, 0, 0, 0, 0, 0]$

$$y_1(n) = \begin{bmatrix} 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} -1 & -1 & 1 & -3 & -1 & +1 & 0 \\ 0 & -1 & -1 & 1 & -3 & -1 & 1 \\ 1 & 0 & -1 & -1 & 1 & -3 & -1 \\ -1 & 1 & 0 & -1 & -1 & 1 & -3 \\ -3 & -1 & 1 & 0 & -1 & -1 & 1 \\ 1 & -3 & -1 & 1 & 0 & -1 & -1 \\ -1 & 1 & -3 & -1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -7 \\ 0 \\ 1 \\ 0 \\ -6 \\ -3 \\ 3 \end{bmatrix}$$

$$y_3(n) \begin{bmatrix} -3 & 0 & 1 & 0 & 3 & -1 & -1 \\ 1 & -3 & 0 & 1 & 0 & 3 & -1 \\ -1 & 1 & -3 & 0 & 1 & 6 & 3 \\ 3 & -1 & 1 & -3 & 0 & 1 & 0 \\ 0 & 3 & -1 & 1 & -3 & -3 & 1 \\ 1 & 0 & 3 & -1 & 1 & 1 & -3 \\ 0 & 1 & 0 & 3 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 4 \\ -3 \\ 8 \\ -3 \\ 5 \end{bmatrix}$$

$$y_4(n) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$y(n) = [1, 1, -3, 3, 0, -6, -3, 3, -3, 8, -3, 5, -1, 1, 0, 0]$$

4.9, $\delta_1 = 0.9,\quad \delta_2 = 0.2,\quad \omega_p = \pi/2,\quad \omega_s = 3\pi/4$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 2\tan(\pi/4) = 2$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 2\tan(3\pi/8) = 4.828$$

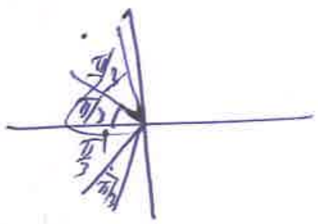
$$\left(\frac{\Omega_s}{\Omega_p}\right) = \frac{4.828}{2} = 2.414$$

$$N \geq \frac{1}{2}\frac{\log\left[\dfrac{\frac{1}{\delta_s^2}-1}{\left(\frac{1}{\delta_p^2}-1\right)}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\frac{1}{2}\log\left(\frac{24}{0.2345}\right)}{\log(2.414)} = 2.626 = 3$$

order of the filter is 3.

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{\delta_1^2}-1\right)^{1/2N}} = \frac{2}{\left(\frac{1}{0.9^2}-1\right)^{1/6}} = \underline{\underline{2.5467}}$$

The



$$H_a(s) = \frac{1}{(s+1)(s^2+2s\cos\pi/3+1)}$$

$$= \frac{1}{(s+1)(s^2+s+1)}$$

denormalising $\quad s \to \left(\frac{s}{\Omega_c}\right)$

$$H(s) = \frac{2.5467^3}{(s+2.5467)(s^2+2.5467s+2.5467^2)}$$

$$= \frac{16.517}{(s+2.5467)(s^2+2.5467s+6.4857)}$$

$$H(z) \quad s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right) = \frac{16.517(z+1)^3}{(z-1)+2.5467(z+1)+\left[2\left(\frac{z-1}{z+1}\right)\right]^2 + 2.5467\,2\left(\frac{z-1}{z+1}\right)^2 + 6.4857(z+1)^2}$$

$$H(z) = \frac{16.517}{\left[2\left(\frac{z-1}{z+1}\right) + 2.5467\right]\left[\left[2\left(\frac{z-1}{z+1}\right)\right]^2 + 2.5467 \cdot 2\left(\frac{z-1}{z+1}\right) + 6.4857\right]}$$

$$= \frac{16.517(z+1)^3}{\left[2(z-1) + 2.5467(z+1)\left[4(z-1)^2 + 2.5467 \cdot 2(z^2-1) + 6.4857(z+1)^2\right]\right.}$$

$$= \frac{16.517(z+1)^2}{\left[2z - 2 + 2.5467z + 2.5467\right] \times}$$

$$2\left[z^2 - 2z + 1\right] + \cancel{2.5467z^2 - 4z + 2}] + 5.0934z^2 - 5.0934$$

$$+ 6.4857z^2 + 12.9714z + 6.4857]$$

$$= \frac{(16.517)(z+1)^3}{(4.5467z + 0.5467)(13.5791z^2 + 8.9714z + 3.3923)}$$

$$= \frac{16.517(z+1)^3}{\left[2z - 2 + 2.5467(z+1)\right]\left[4(z-1)^2 + 5.0934(z^2-1) + 6.4857(z^2+2z+1)\right]}$$

$$= \frac{16.517(z+1)^3}{(4.5467z + 0.5467)(15.579z^2 + 4.9714z + 5.3923)}$$

$$H(z) = \frac{16.517(z+1)^3}{(70.83z^3 + 31.12z^2 + 27.2351z + 2.948)}$$

or expren in terms of (z^{-1}).

$$= \frac{0.2332(1+z^{-1})^3}{1 + 0.4394z^{-1} + 0.3845z^{-2} + 0.0416z^{-3}}$$

4b. $\delta_p = 0.707$, $\delta_s = 0.1$, $w_p = 0.2\pi$ $w_s = 0.5\pi$

$$\Omega_p = \frac{2}{T}\tan(0.1\pi) = 0.6498$$

$$\Omega_s = \frac{2}{T}\tan(0.25\pi) = 2.$$

For chebyshev filter:

$$|H(\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2(\Omega)}. \quad \text{at } \Omega = \Omega_p$$

$$\delta_p^2 = \frac{1}{1+\varepsilon^2} \qquad \varepsilon^2 = \frac{1}{\delta_p^2} - 1$$

$$\varepsilon = \frac{1}{0.707^2} - 1 = 1$$

$$N \geq \frac{\cosh^{-1}\left[\frac{\left(\frac{1}{\delta_s^2}-1\right)}{\left(\frac{1}{\delta_p^2}-1\right)}\right]^{1/2}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}.$$

$$N \geq \frac{\cosh^{-1}\left[\left[\frac{\frac{1}{0.1^2}-1}{\left(\frac{1}{0.5}-1\right)}\right]^{1/2}\right]}{\cosh^{-1}\left(\frac{2}{0.6498}\right)} \geq \frac{\cosh^{-1}[9.95]}{\cosh^{-1}(3.078)}$$

$$\geq 1.669 = \underline{\underline{2}}$$

4c

$$H(z) = \frac{1}{4} + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$$

$$H(w) = \frac{1}{4} + \frac{1}{2}e^{-jw} + \frac{1}{4}e^{-j2w}$$

$$= \frac{1}{4}\left[1 + e^{-j2w}\right] + \frac{1}{2}e^{-jw}$$

$$e^{-jw}\left[\frac{1}{4}\left[2\cos w\right] + \frac{1}{2}\right]$$

$$= \frac{1}{2}e^{-jw}\left[1 + \cos w\right]$$

$$H(w) = \frac{1}{2}e^{-jw}\left[1 + \cos w\right]$$

$$|H(w)| = \left(\frac{1+\cos w}{2}\right)$$

$$\angle H(w) = \varphi(w) = -w$$

$$\tau_p = -\frac{\varphi(w)}{w} = 1, \quad \tau_g = -\frac{d[\varphi(w)]}{dw} = \underline{1}$$

Q5 (a)

$$H_d(w) = 0 \quad -\pi/4 \le w \le \pi/4$$
$$= e^{-j2w} \quad \pi/4 \le |w| \le \pi$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw}) \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{-j2w} \cdot e^{jwn}\, dw + \int_{\pi/4}^{\pi} e^{-j2w} \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j(n-2)w}\, dw + \int_{\pi/4}^{\pi} e^{j(n-2)w}\, dw\right.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(n-2)w}}{j(n-2)}\right]_{-\pi}^{-\pi/4} + \left.\frac{e^{j(n-2)w}}{j(n-2)}\right]_{\pi/4}^{\pi}$$

$$h_d(n) = \frac{1}{\pi(n-2)}\left[\sin \pi(n-2) - \sin(n-2)\frac{\pi}{4}\right] \quad n \ne 2$$

$$h_d(0) = \frac{-1}{2\pi} = h_d(4)$$

$$h_d(1) = \frac{-1}{\sqrt{2}\,\pi} = h_d(3)$$

$$\frac{\sin \pi(n-2)}{\pi(n-2)} - \frac{\sin(n-2)\pi/4}{\pi(n-2)}$$

$$h_d(2) = 1 - \frac{1}{4} = \underline{\frac{3}{4}}$$

$$w_H(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \le n \le M-1$$

$w_H(0) = 0.08$, $w_H(1) = 0.54$, $w_H(2) = 1$, $w_H(3) = 0.54$, $w_H(4) = 0.08$

$$h_d(0) \times w_H(0) = -0.0127 = h(4)$$

$$h(1) = -0.1215 = h(3)$$

$$h(2) = 0.75$$

$$H(\omega) = e^{-j2\omega}\left[0.75 - 0.243\cos\omega - \underset{0.025}{\cancel{0.125}}\cos 2\omega\right]$$

Ans b

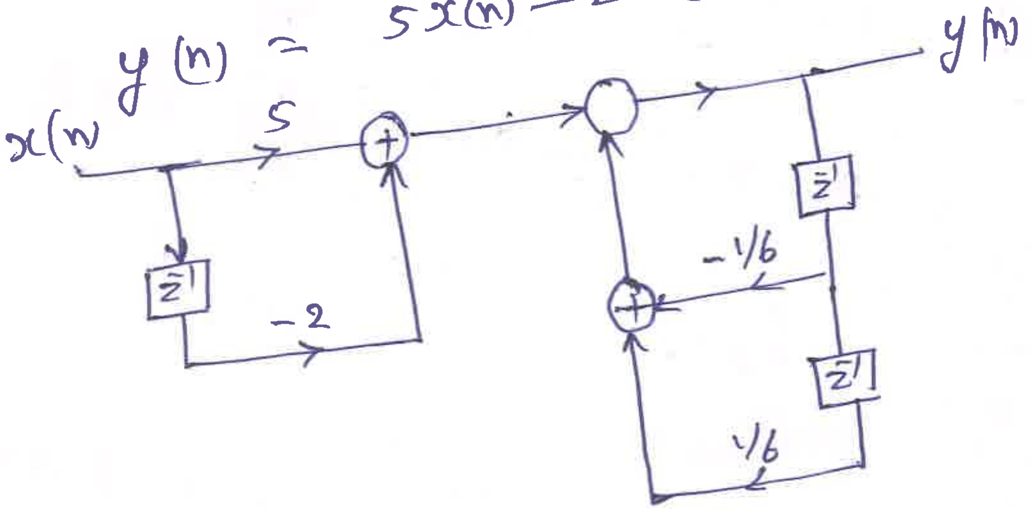
$$H(z) = \frac{3z(5z-2)}{(z+\frac{1}{2})(3z-1)} = \frac{15z^2-6z}{3z^2+\frac{3}{2}z - z - \frac{1}{2}}$$

$$H(z) = \frac{15z^2-6z}{3z^2+\frac{1}{2}z-\frac{1}{2}} = \frac{\cancel{15}\,3(5z^2-2z)}{3(z^2+\frac{1}{6}z-\frac{1}{6})}$$

$$= \frac{5z^2-2z}{z^2+\frac{1}{6}z-\frac{1}{6}} = \frac{5-2z^{-1}}{1+\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{5-2z^{-1}}{1+\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$$

$$y(n) = 5x(n) - 2x(n-1) - \frac{1}{6}y(n-1) + \frac{1}{6}y(n-2)$$



Direct form-I

cascade form

$$H(z) = \frac{3z}{z+1/2} \cdot \frac{5z-2}{3z-1}$$

$$= \left(\frac{3}{1+\frac{1}{2}z^{-1}}\right)\left(\frac{5-2z^{-1}}{3-z^{-1}}\right)$$

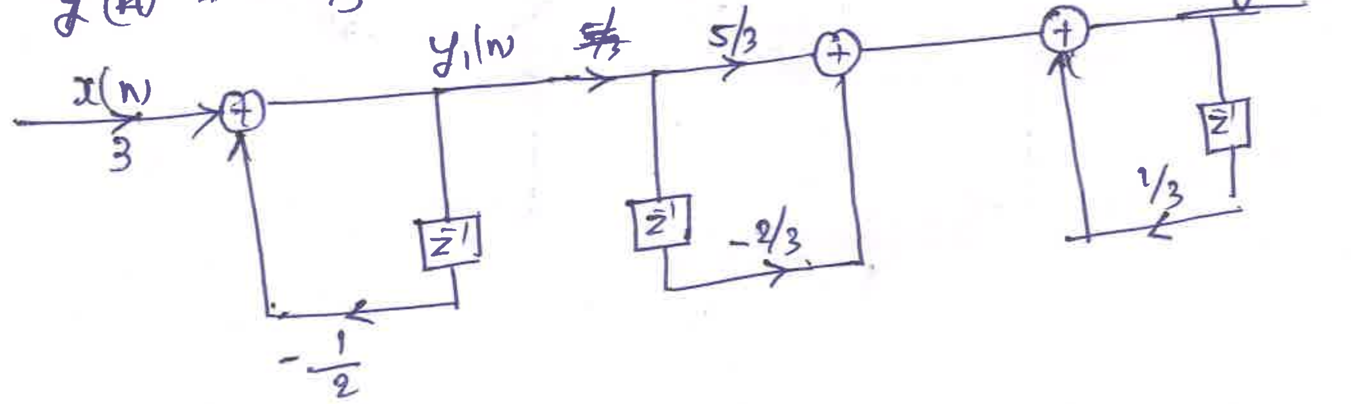
$$H(z) \qquad H_1(z) \cdot H_2(z)$$

$$H_1(z) = \frac{3}{1+\frac{1}{2}z^{-1}}$$

$$\frac{Y_1(z)}{X(z)} = \frac{3}{1+\frac{1}{2}z^{-1}} \qquad y_1(n) = 3x(n) - \frac{1}{2}y_1(n-1).$$

$$H_2(z) = \frac{5-2z^{-1}}{3-z^{-1}} = \frac{Y(z)}{Y_1(z)} = \frac{5-2z^{-1}}{3-z^{-1}} = \frac{5/3 - \frac{2}{3}z^{-1}}{1-\frac{1}{3}z^{-1}}$$

$$y(n) = 5/3\, y_1(n) - \frac{2}{3}y_1(n-1) + \frac{1}{3}y(n-1)$$



Parlell form

$$\frac{H(z)}{z} = \frac{(5z-2)}{(z+1/2)(z-\frac{1}{3})} = \frac{A}{z+1/2} + \frac{B}{z-1/3}.$$

$$A = \left.\frac{5z-2}{z-1/3}\right|_{z=-1/2} = \frac{-\frac{5}{2}-2}{-\frac{1}{2}-\frac{1}{3}} = \frac{-9/2}{-5/6} = \frac{27}{5}$$

$$B = \left.\frac{5z-2}{z+1/2}\right|_{z=\frac{1}{3}} = \frac{\frac{5}{3}-2}{\frac{1}{3}+\frac{1}{2}} = \frac{-1/3}{5/6} = -\frac{2}{5}$$

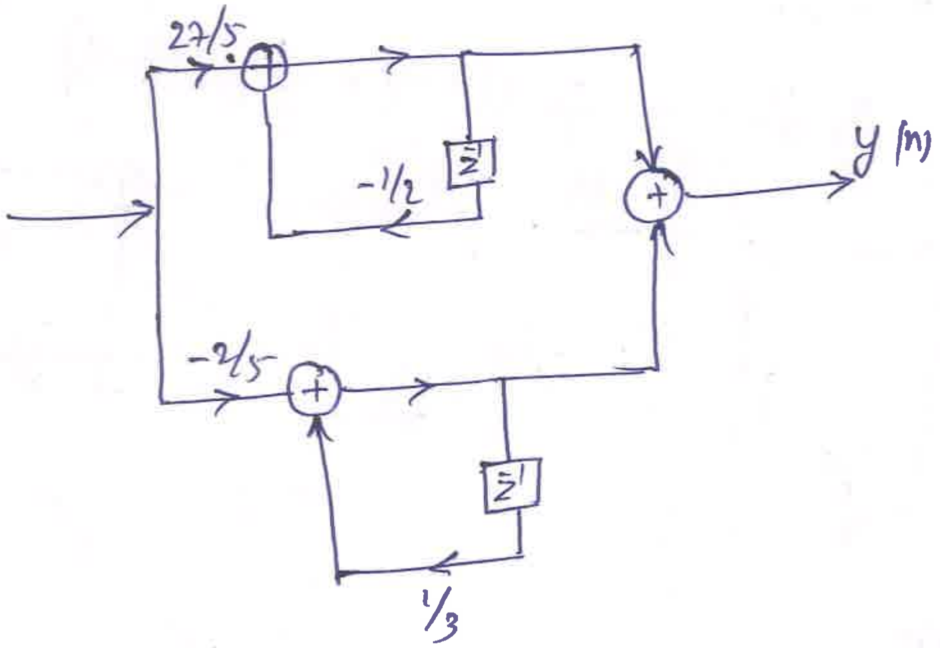
$$\frac{H(z)}{z} = \frac{27/5}{z+1/2} - \frac{\frac{2}{5}}{z-1/3}.$$

$$H(z) = \frac{27}{5}\frac{z}{z+1/2} - \frac{2}{5}\left(\frac{z}{z-1/3}\right)$$

$$= \frac{27}{5}\frac{1}{1+\frac{1}{2}z^{-1}} - \frac{2}{5}\frac{1}{1-\frac{1}{3}z^{-1}}$$

$H(z) = H_1(z) + H_2(z)$

$H_1(z) = \dfrac{27/5}{1 + \frac{1}{2} z^{-1}}$ $\quad H_2(z) = -\frac{2}{5} \cdot \dfrac{1}{1 - \frac{1}{3} z^{-1}}$



Q6 c) $\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+j2 \\ -2 \\ -2-j2 \end{bmatrix}$$

$\sum |X(k)|^2 = \frac{1}{10} \sum 10^2 + |-2+j2|^2 + |-2|^2 + |-2-j2|^2$

$= \underline{\underline{30}}$ units