Q.2 a) $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)(s+5)}$.

(i) number of branches = 4

(ii) Number of asymptotes = 4

$\theta_0 = 45°$, $\theta_1 = 135°$, $\theta_2 = 225°$ and $\theta_3 = 315°$ — (1 mk).

(iii) Centroid $= \dfrac{-1-3-5-0}{} = -2.25$ — (1 mk.)

(iv) Break away point:-

$\dfrac{dK}{ds} = 0$

$1 + G(s)H(s) = 0$

$1 + \dfrac{K}{s(s+1)(s+3)(s+5)} = 0$

$s(s+1)(s+3)(s+5) + K = 0$

$\therefore K = -[s^4 + 9s^3 + 23s^2 + 15s]$

$\dfrac{dK}{ds} = -[4s^3 + 27s^2 + 46s + 15] = 0$

$\therefore s = -0.425, -4.253, -2.07$

$K = 2.878$ for $s = -0.425$

$K = 12.949$ for $s = -4.253$.

$K = -6.035$ for $s = -2.07$

$\therefore$ valid breakaway points are $s = -0.425$ & $s = -4.253$. — (2 mks)

(v) $s^4 + 9s^3 + 23s^2 + 15s + K = 0$

$\therefore \omega = 1.29$

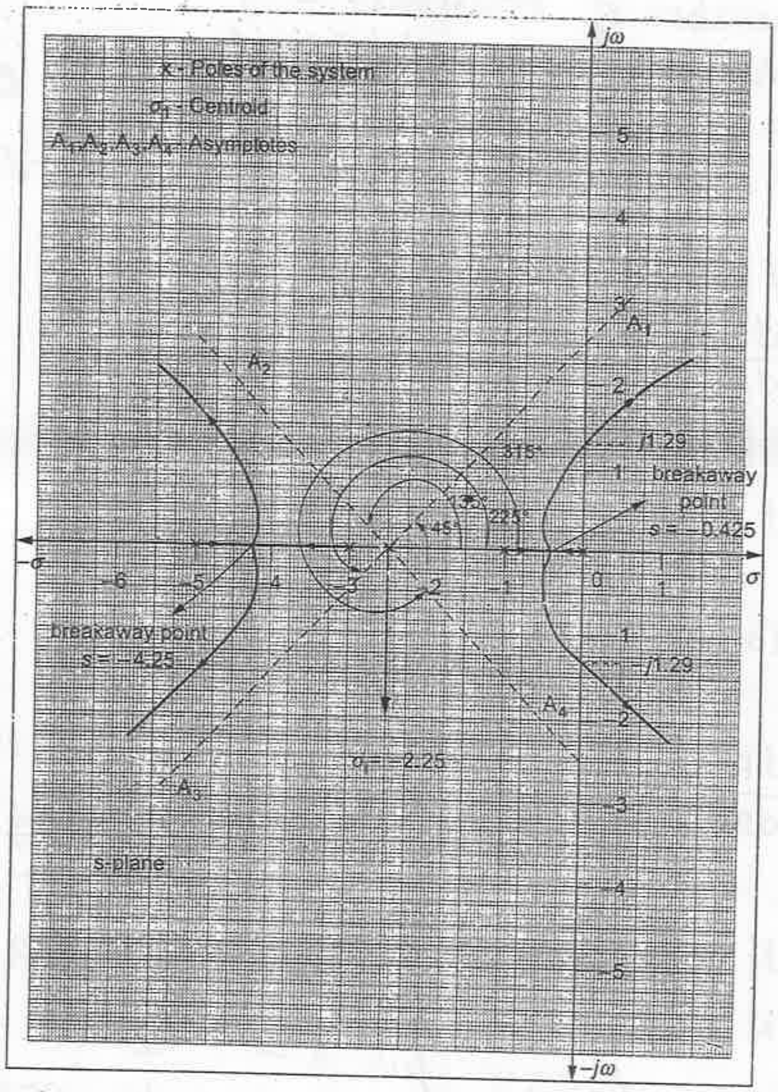
$\therefore$ root locus intersects at $\pm j1.29$

$\omega^4 - 23\omega^2 + K = 0$ — (1 mk.)

$\therefore K = 35.55$

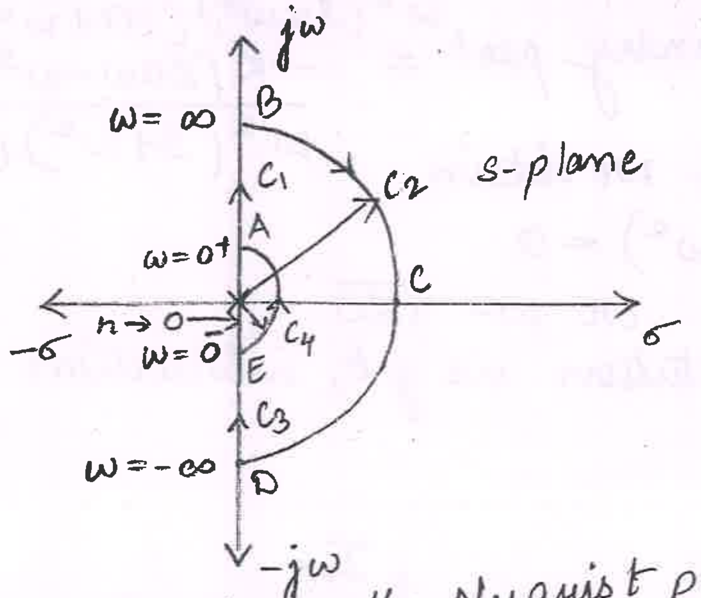
(2)

OL


x - Poles of the system
σ_1 - Centroid
A_1, A_2, A_3, A_4 - Asymptotes
jω
-jω
σ
-σ
A_1
A_2
A_3
A_4
315°
225°
135°
45°
j1.29
-j1.29
breakaway point s = -0.425
breakaway point s = -4.25
σ_1 = -2.25
s-plane

— (5mks.)

Q.3b)



Sections present in the Nyquist path;

Section	Parameter
AB	$s = j\omega$ & $\omega = 0^+$ to ∞
Semicircle BCD	$s = \lim_{R\to\infty} Re^{j\theta}$ and $\frac{\pi}{2} \leq \theta \leq -\pi/2$
DE	$s = -j\omega$ & $\omega = -\infty$ to 0^-
Semicircle EFA	$s = \lim_{r\to 0} re^{j\theta}$ & $\frac{-\pi}{2} \leq \theta \leq \pi/2$

(2mks)

Construct contours in $G(s)H(s)$-plane for each section & the individual contours.

To determine the intersection point of the contour in the real axis;

The loop T.F. of the given system is $G(s)H(s) =$

$$\frac{K}{s(s+2)(s+10)}$$

$$\therefore G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega+2)(j\omega+10)}$$

$$= \frac{-Kj\omega\left[20 - 12j\omega - \omega^2\right]}{\omega^2(3+\omega^2)(100+\omega^2)}$$

(4)

04

$\therefore$ real part of T.F $= \dfrac{-12\omega^2}{\omega^2(3+\omega^2)(100+\omega^2)}$

and imaginary part $= \dfrac{-K(20\omega-\omega^3)}{\omega^2(3+\omega^2)(100+\omega^2)}$

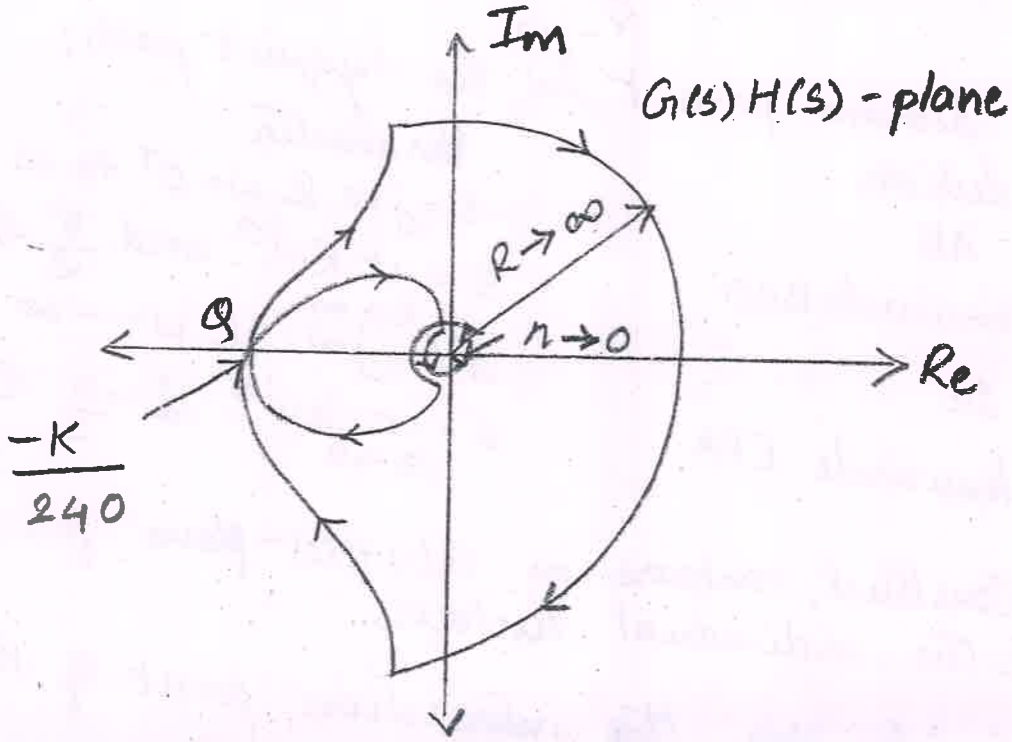
equating we obtain,

$\omega(20-\omega^2)=0$

$\omega^2 = 20$ or $\omega = \sqrt{20}$

(3mks.) by substitution we get, intersection point Q as $\dfrac{-K}{240}$

(5mks)



Q4

05

Q4a) Number of forward paths $K = 2$

$T_1 = G_1 G_2 G_3 G_4$

$T_2 = G_1 G_5 G_8 G_4$ — (1mk.)

$L_1 = G_1 G_2 H_1$

$L_2 = G_3 G_4$ — (2mks.)

$L_3 = G_5 G_6$

$L_4 = G_1 G_4 G_5 G_8 H_1$

$L_5 = G_7$

$L_1 L_5 = G_1 G_2 G_7 H_1$

$L_2 L_5 = G_3 G_4 G_7$ — (2mks.)

$L_2 L_3 = G_3 G_4 G_5 G_6$

$\Delta = 1 - [G_1 G_2 H_1 + G_3 G_4 + G_5 G_6 + G_1 G_4 G_5 G_8 H_1 + G_7] + [G_1 G_2 G_7 H_1 + G_3 G_4 G_7 + G_3 G_4 G_5 G_6]$

— (1mk.)

$\left.\begin{array}{l}\Delta_1 = 1 - G_7 \\ \Delta_2 = 0\end{array}\right\}$ (2mks.)

$\therefore T.F = \dfrac{\sum T_K \Delta_K}{\Delta}$

$= \dfrac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$

$\therefore T.F = \dfrac{G_1 G_2 G_3 G_4 (1 - G_7) + G_1 G_5 G_4 G_8}{1 - [G_1 G_2 H_1 + G_3 G_4 + G_5 G_6 + G_1 G_4 G_5 G_8 H_1 + G_7] + [G_1 G_2 G_7 H_1 + G_3 G_4 G_7 + G_3 G_4 G_5 G_6]}$

— (2mks.)

Q.4b) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s^2+10s+K}$ — (1mk.)

$\omega_n^2 = K$

$\omega_n = \sqrt{K}$ — (1mk.)

$2\xi\omega_n = 10$ — (1mk.)

$\therefore \xi = 5/\sqrt{K}$ — (1mk.)

for $\xi = 0.5$, $K = 100$

$\therefore \omega_n = 10$ rad/sec — (1mk.)

$\omega_d = \omega_n\sqrt{1-\xi^2} = 8.66$ rad/sec — (1mk)

$t_s = \frac{4}{\xi\omega_n} = 0.8$ sec. — (1mk.)

$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 16.303\%$ — (2mks.)

$T_p = \pi/\omega_d = 0.3627$ secs. — (1mk.)

Q.5a)

s^5	1	15	44
s^4	6	30	24
s^3	10	40	0
s^2	6	24	0
s^1	0	0	

— (2mks)

$A(s) = 6s^2 + 24$

$\frac{dA(s)}{ds} = 12s$

— (2mks.)

s^5	1	15	44
s^4	6	30	24
s^3	10	40	0
s^2	6	24	0
s^1	12	0	
s^0	24		

— (2mks.)

Although there is no sign change in the 1st column of Routh's array, the system cannot be suggested as a stable system. — (1mk.)

$A(s) = 6s^2 + 24 = 0$

$\therefore 6s^2 = -24$

$s = \pm j2$ } (2mks.)

The given system is marginally stable as the dominant roots lie on the imaginary axis of the s-plane. — (1mk.)

Q.5 b) $G(s)H(s) = \dfrac{80}{s(s+2)(s+20)}$

$= \dfrac{2}{s(1+s/2)(1+s/20)}$

$G(j\omega)H(j\omega) = \dfrac{2}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)}$

Two corner frequencies,

$\omega_{c_1} = 2$ rad/sec.

(2mks) $\omega_{c_2} = 20$ rad/sec.

Term	Corner freq.	slope of the term (dB/dec)	change in slope (dB/dec)
$\frac{1}{j\omega}$	—	−20 dB/dec.	—
$\frac{1}{1+\frac{j\omega}{2}}$	$\omega_{c_1} = 2$	−20	$-20-20=-40$
$\frac{1}{1+\frac{j\omega}{20}}$	$\omega_{c_2} = 20$	−20	$-40-20=-60$

(2mks.)

Phase Angle Plot can be obtained as,

$\phi = -90° - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/20)$

Freq. ω (rad/sec)	$-\tan^{-1}(\omega/2)$	$-\tan^{-1}(\omega/20)$	ϕ
0.2	-5.7°	-0.57°	-96.27°
2	-4.5°	-5.7°	-140.7°
8	-75°	-21.8°	-187.76°
10	-78.69°	-26.56°	-195.29°
20	-84.28°	-45°	-219.28°
40	-87.13°	-63.43°	-240.58°
∞	-90°	-90°	-270°

(2mks)

(4mks)