Q.2 a) $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)(s+5)}$.

(i) number of branches = 4

(ii) Number of asymptotes = 4

$\theta_0 = 45°$, $\theta_1 = 135°$, $\theta_2 = 225°$ and $\theta_3 = 315°$ — (1mk).

(iii) Centroid = $\dfrac{-1-3-5-0}{4} = -2.25$ — (1mk.)

(iv) Break away point:-

$\dfrac{dK}{ds} = 0$

$1 + G(s)H(s) = 0$

$1 + \dfrac{K}{s(s+1)(s+3)(s+5)} = 0$

$s(s+1)(s+3)(s+5) + K = 0$

$\therefore K = -[s^4 + 9s^3 + 23s^2 + 15s]$

$\dfrac{dK}{ds} = -[4s^3 + 27s^2 + 46s + 15] = 0$

$\therefore s = -0.425, -4.253, -2.07$

$K = 2.878$ for $s = -0.425$

$K = 12.949$ for $s = -4.253$.

$K = -6.035$ for $s = -2.07$

$\therefore$ valid breakaway points are $s = -0.425$ & $s = -4.253$. — (2mks)

(v) $s^4 + 9s^3 + 23s^2 + 15s + K = 0$

$\therefore \omega = 1.29$

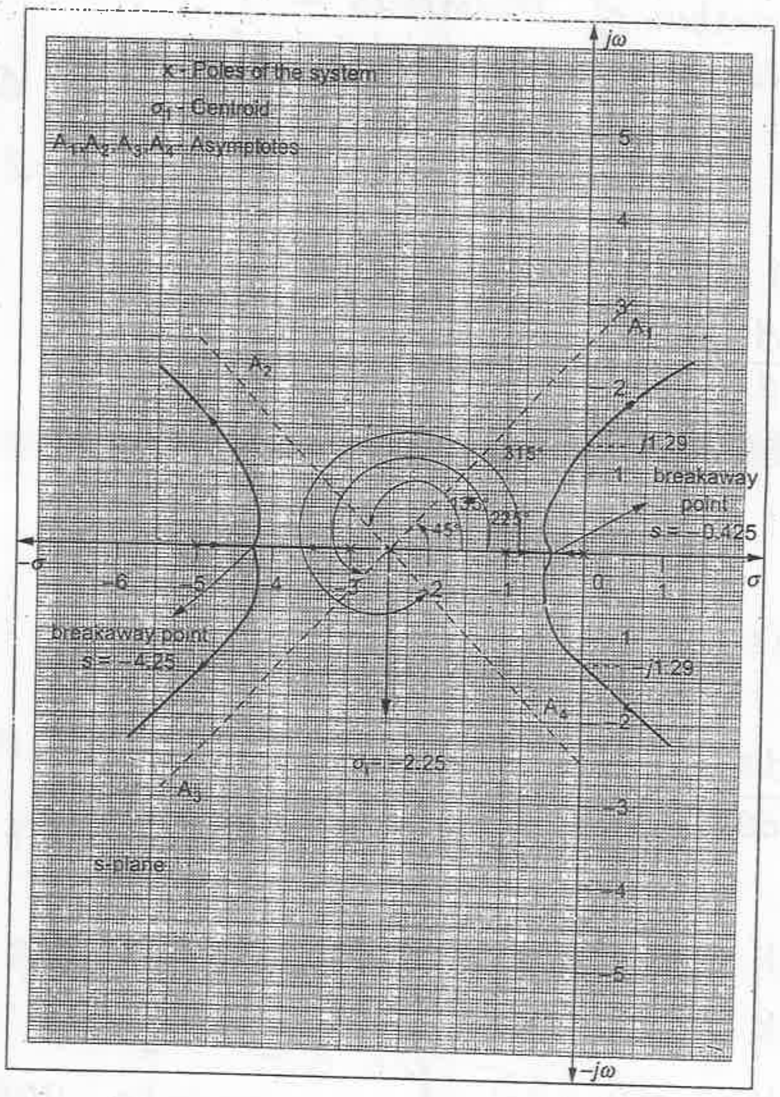
$\therefore$ root locus intersects at $\pm j1.29$

$\omega^4 - 23\omega^2 + K = 0$ — (1mk.)

$\therefore K = 35.55$

(2)

OL

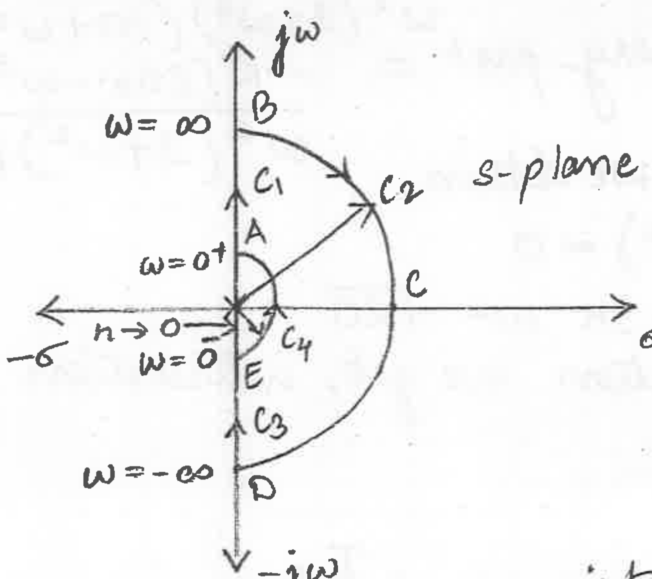


x - Poles of the system
σ_1 - Centroid
A_1, A_2, A_3, A_4 - Asymptotes
$j\omega$
$-j\omega$
σ
$-\sigma$
A_1
A_2
A_3
A_4
315°
135°
225°
45°
$j1.29$
$-j1.29$
breakaway point $s = -0.425$
breakaway point $s = -4.25$
$\sigma_1 = -2.25$
s-plane

— (5mks.)

03

Q.3b)



Sections present in the Nyquist path;

Section	Parameter
AB	$s = j\omega$ & $\omega = 0^+$ to ∞
Semicircle BCD	$s = \lim_{R\to\infty} Re^{j\theta}$ and $\frac{\pi}{2} \leq \theta \leq -\pi/2$
DE	$s = -j\omega$ & $\omega = -\infty$ to 0^-
Semicircle EFA	$s = \lim_{r\to 0} re^{j\theta}$ & $\frac{-\pi}{2} \leq \theta \leq \pi/2$

(2mks)

Construct contours in $G(s)H(s)$ - plane for each section & the individual contours.

To determine the intersection point of the contour in the real axis;

The loop T.F. of the given system is $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$

$$\therefore G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega+2)(j\omega+10)}$$

$$= \frac{-Kj\omega\left[20 - 12j\omega - \omega^2\right]}{\omega^2(3+\omega^2)(100+\omega^2)}$$

04 Q4

$\therefore$ real part of T.F $= \dfrac{-12\omega^2}{\omega^2(3+\omega^2)(100+\omega^2)}$

and imaginary part $= \dfrac{-K(20\omega-\omega^3)}{\omega^2(3+\omega^2)(100+\omega^2)}$

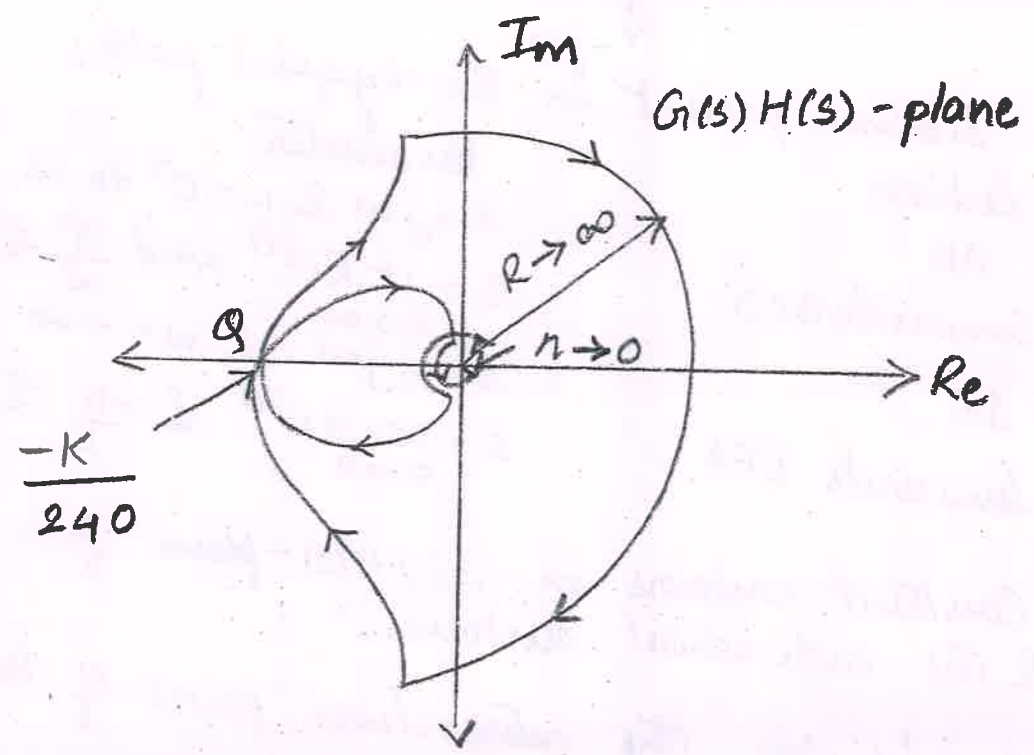
equating we obtain,

$\omega(20-\omega^2)=0$

$\omega^2 = 20$ or $\omega = \sqrt{20}$

(3mks.) by substitution we get, intersection point Q as $\dfrac{-K}{240}$

(5mks)



$A(s) = 6s^2 + 24 = 0$ (7)

$\therefore 6s^2 = -24$

$s = \pm j2$

The aim... (5) | s.)

...as the axis of the - (1mk.)

1. 3

Q4a) Number of forward paths $K = 2$ (05)

$T_1 = G_1 G_2 G_3 G_4$ — (1mk.)

$T_2 = G_1 G_5 G_8 G_4$

$L_1 = G_1 G_2 H_1$ — (2mks.)

$L_2 = G_3 G_4$

$L_3 = G_5 G_6$

$L_4 = G_1 G_4 G_5 G_8 H_1$

$L_5 = G_7$

$L_1 L_5 = G_1 G_2 G_7 H_1$ — (2mks.)

$L_2 L_5 = G_3 G_4 G_7$

$L_2 L_3 = G_3 G_4 G_5 G_6$

$\Delta = 1 - [G_1 G_2 H_1 + G_3 G_4 + G_5 G_6 + G_1 G_4 G_5 G_8 H_1 + G_7] + [G_1 G_2 G_7 H_1 + G_3 G_4 G_7 + G_3 G_4 G_5 G_6]$ — (1mk.) ec)

40

$\Delta_1 = 1 - G_7$
$\Delta_2 = 0$ } (2mks.)

$\therefore T.F = \dfrac{\sum T_K \Delta_K}{\Delta}$

$= \dfrac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$

$\therefore T.F = \dfrac{G_1 G_2 G_3 G_4 (1 - G_7) + G_1 G_5 G_4 G_8}{1 - [G_1 G_2 H_1 + G_3 G_4 + G_5 G_6 + G_1 G_4 G_5 G_8 H_1 + G_7] + [G_1 G_2 G_7 H_1 + G_3 G_4 G_7 + G_3 G_4 G_5 G_6]}$

— (2mks.)

(8).

08

Freq. ω (rad/sec)	$-\tan^{-1}(\omega/2)$	$-\tan^{-1}(\omega/20)$	ϕ
0.2	-5.7°	-0.57°	-96.27°
2	-4.5°	-5.7°	-140.7°
8	-75°	-21.8°	-187.76°
10	-78.69°	-26.56°	-195.29°
20	-84.28°	-45°	-219.28°
40	-87.13°	-63.43°	-240.58°
∞	-90°	-90°	-270°

(2mks)

(4mks)