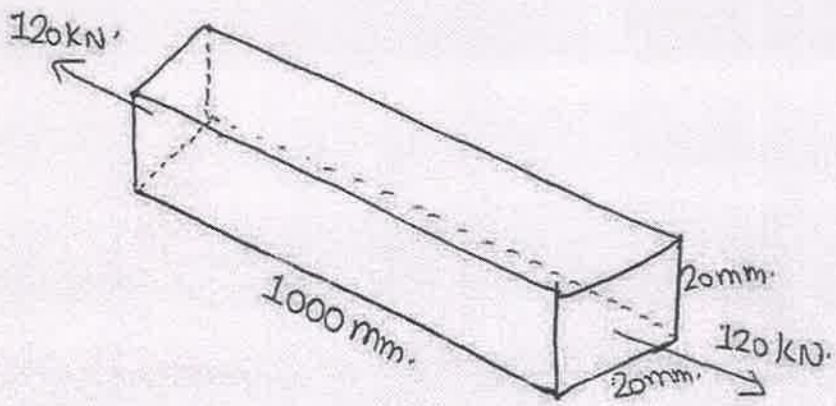
QP code: 25760

Q:1 a)



$\Delta l = 0.5 mm,$

$\Delta t = \Delta b = -0.003 mm.$

$$\text{Poisson's Ratio} = \left| \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} \right|$$

$$= \left| \frac{\frac{-0.003}{20}}{\frac{0.5}{1000}} \right|$$

$$\boxed{\text{Poisson's Ratio} = 0.3}$$

Young's Modulus,

By hook's law,

$$dl = \frac{PL}{AE}$$

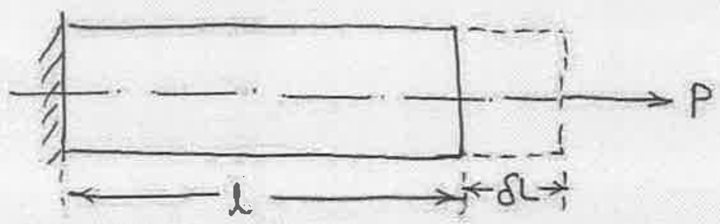
$$\therefore\ 0.5 = \frac{120 \times 10^3 \times 1000}{20 \times 20 \times E}$$

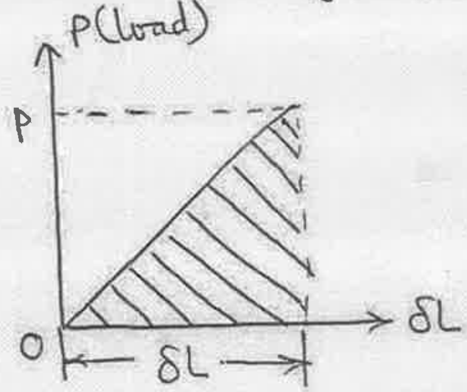
$$\therefore\ E = 600 \times 10^3\ N/mm^2$$

$$\boxed{E = 600\ GPa.}$$

Q.1 b) Expression for Strain Energy due to Gradually Applied Load.

Consider a simple uniform bar subjected to gradually applied load.





Let P = Gradually applied load in axial direction

A = Area of cross-section of bar

l = length of bar

δL = Deformation

V = Volume of bar = Area × length

E = Young's Modulus or Modulus of Elasticity or Hooke's Constant

When the load is applied from 0 to P, deformation increased from 0 to δL in a linear manner

Work done is given by area of load deformation diagram.

Work is equal to strain stored in the member.

Work done by axial load = Strain Energy = Area under load deformation diagram.

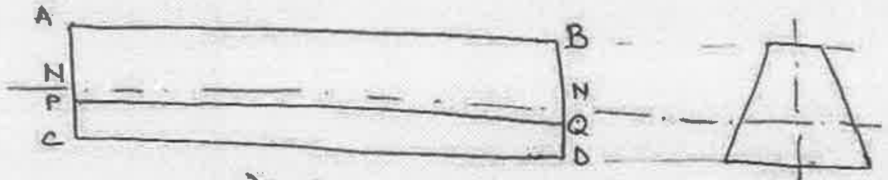
$$\text{Strain Energy, } U = \frac{1}{2} \times P \times \delta L = \frac{1}{2} P \times \frac{PL}{AE} = \frac{1}{2}\frac{P^2 L}{AE}$$

$$= \frac{1}{2}\frac{(\sigma A)^2 \times L}{AE} = \frac{1}{2}\frac{\sigma^2 AL}{E} = \frac{1}{2}\frac{\sigma^2}{E} \times \text{volume}$$

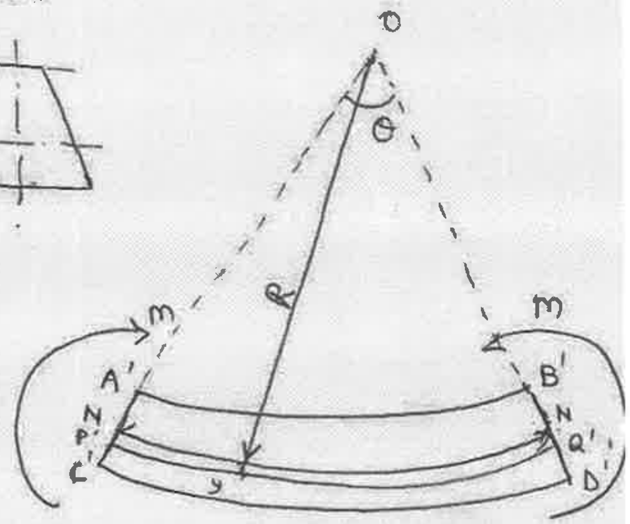
$$S.E. = U = \frac{\sigma^2}{2E} \times V$$

Q·1 c) <u>Derivation of flexural formula.</u>

Consider a beam AB - CD which is straight when not loaded. Let the beam is subjected to bending moment 'M' at the ends. Due to the moment 'm' at the ends, the beam bends.



a) Beam before bending



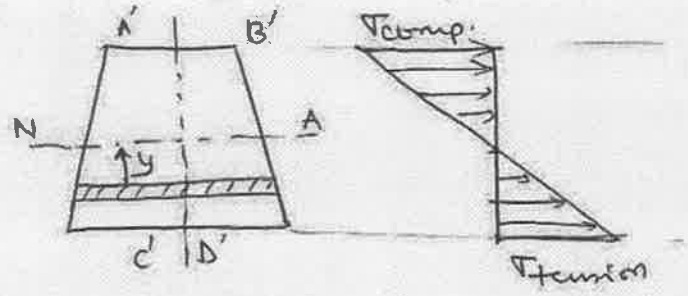
Strain in the layer PQ

$$\varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{P'Q' - PQ}{PQ}$$

$$\varepsilon = \frac{P'Q' - NN}{NN} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

$$\varepsilon = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \quad -①$$

Since E/R is constant, σ is proportional to radial distance y from NN



Consider a small area dA at a distance y from neutral axis.

Stress on this small area ~~dA at a distance y from NA~~.

$$\sigma = \frac{E}{R} \cdot y$$

Small force on this area dF = Stress x area = $\sigma \cdot dA \cdot \frac{E}{R} \cdot y \cdot dA$

Moment of this small force about N.A., $dm = dF \times y = \left(\frac{E}{R} \cdot y \cdot dA\right) \cdot y$

$= \frac{E}{R} \cdot y^2 \cdot dA$

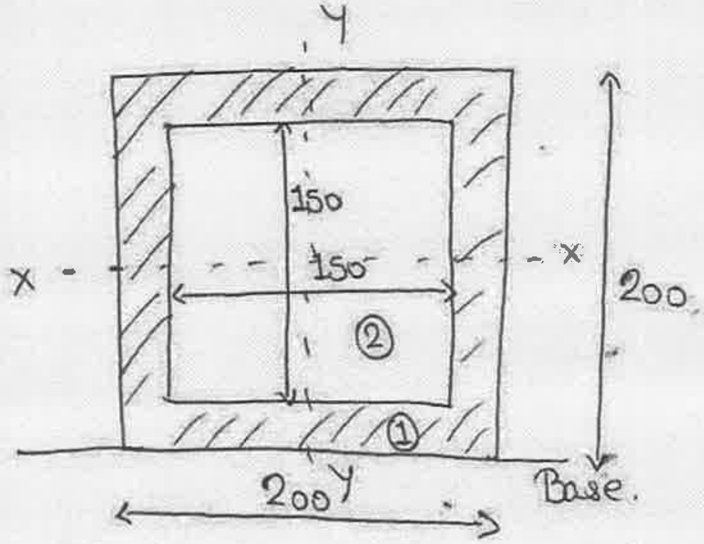
Total moment $m = \int \frac{E}{R} \cdot y^2 \cdot dA = \frac{E}{R} \int y^2 \cdot dA$

$$m = \frac{E}{R} \cdot I \text{ or } \frac{m}{I} = \frac{E}{R} \quad -②$$

combining ① & ②

$$\frac{m}{I} = \frac{\sigma}{y} = \frac{E}{R}.$$

Q:1 d)



By Parallel Axis Theorem,

$$I_{base} = I_G + Ah^2 = [I_G + Ah^2]_1 - [I_G + Ah^2]_2$$

$$= \left[\frac{200 \times 200^3}{12} + (200 \times 200) \times 100^2\right] - \left[\frac{150 \times 150^3}{12} + (150 \times 150) \times (25+75)^2\right]$$

$$= 533.33 \times 10^6 - 267.18 \times 10^6$$

$$\boxed{I_{base} = 266.15 \times 10^6 \ mm^4}$$

As,

$I = Ak^2$

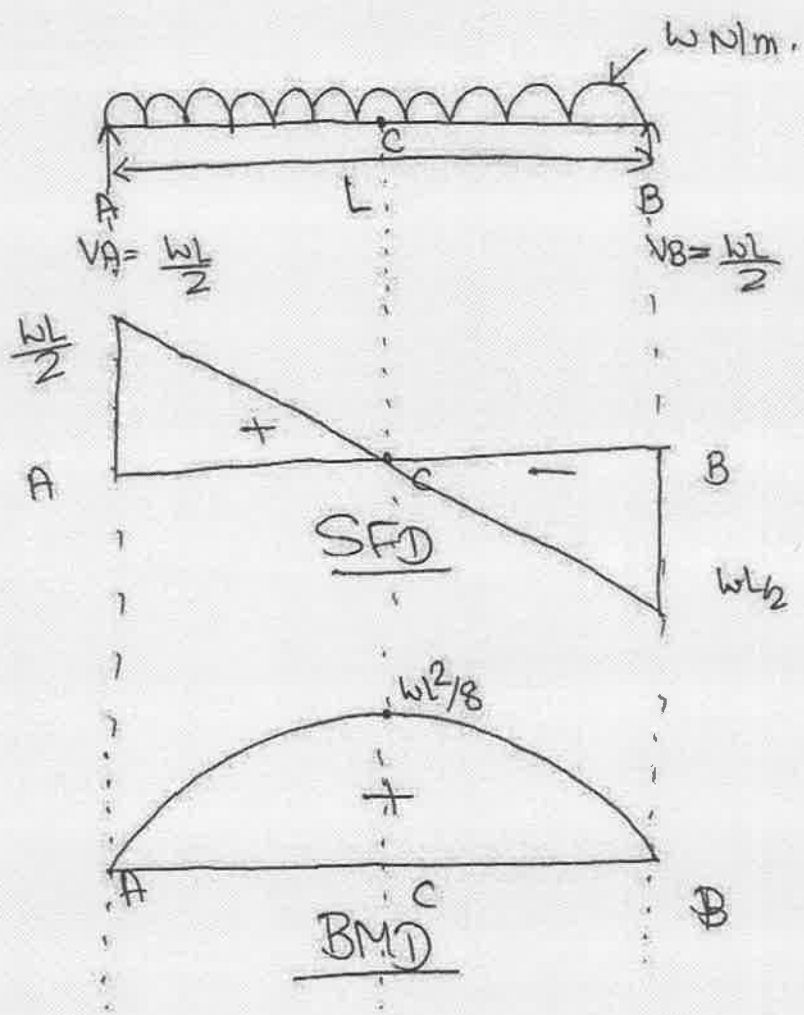
k = Radius of Gyration.

$A = 200 \times 200 - 150 \times 150^2 = 17.5 \times 10^3 \ mm^2$

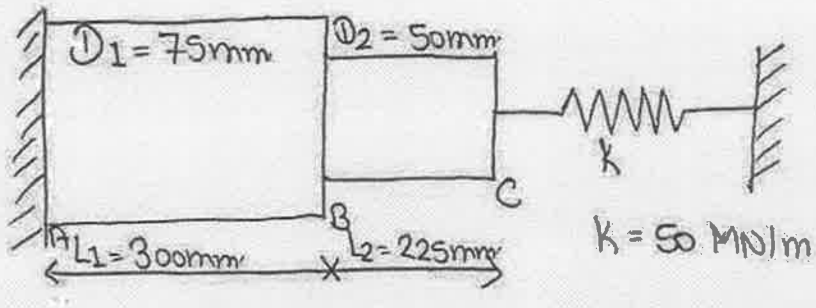
$\therefore \quad 266.15 \times 10^6 = 17.5 \times 10^3 \times k^2$

$\therefore \quad \boxed{k = 123.32 \ mm.}$

Q:1 e)



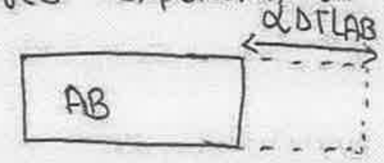
Q:2 a)



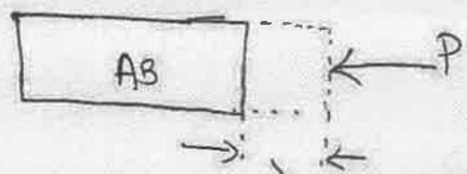
$E = 6\ GPa$
$= 6\times10^3\ N/mm^2$

$K = 50\ MN/m. = 50\ KN/mm$

Free expansion of AB,



Final Expansion of AB,



$\alpha \Delta T L_{AB} - \dfrac{P L_{AB}}{A_{AB} \cdot E_{AB}}$

Final Expansion of BC,

$\alpha \Delta T L_{BC} - \dfrac{P \cdot L_{BC}}{A_{BC} \cdot E_{BC}}$

As both shafts are in Series.

$\therefore$ Total Deflection = Summation of deflection of each shaft
= Deflection of spring.

$$\therefore \left(\alpha \Delta T L_{AB} - \frac{P L_{AB}}{A_{AB} \cdot E_{AB}}\right) + \left(\alpha \Delta T L_{BC} - \frac{P L_{BC}}{A_{BC} \cdot E_{BC}}\right) = \frac{P}{K}$$

On Solving,

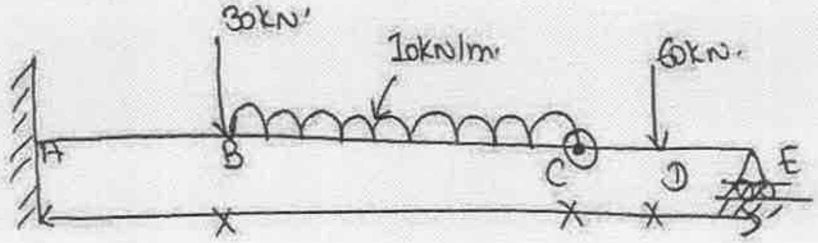
$$P = 31.25\ KN$$

$$dl_{AB} = \cancel{\frac{P L_{AB}}{A_{AB} \times E_{AB}}} = \cancel{0.333}\ \alpha \Delta T L_{AB} - \frac{P L_{AB}}{A_{AB} \cdot E_{AB}} = 0.5463\ mm.$$

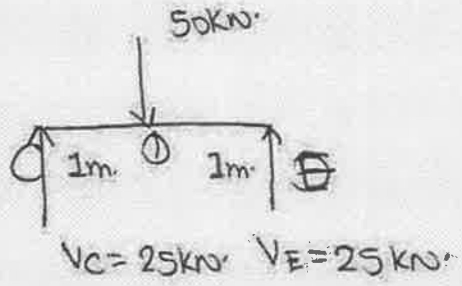
$$dl_{BC} = 0.07816\ mm.$$

$\sigma_{AB} = 7.07\ N/mm^2\ (C)$ $\sigma_{BC} = 15.915\ N/mm^2\ (C).$

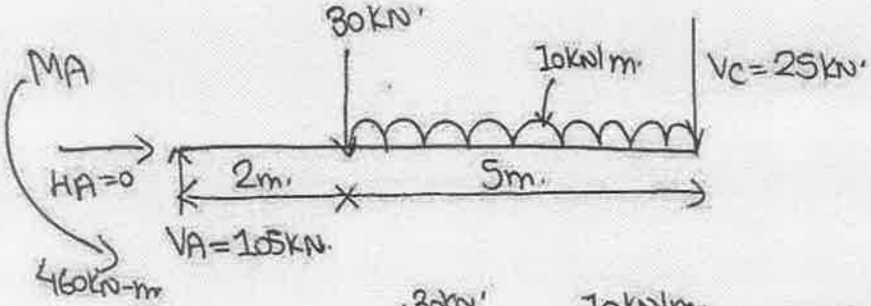
Q:2 b)



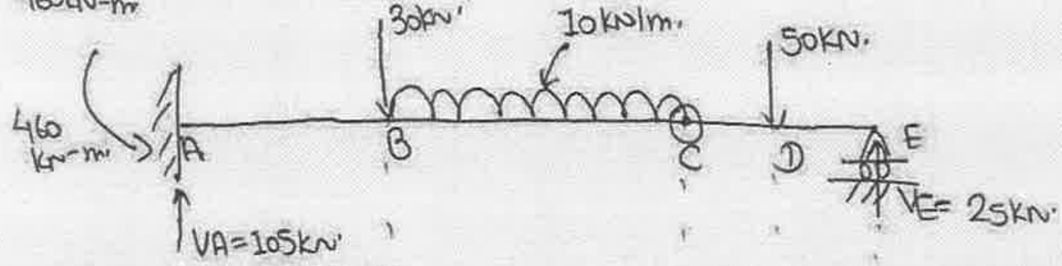
30KN
10KN/m
60KN
A
B
C
D
E



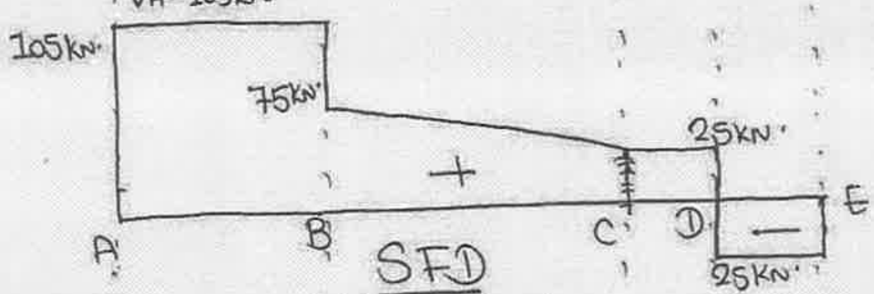
50KN
1m
1m
D
E
VC = 25KN
VE = 25KN



MA
30KN
10KN/m
VC = 25KN
HA = 0
2m
5m
VA = 105KN
460KN-m

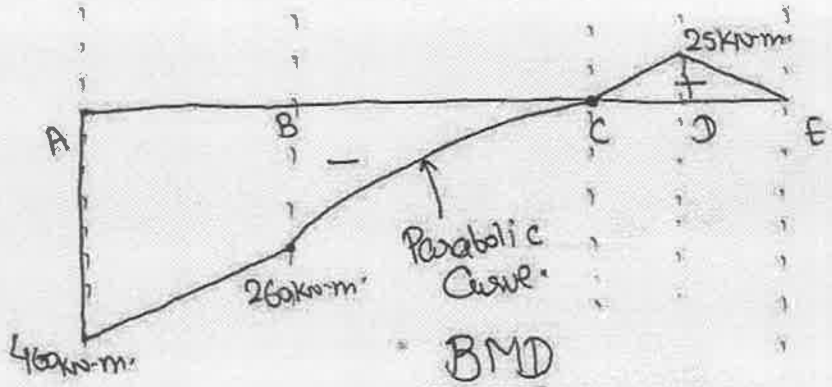


460 KN-m
30KN
10KN/m
50KN
A
B
C
D
E
VA = 105KN
VE = 25KN



105KN
75KN
25KN
+
−
25KN
A
B
C
D
E

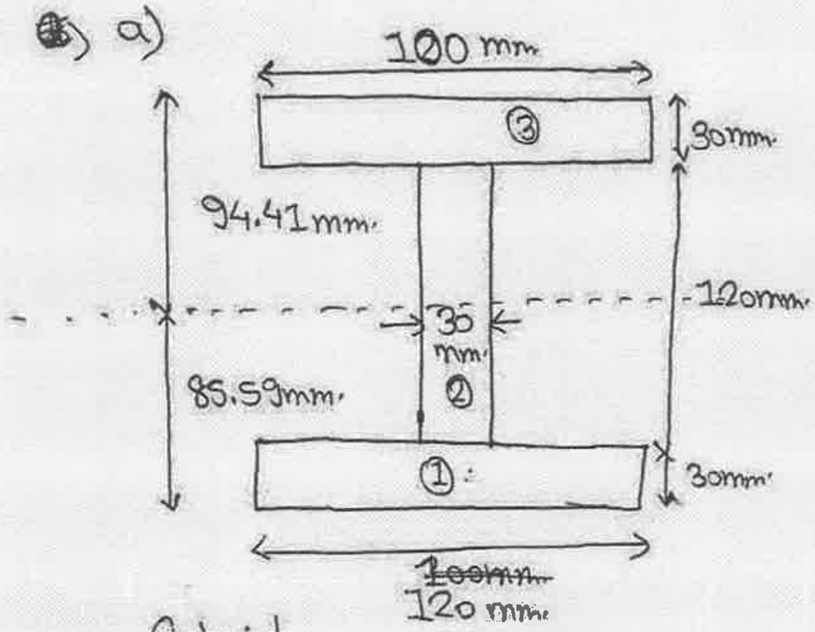
SFD

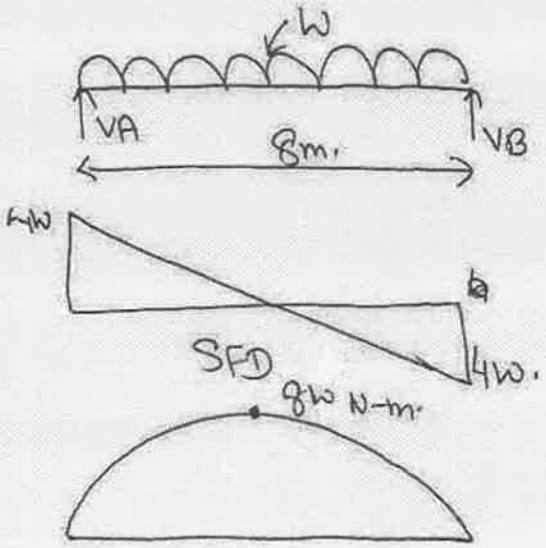


25KN-m
+
A
B
C
D
E
−
Parabolic Curve
260KN-m
460KN-m

BMD

Q.3 a)





Centroid

$$\bar{y} = \frac{120\times30\times15 + 120\times30\times90 + 100\times30\times165}{120\times30 + 120\times30 + 100\times30}$$

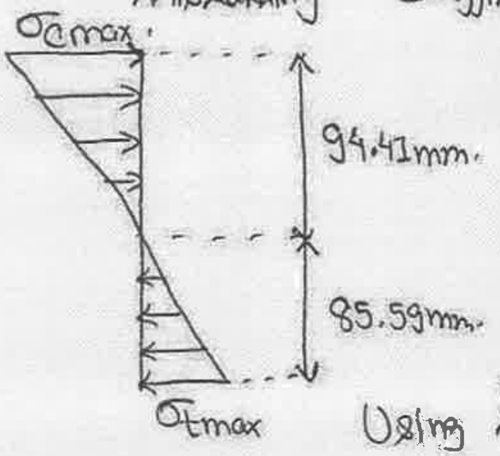
$= 85.59\,mm$

MI @ X-X

$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$

$I_{xx} = 41.77\times10^{6}\,mm^4$

~~Hogging~~ Sagging B.M.,



In this top fibers are under compression & bottom fibers under tension. let assume that tension reaches maximum value first,

$\therefore \sigma_{Tmax} = 30\,N/mm^2$

Using Symmetry of triangles. $\frac{\sigma_{Cmax}}{y_{cmax}} = \frac{\sigma_{Tmax}}{y_{tmax}}$

$\therefore \sigma_C = 33.09\,N/mm^2$,

As $\sigma_C = 33.09\,N/mm^2 < \sigma_{C\,allowable} = 45\,N/mm^2$

Hence Design is safe & assumption correct.

Using, $\frac{M}{I} = \frac{\sigma_{Cmax}}{y_{cmax}}$

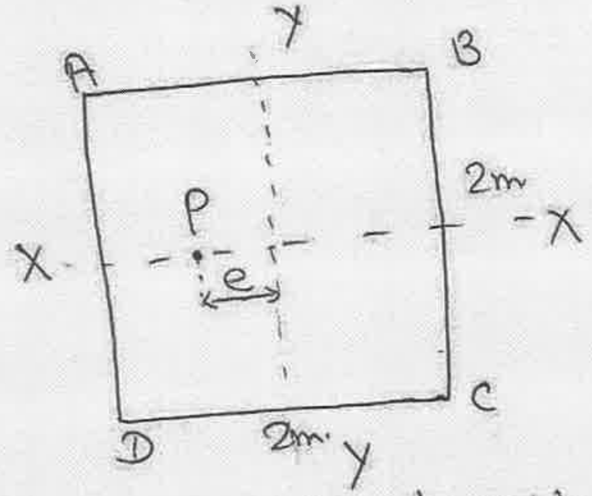
$\therefore M = \frac{33.09\times41.77\times10^{6}}{94.41} = 14.64\times10^{6}\,N/mm^2$

$M_{max} = 8\times W\ N\text{-}m$
$= 8000W\ N\text{-}mm$

$8000W = 14.64\times10^{6}$

$\therefore W = 1830\,N/mm$

Q:3 b)



Max. Compressive stress at point A,D & Max. Tensile stress at B,C [Min. Compressive stress].

Direct compressive Stress $\sigma d = \frac{110}{4} = 27.5 \times 10^3 \ N/m^2 = 27.5 \ kN/m^2$

$$Z_{yy} = \frac{I_{yy}}{x_{max}} = \frac{\frac{2 \times 2^3}{12}}{1} = 1.33 \ m^3$$

Bending Stress $\sigma_b = \frac{M_y \cdot x_{max}}{I_{yy}} = \frac{M_y}{Z_{yy}} = \frac{20 \times e}{1.33}$

Along AD, $\sigma = 27.5 + 15e$

Along BC, $\sigma = 27.5 - 15e$

$$\sigma|_{AD} = 2 * \sigma|_{BC}$$

$$(27.5 + 15e) = 2 * (27.5 - 15e)$$

On Solving,

$e = 0.61m.$

Q. 4 a) Let the internal and external diameters of the shaft be 'd' & 'D' respectively.

$$\frac{d}{D} = \frac{3}{5}$$

$$P = \frac{2\pi N T}{60000} \Rightarrow 450 = \frac{2\pi \times 120 \times T}{60000}$$

$$T = \frac{450 \times 60000}{2\pi \times 120} = 35810\,Nm = 35810 \times 1000\,Nmm$$

Polar moment of Inertia $= I_p = \frac{\pi}{32}(D^4 - d^4)$

Polar Modulus $= \frac{I_p}{(D/2)} = \frac{2I_p}{D} = \frac{2\pi}{32}(D^4 - d^4) = \frac{\pi}{16D}(D^4 - d^4)$

$$= \frac{\pi}{16D}\left(D^4 - \frac{81}{625}D^4\right) = \frac{\pi D^3}{16} \times \frac{554}{625} = 0.1709 D^3$$

When the maximum shear stress is $60 N/mm^2$

$T = \tau_s \times$ polar modulus

Polar modulus $= \frac{T}{\tau_s}$

$$0.1709 D^3 = \frac{35810 \times 1000}{60}$$

$$D = 151.7\,mm$$

When the twist in a length of 2.5 m is 1°

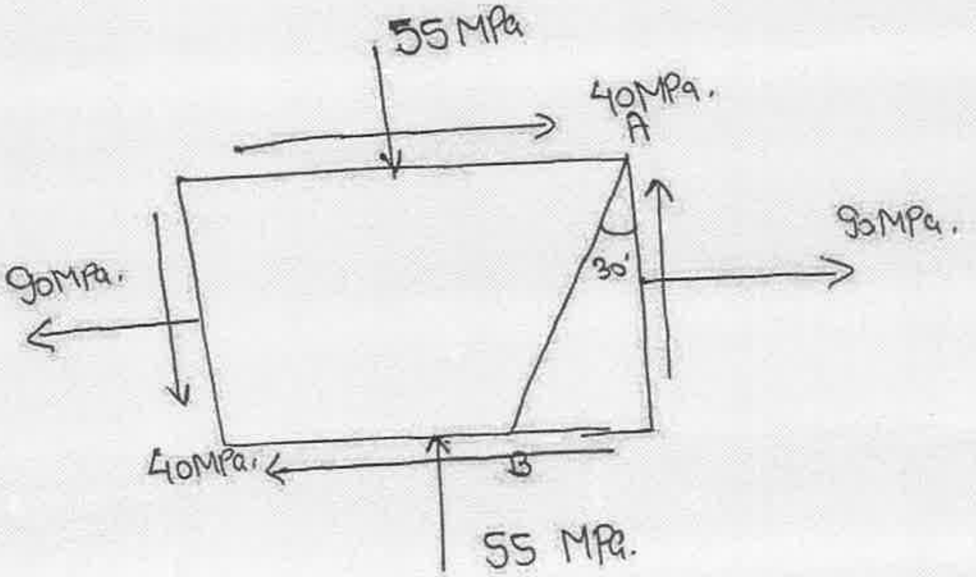
$$I_p = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}\left(D^4 - \frac{81}{625}D^4\right) = \frac{544}{32 \times 625}\pi D^4$$

$$\frac{T}{I_p} = \frac{G\theta}{l} \Rightarrow \theta = \frac{l}{G} \cdot \frac{T}{I_p}$$

$$\frac{\pi}{180} = \frac{2500}{8 \times 10^4} \times \frac{35810 \times 1000 \times 32 \times 625}{544 \pi D^4}$$

$$D^4 = 7.5 \times 10^8$$

Q:4 b)



Principal Stress,

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

$$= 17.5 \pm 82.8$$

$\therefore \sigma_1 = 100$ MPa,

$\sigma_2 = -65$ MPa.

Max. Shear ~~Stress~~ plane,

$$\tan 2\theta = \frac{\sigma_x-\sigma_y}{2\tau_{xy}} \qquad \tau_{max.} = \frac{\sigma_1-\sigma_2}{2} = 82.5 \text{ MPa}.$$

$$= \frac{90+55}{2*40}$$

$2\theta_1 = 61.11°$ $\qquad$ $2\theta_2 = 241.11$

$\theta_1 = 30.55°$ $\qquad$ $\theta_2 = 120.55°$

$$\sigma_n\big|_{\theta=30°} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right).\cos2\theta + \tau_{xy}\sin2\theta$$

$$= 17.5 + 36.25 + 34.64$$

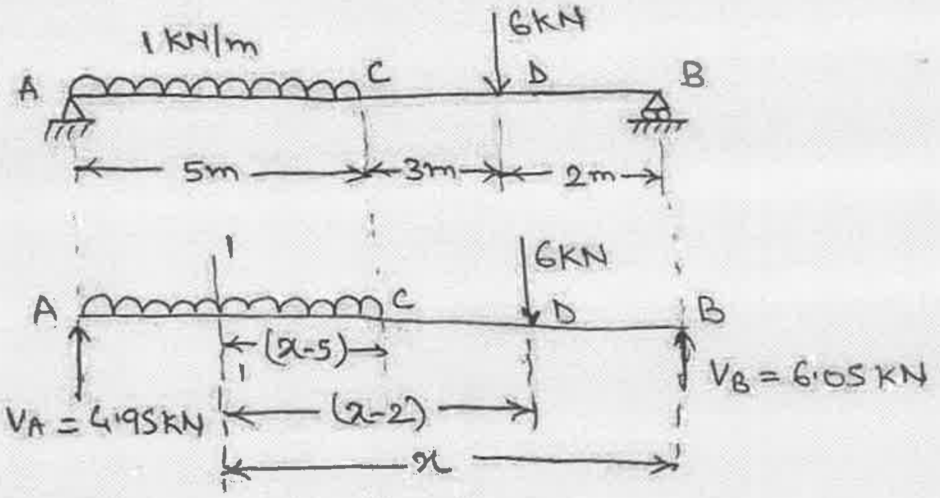
$\sigma_n = 88.39$ MPa.

$$\tau\big|_{\theta=30°} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\cdot\sin2\theta + \tau_{xy}\cdot\cos2\theta$$

$$= -15.15 + 20$$

$\therefore \tau\big|_{\theta=30°} = 4.85$ MPa.

Q. 5 a)



Support Reactions.

$$V_A + V_B = 5 + 6 = 11$$

$$10V_A = 5\times7.5 + 6\times2 \Rightarrow V_A = 4.95\,kN\,(\uparrow),\ V_B = 6.05\,kN\,(\uparrow$$

Take section 1-1 at a distance x from B.

$$M_x = EI\frac{d^2y}{dx^2} = 6.05x\Big|_0^2 - 6(x-2)\Big|_2^5 - \frac{1(x-5)^2}{2}\Big|_5^{10}$$

Integrating twice

$$EI\cdot y = C_1x + C_2 + \frac{6.05x^3}{6}\Big|_0^2 - \frac{(x-2)^3}{6}\Big|_2^5 - \frac{(x-5)^4}{24}\Big|_5^{10}$$

at $x=0, y=0, C_2=0$.

$x=10, y=0, C_1=-47.03$

General eqⁿ of slope & deflection are.

$$EI\frac{dy}{dx} = -47.03 + \frac{6.05x^2}{2}\Big|_0^2 - 3(x-2)^2\Big|_2^5 - \frac{(x-5)^3}{6}\Big|_5^{10}$$

$$EI\cdot y = -47.03x + \frac{6.05x^3}{6}\Big|_0^2 - (x-2)^3\Big|_2^5 - \frac{(x-5)^4}{24}\Big|_5^{10}.$$

Slope at D = $(dy/dx)_D = 2.351\times10^{-3}$ radians $(\;)$

Slope at A = $(dy/dx)_A = 2.132\times10^{-3}$ radians (C)

Deflection at D = $y_D = -4.3\times10^{-3}$ m or -4.3 mm & 4.3 mm $(\downarrow)$

" " C $= y_C = -6.80\times10^{-3}$ m or -6.8 mm or 6.8 mm $(\downarrow)$

max. deflection occurs at, $x = 4.87$ m from end B.

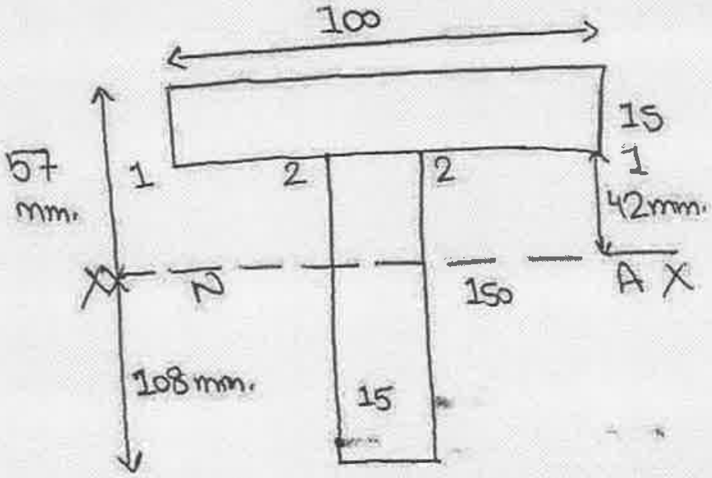
or $x = 5.13$ m from end A.

at $x = 4.87$ m, $EIy_{max} = -47.03\times4.87 + \left(\frac{6.05\times4.87^3}{6}\right) - (4.87$

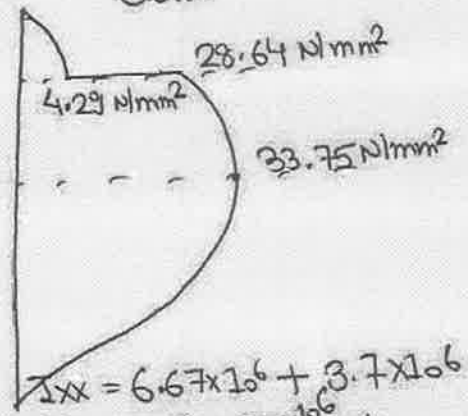
$$y_{max} = \frac{-136.212}{EI} = \frac{-136.212}{2\times10^8\times10^{-4}} = -6.810\times10^{-3}\,m$$

$$y_{max} = 6.810\,mm\,(\downarrow)$$

Q:5 b)



Shear Stress Distribution Curve



$I_{xx} = 6.67 \times 10^6 + 3.7 \times 10^6$
$= 10.37 \times 10^6$ mm⁴

$$\bar{y} = \frac{150 \times 15 \times 75 + 100 \times 15 \times (150 + 7.5)}{150 \times 15 + 100 \times 15}$$

$= 108$ mm.

$$I_{xx} = \frac{15 \times 150^3}{12} + (150 \times 15) \times (75 - 108)^2 + \frac{100 \times 15^3}{12} + (100 \times 15) \times \cancel{(157.5 -} (42 + 7.5)^2$$

Shear Stress at Top of flange, $= 0$

Shear Stress at junction of flange & web in flange Portion,

$$q = \frac{Sa\bar{y}}{Ib}$$

$$= \frac{60 \times 10^3 \times (100 \times 15) \times \left(42 + \frac{15}{2}\right)}{10.37 \times 10^6 \times 100}$$

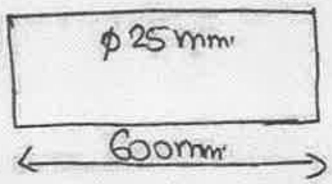
$q_{1-1} = 4.29$ N/mm²

$q_{2-2} = \dfrac{4.29 \times 100}{15} = 28.64$ N/mm²

$$q_{N-A} = \frac{60 \times 10^3 \times \left[(100 \times 15) \times (42 + 7.5) + (42 \times 15) \times 21\right]}{10.37 \times 10^6 \times \cancel{100}\ 15}$$

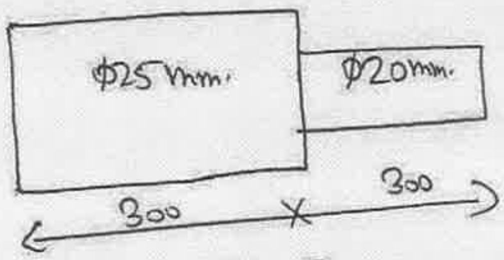
$= 33.75$ N/mm²

Q:6 (a) Case (a)



Strain energy stored in bar A,

$$U_A = \frac{P^2 L}{2AE} = \frac{P^2 \times 600}{2 \times \frac{\pi}{4} \times 25^2 \times E} = 0.611 \frac{P^2}{E}$$



Case (b)

$$U_B = \frac{P^2 \times 300}{2 \times \pi/4 \times 25^2 \times E} + \frac{P^2 \times 300}{2 \times \pi/4 \times 20^2 \times E} =$$

$$= \left[0.3056 + 0.4775\right] \frac{P^2}{E} = 0.7831 \frac{P^2}{E}$$

$$\therefore \frac{\text{Strain Energy in B}}{\text{Strain Energy in A}} = \frac{\frac{P^2}{E} \times 0.7831}{\frac{P^2}{E} \times 0.611} = 1.28$$

Q.6 b) $A = \frac{\pi}{4}(40^2 - 30^2) = 549.78 \text{ mm}^4$

$I = \frac{\pi}{64}(40^4 - 30^4) = 85902.924 \text{ mm}^4$

$I = AK^2 \Rightarrow 85902.924 = 549.78 K^2$

$k^2 = 156.25 \text{ mm}^2$

Crippling load by Euler's formula is given by

$$P_E = \frac{\pi^2 EI}{l_E^2} = \frac{\pi^2 \times 2\times10^3 \times 85902.924}{(2500)^2}$$

$P_E = 27130.5 N = 27.1305 KN$

Crippling load by Rankine's formula is given by

$$P_R = \frac{\sigma_c \cdot A}{1+\alpha\left(\frac{L_E}{K}\right)^2} = \frac{320 \times 549.78}{1+\frac{1}{7500}\left(\frac{2500}{156.25}\right)^2}$$

$= 27778.36 N = 127.778 KN$

For comparision $\frac{P_R}{P_E} = \frac{27.778}{27.1305} = 1.024$