Model Answer

Set - 2 T 1734 A Sem IV (IT)

Q.1

[a] (i) since $\sum_{i=-2}^{3} p_i = 1 \Rightarrow 0.2 + k + 0.1 + \dots + 2k = 1$

$\Rightarrow 5k = 1 - 0.4 = 0.6$

$\Rightarrow k = \frac{3}{25}$

(ii) $E(X) = \sum_{i=-2}^{3} x_i p_i = (-2 \times 0.2) + (-1 \times 3/25) + \dots + (3 \times 6/25)$

$= -0.4 + (-\frac{3}{25}) + 0 + \frac{6}{25} + 0.2 + \frac{18}{25}$

$= -26/15$

(iii) $V(X) = \sum_{i=-2}^{3} x_i^2 p_i - \bar{x}^2$

$= \left[(4 \times 0.2) + (1 \times \frac{3}{25}) + 0 + (1 \times \frac{6}{25}) + (4 \times 0.1) + (9 \times \frac{6}{25})\right] - (-26/25)^2$

$= \frac{18}{5} - \left(-\frac{26}{25}\right)^2 = 2.5184$

[b] since $3^2 \equiv 1 \pmod 8$

$3^3 \equiv 3 \pmod 8$

$3^4 \equiv 1^2 \pmod 8$

$3^{10} = (3^2)^5 \equiv 1^5 \pmod 8 = 1 \pmod 8$

Thus, $3^2 \cdot 3^3 \cdot 3^4 \cdot 3^{10} \equiv 1 \cdot 3 \cdot 1 \cdot 1 \pmod 8 \equiv 3 \pmod 8$

The required smallest +ve integer is 3.

[c] Let $y = \frac{5}{6}x + \frac{90}{6}$ & $x = \frac{8}{15}y + \frac{130}{15}$

$\Rightarrow b_{yx} = \frac{5}{6}$ & $b_{xy} = \frac{8}{15}$

Now, $r^2 = b_{yx} \times b_{xy} = \frac{5}{6} \times \frac{8}{15} = \frac{4}{9} < 1$

$\Rightarrow r = \pm \frac{2}{3}$

But as b_{yx} is +ve $\Rightarrow r = 2/3$.

[d]

$*$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

From the composition table

1. for every pair $a, b \in G$ $\exists$ an ell. $a * b \in G$.
   $\therefore$ $*$ is an binary operation in G.
2. since multiplication of comp. nos is associative, so $*$ is associative in G.
3. $1 \in G$ is an identity ell in G.
4. $1^{-1} = 1$, $(-1)^{-1} = 1$, $i^{-1} = -i$, $(-i)^{-1} = i$

   $\therefore \forall a \in G \ \exists \ a^{-1} \in G$.

$\therefore$ $(G, *)$ is a group.

Q.2

[a] $111 \equiv 6 \pmod 7$

$111^{333} \equiv 6^{333} \pmod 7$

$= (6^2)^{166} \cdot 6 \pmod 7$

$\equiv 1^{166} \cdot 6 \pmod 7$ as $6^2 \equiv 1 (7)$

$= 6 \pmod 7$

$333 \equiv 4 \pmod 7$

$\Rightarrow 333^{111} \equiv 4^{111} \pmod 7$

$\equiv (4^3)^{37} \pmod 7$

$\equiv 1^{37} \pmod 7$ as $4^3 \equiv 1 (7)$

$\equiv 1 \pmod 7$

Thus, $111^{333} + 333^{111} \equiv 6+1 \pmod 7 \equiv 0 \pmod 7$

$\Rightarrow 7 \mid 111^{333} + 333^{111}$

[b] Ho: The accidents are uniformly distributed over a week.

Exp. freq. $= E = \frac{1}{7}(13+15+9+11+12+10+14) = \frac{84}{7} = 12$

12 accidents occur in a day.

O	E	$\frac{(O-E)^2}{E}$
13	12	$(13-12)^2/12$
15	12	$(15-12)^2/12$
9	12	$(9-12)^2/12$
11	12	$(11-12)^2/12$
12	12	$(12-12)^2/12$
10	12	$(10-12)^2/12$
14	12	$(14-12)^2/12$
		$\Sigma\ 28/12$

$\therefore \chi^2 = \Sigma \frac{(O-E)^2}{E}$

$= 28/12$

$= 2.33$

$\chi^2_\alpha = 12.59$

as $d.f = 7-1 = 6$

As $\chi^2 < \chi^2_\alpha$

Accept Ho.

$\therefore$ The accidents are uniformly distributed over a week.

[c] (i) $f = (1\ 3\ 2\ 5)(1\ 4\ 3)(2\ 5\ 1)$

$= \begin{pmatrix} 1 & 3 & 2 & 5 \\ 3 & 2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 3 \\ 4 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 & 1 \\ 5 & 1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$$

$$= (1\ 2)(3\ 4\ 5)$$

(ii) $(A+B)(A+B')(A'+B)(A'+B')$

$= (AA + AB' + BA + BB')(A'A' + A'B' + BA' + BB')$

$= (A + A(B+B') + 0)(A' + A'(B'+B) + 0)$

$= (A + A(1))(A' + A'(1))$

$= (A+A)(A'+A') = AA' = 0$

Q.3

[a]

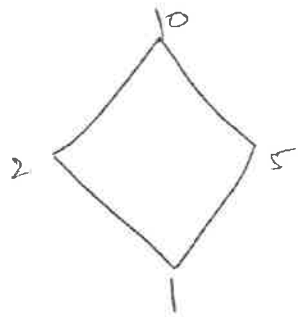
i	q_{i+1}	r_i	x_i	y_i
-1	-	2378	1	0
0	1	1769	0	1
1	2	609	1	-1
2	1	551	-2	3
3	9	58	3	-4
4	2	$\boxed{29}$	-29	39
5	-	0	-	-

$\therefore \gcd(2378, 1769) = 29 = (-29)2378 + (39)1769$

[b] —

[c] Here $D_{10} = \{1, 2, 5, 10\}$

$(D_{10}, \leq) = \{(1,1), (1,2), (1,5), (1,10), (2,10), (2,2), (5,5), (5,10), (10,10)\}$



Here each pair has lub & glb

so $(D_{10}, \leq)$ is a lattice.

Q.5

[a] since $M_0(t) = E(e^{tx})$

$$= \sum_{i=1}^{n} e^{tx_i} p_i$$

$$= \sum_{i=1}^{n} e^{tx_i}\, {}^nC_{x_i}\, p^{x_i} q^{n-x_i}$$

$$= \sum_{i=1}^{n} (e^t p)^{x_i} q^{n-x_i}\, {}^nC_{x_i}$$

$$= (e^t p + q)^n$$

Now, $E(X) = [M_0'(t)]_{t=0}$

$$= \left[\frac{d}{dt}(e^t p + q)^n\right]_{t=0}$$

$$= \left[n(e^t p + q)^{n-1}(e^t p + 0)\right]_{t=0}$$

$$= np$$

Again $V(X) = E(X^2) - (\bar{X}^2)$

Also, $E(X^2) = [M_0''(t)]_{t=0}$

$$= \left[\frac{d^2}{dt^2}(e^t p + q)^n\right]_{t=0}$$

$$= \left[n(n-1)(e^t p + q)^{n-2}(e^t p)^2 + npe^t(e^t p + q)^{n-1}\right]_{t=0}$$

$$= n(n-1)p^2 + np$$

$\therefore V(X) = (n^2p^2 - np^2 + np) - (np)^2$

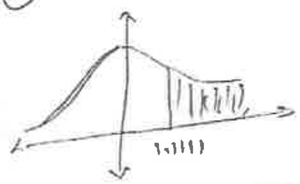
$$= np(1-p) = npq$$

[b] G.t. $\bar{x} = 1200$ & $\sigma = 90$, $N = 2500$

(1) $P(X > 1300) = P(z > 1.1111) = 0.5 - P(0 < z < 1.1111)$

$= 0.5 - 0.3665$

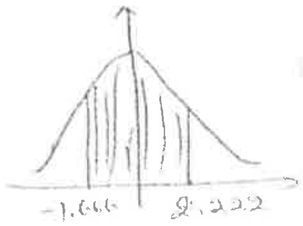
$= 0.1335$



$\therefore$ No. of bulbs $= 2500 \times 0.1335 \simeq 334$

(7)

(ii) $P(1050 < X < 1400) = P\left(-\frac{15}{9} < z < \frac{20}{9}\right)$



$= P(-1.666 < z < 2.2222)$

$= P(0 < z < 1.666) + P(0 < z < 2.2222)$

$= 0.4515 + 0.4868$

$= 0.9283$

$\therefore$ No. of bulbs $= 2500 \times 0.9283 \cong 2321$

[c]

(i) $(a, n) = (8, 77) = 1$

Using Euler's Theorem $a^{-1} \equiv a^{\phi(n)-1} \pmod{n} \rightarrow (1)$

Also, $\phi(n) = \phi(77) \Rightarrow \phi(7 \cdot 11) = 60$

So from (1), $8^{-1} = 8^{60-1} \pmod{77}$

$\Rightarrow 8^{-1} = 8^{59} \pmod{77}$

Again, $8^2 \equiv -13\ (77)$, $8^3 \equiv -27\ (77)$, $8^4 \equiv 15\ (77)$,

$8^5 \equiv 43\ (77)$, $8^{10} \equiv 43^2\ (77) \equiv 1\ (77)$

$\therefore 8^{59} = 8^{50} \cdot 8^5 \cdot 8^4 \equiv 1 \cdot 43 \cdot 15\ (77) = 29\ (77)$

$\therefore 8^{-1} \equiv 29 \pmod{77}$

(ii) $\left(\frac{32}{15}\right) = \left(\frac{32}{3 \cdot 5}\right) = \left(\frac{32}{3}\right)\left(\frac{32}{5}\right) = \left(\frac{2}{3}\right)\left(\frac{2}{5}\right)$

$= (-1)^{\frac{3^2-1}{8}} (-1)^{\frac{5^2-1}{8}}$

$= (-1)^1 (-1)^3 = 1$

Q4.

[a] ~~...~~ Prepare the table.

$\Sigma x = 311$ $\qquad n = 10$

$\Sigma y = 257$

$\Sigma x^2 = 9875$ $\qquad$ Since $r = \dfrac{cov(x,y)}{\sigma_x \sigma_y}$

$\Sigma y^2 = 6763$ $\qquad \bar{x} = \dfrac{\Sigma x}{n} = 31.1$

$\Sigma xy = 8171$ $\qquad \bar{y} = \dfrac{\Sigma y}{n} = 25.7$

$cov(x,y) = \frac{1}{n}\Sigma xy - (\bar{x}\bar{y}) = 17.83$

$\sigma_x^2 = \frac{1}{n}\Sigma x^2 - (\bar{x})^2 = 20.29 \Rightarrow \sigma_x = 4.50$

$\sigma_y^2 = \frac{1}{n}\Sigma y^2 - (\bar{y})^2 = 15.81 \Rightarrow \sigma_y = 3.97$

$\Rightarrow r = 0.9948$

[b] G.T. $G = \{I, (1\,2), (2\,3), (3\,1), (1\,2\,3), (1\,3\,2)\}$

Let $f_1 = I$, $f_2 = (1\,2)$, $f_3 = (2\,3)$ and so on

Also given that $H = \{I, (1\,2)\} = \{f_1, f_2\}$

cosets of H in G:

$f_1 H = \{f_1 f_1, f_1 f_2\} = \{f_1, f_2\} = H$

$f_2 H = \{f_2 f_1, f_2 f_2\} = \{f_2, f_1\} = H$

$f_3 H = \{f_3 f_1, f_3 f_2\} = \{f_3, f_5\}$

$f_4 H = \{f_4 f_1, f_4 f_2\} = \{f_4, f_6\}$

$f_5 H = \{f_5 f_1, f_5 f_2\} = \{f_5, f_3\}$

$f_6 H = \{f_6 f_1, f_6 f_2\} = \{f_6, f_4\}$

Here $G = \{f_1, f_2\} \cup \{f_3, f_5\} \cup \{f_6, f_4\}$

So $[G:H] = 3$

[c] (i) $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{SE} = \dfrac{2}{\sqrt{3}} = 1.15$

where $SE = \sqrt{s_1^2/n_1 + s_2^2/n_2}$ as $\sigma_1 \neq \sigma_2$ not given

$SE = \sqrt{\frac{\geq^2}{32} + \frac{6^2}{36}} = \sqrt{3}$

Now LOS = 5% = 0.05 for TTT

for OTT, LOS = 0.5 − 0.05 = 0.45

$\Rightarrow z_\alpha = 1.64$

Here $|3| > z_\alpha$ ⇒ Accept Ho.

Thus, boys donot perform better than girls.

(ii) $H_0: \mu = 5.4$

$H_1: \mu \neq 5.4$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{6.2 - 5.4}{\sqrt{10.24}/\sqrt{10-1}} = 0.75$

LOS = α = 5% = 0.05

$t_{\alpha,\nu} = t_{0.05,9} = 2.262$

Here $|t| = 0.75$ & $|t| < t_\alpha$

Accept Ho.

∴ mean of population can be regarded as 5.4.

Q.6

80

[a] Here $(3,5) = (5,7) = (7,3) = 1$

Also, $a_1 = 2 \quad m_1 = 3 \Rightarrow M_1 = \frac{n}{m_1} = 35$

$a_2 = 3 \quad m_2 = 5 \quad M_2 = \frac{n}{m_2} = 21$

$a_3 = 2 \quad m_3 = 7 \quad M_3 = \frac{n}{m_3} = 15$

$n = m_1 m_2 m_3 = 105$

$M_1^{-1} \pmod 3 = 35^{-1} \pmod 3 = 35^{3-2} \pmod 3 = 35 \pmod 3$
$\equiv 2 \pmod 3$

$M_2^{-1} \pmod 3 = 21^{-1} \pmod{\cancel{3}\,5} = 21^{5-2} \pmod 5 = 21^3 \pmod 5$
$\equiv 1^3 (5) \equiv \cancel{1}\,1 \pmod 5$

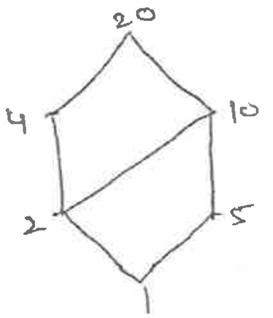
$M_3^{-1} \pmod 3 = 15^{-1} \pmod 7 = 15^{7-2} \pmod 7 = 15^5 \pmod 7$
$\equiv 1^5 \pmod 7$
$\equiv 1 \pmod 7$

$x = a_1 M_1 M_1^{-1} + a_2 M_2 M_2^{-1} + a_3 M_3 M_3^{-1} \pmod{m_1 m_2 m_3}$

$\Rightarrow x = 233 \pmod{105} \equiv 23 \pmod{105}$

[b] $L = \{1, 2, 4, 5, 10, 20\}$

$(L, \le)$ Hasse diagram is

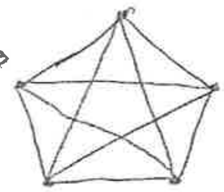


Ele. :	1	2	4	5	10	20
comp. :	20	No comp.	10	No comp.	4	1

This is a distributive lattice as either every ele. has one compliment or no compliment.

This is not a complimented lattice as ele. 2 & 5 ~~does~~ not have compliment.

[c] (i) ~~—~~ $K_5 \rightarrow$



(ii) —

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