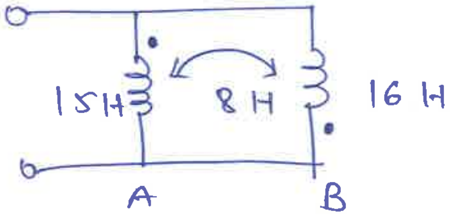
Q P code. 22709

Solution set

1 ⓐ



$$L_A = L_1 - M_{12} = 15 - 8 = 7\ H$$

$$L_B = L_2 - M_{12} = 16 - 8 = 8\ H$$

$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B}$$

$$= \frac{1}{7} + \frac{1}{8} = \frac{15}{56}$$

$$L = \frac{56}{15} = 3.73\ H$$

ⓑ

$$m(s) = s^4 + 6s^2 + 8$$

$$n(s) = 7s^3 + 21s$$

$$7s^3 + 21s \,\big)\, s^4 + 6s^2 + 8 \,\big(\, \frac{1}{7}s$$

$$s^4 + 3s^2$$

$$3s^2 + 8 \,\big)\, 7s^3 + 21s \,\big(\, \frac{7}{3}s$$

$$7s^3 + \frac{56s}{3}$$

$$\frac{7}{3}s \,\big)\, 3s^2 + 8 \,\big(\, \frac{9}{7}s$$

$$3s^2$$

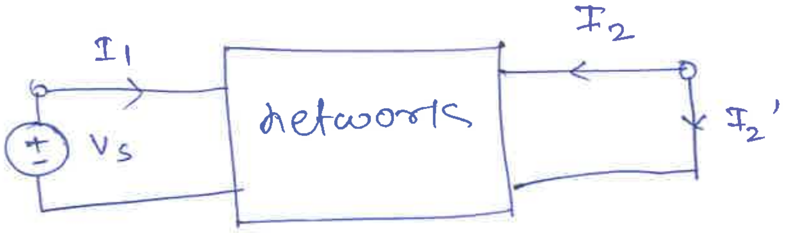
$$8 \,\big)\, \frac{7}{3}s \,\big(\, \frac{7}{24}s$$

$$\frac{7}{3}s$$

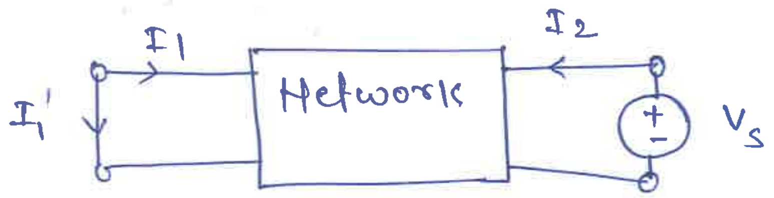
$$0$$

since all the quotient terms are positive, the polynomial P(S) is Hurwitz.

ⓒ



$$\frac{V_s}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$



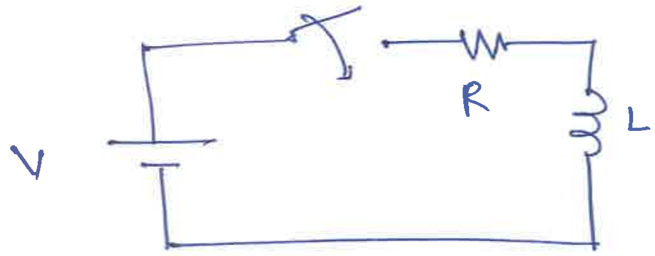
$$\frac{V_s}{I_1'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}$$

For the network to be reciprocal

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

$$\boxed{Z_{12} = Z_{21}}$$

(d)



$$V - Ri - L\frac{di}{dt} = 0$$

$$\frac{L\,di}{V - Ri} = dt$$

Integrating both the sides

$$-\frac{L}{R}\ln(V - Ri) = t + K \quad ①$$

setting $i = 0$ at $t = 0$

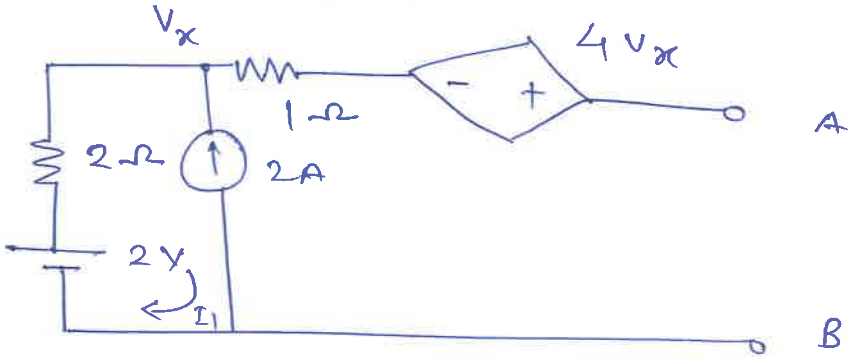
$$\boxed{-\frac{L}{R}\ln V = K} \text{ put in } ①$$

$$-\frac{L}{R}\left[\ln(V - Ri)\right] = t - \frac{L}{R}\ln V$$

$$\frac{V - Ri}{V} = e^{-\frac{R}{L}t}$$

$$\boxed{i = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}}$$

2a



Step-I Calculation of V_{TH}

$$2-2I_1-V_x=0$$

$$V_x = 2-2I_1$$

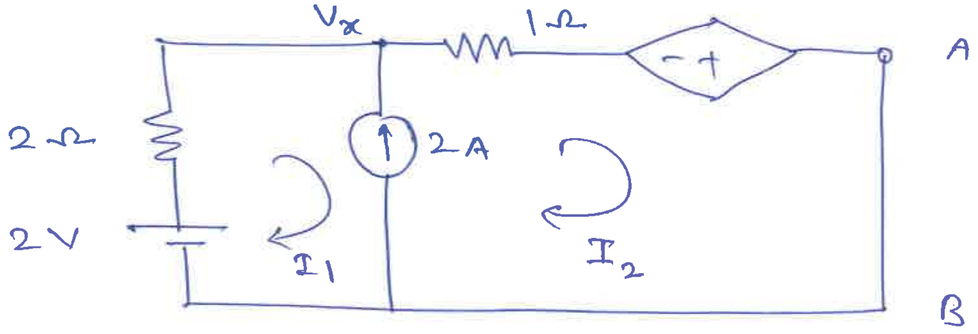
For mesh I $\quad I_1 = -2\ A$

$$V_x = 2-2(-2) = 6V$$

$$\therefore \quad 2-2I_1-0+4V_x-V_{TH}=0$$

$$V_{TH} = 30V.$$

Step-II Calculation of I_N



$$V_x = 2-2I_1$$

$$I_2 - I_1 = 2$$

Applying KVL

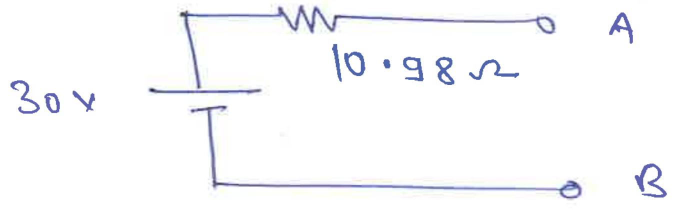
$$2-2I_1-I_2+4V_x=0$$

$$10I_1+I_2=10$$

$$I_1 = 0.73 \quad I_2 = 2.73\ A$$

$$I_N = 2.73\ A$$

$$R_{TH} = \frac{V_{TH}}{I_N} = \frac{30}{2.73} = 10.98\Omega$$

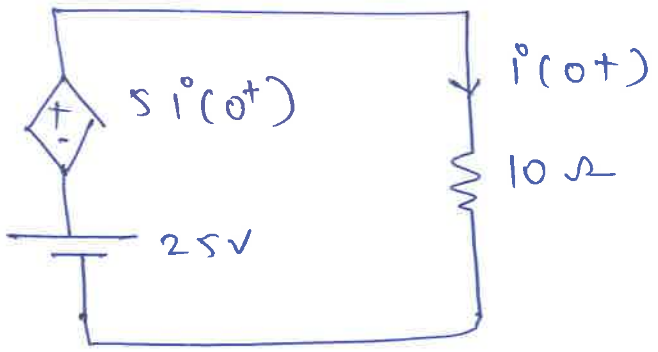


2b

At $t = 0^-$ $\quad i(0^-) = \frac{100}{10+10} = 5\,A$

$$V_c(0^-) = 100 - 10(i(0^-)) - 5\,i(0^-)$$
$$= 100 - 50 - 25 = 25\,V$$

At $t = 0^+$ the network is as shown below



$$25 + 5\,i(0^+) - 10\,i(0^+) = 0$$
$$i(0^+) = 5\,A$$

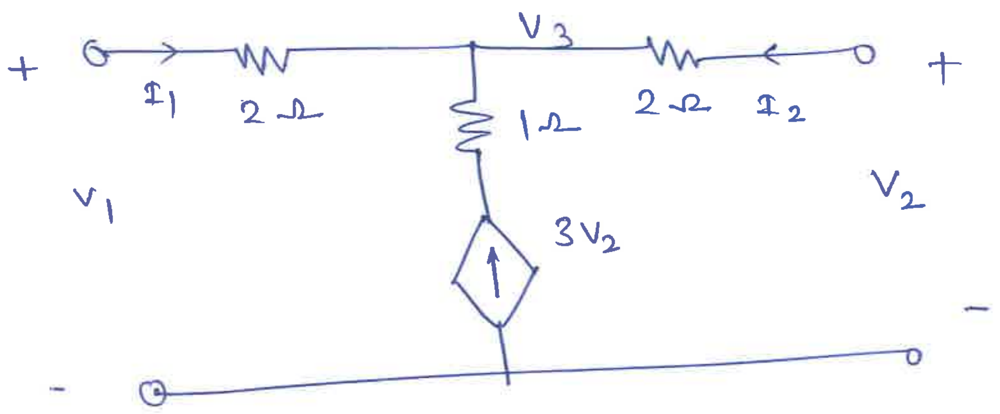
For $t > 0$

$$25 - \frac{1}{4\times10^{-6}}\int_0^t i\,dt + 5i - 10i = 0$$

$$\frac{di}{dt} + 5000\,i = 0$$

$$\boxed{i(t) = e^{-50000t} \quad \text{for } t > 0}$$

3a



$$I_1 = \frac{V_1 - V_3}{2}$$

$$= \frac{V_1}{2} - \frac{V_3}{2}$$

$$I_2 = \frac{V_2 - V_3}{2}$$

$$= \frac{V_2}{2} - \frac{V_3}{2}$$

writing KCL at node

$$I_1 + I_2 + 3V_2 = 0$$

putting in above equation

$$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} + 3V_2 = 0$$

$$V_3 = \frac{V_1}{2} + \frac{7}{2}V_2$$

$$\therefore \quad I_1 = \frac{V_1}{4} - \frac{7}{4}V_2$$

$$I_2 = \frac{V_1}{4} - \frac{5}{4}V_2$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{-7}{4} \\ \frac{-1}{4} & \frac{-5}{4} \end{bmatrix}$$

*

3b

Foster I form

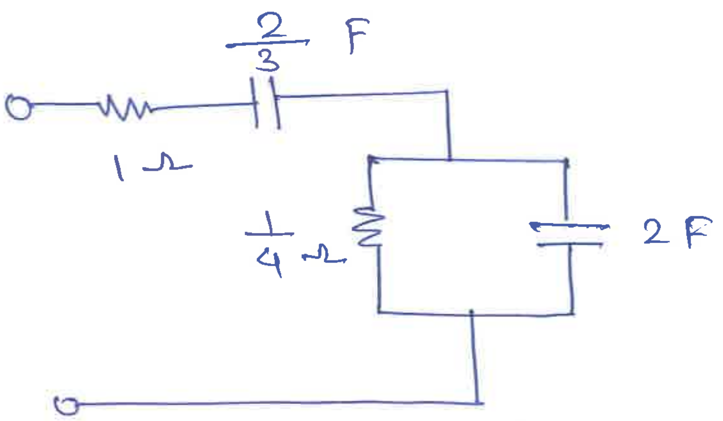
$$Z(s) = \frac{s^2+4s+3}{s^2+2s}$$

$$\begin{array}{r|l} s^2+2s & s^2+4s+3 \;(\,1 \\ & \underline{s^2+2s} \\ & 2s+3 \end{array}$$

$$Z(s) = 1 + \frac{2s+3}{s^2+2s}$$

$$Z(s) = 1 + \frac{K_1}{s} + \frac{K_2}{s+2}$$

$$Z(s) = 1 + \frac{3/2}{s} + \frac{\frac{1}{2}}{s+2}$$



Caur - II

$$Z(s) = \frac{3+4s+s^2}{2s+s^2}$$

By continued fraction method

$$2s+s^3 \overline{)\, 3+4s+s^2 \,}\left(\, 3/2s\right.$$

$$3+\frac{3}{2}s$$

$$\frac{5}{2}s+s^2 \overline{)\, 2s+s^2 \,}\left(\, \frac{4}{5}\right.$$

$$2s+\frac{4}{5}s^2$$

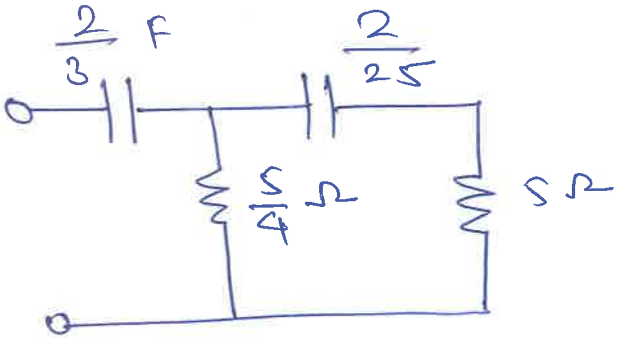
$$\frac{1}{5}s^2 \overline{)\, \frac{5}{2}s+s^2 \,}\left(\, \frac{25}{2s}\right.$$

$$\frac{5}{2}s$$

$$s^2 \overline{)\, \frac{s^2}{5} \,}\left(\, \frac{1}{5}\right.$$

$$\frac{1}{5}s^2$$

$$0$$



4a

$$F(s) = \frac{s^3 + 8s^2 + 15s}{s^2 + 5s + 4}$$

The function has poles at $s=-1$ and $s=-4$ and zeros are at $s=0$, $s=-3$ and $s=-5$

ⓑ There is no pole on $j\omega$ axis, hence residue test is not required

ⓒ $m_1 = 8s^2 \quad n_1 = s^3 + 15s$

$m_2 = s^2 + 4, \quad n_2 = 5s$

$$A(\omega^2) = 3s^4 - 43s^2 \Big|_{s=j\omega}$$

$$= 3\omega^4 + 43\omega^2$$

$A(\omega^2)$ is positive for all $\omega \geq 0$

All conditions are satisfied therefore the function is positive real.

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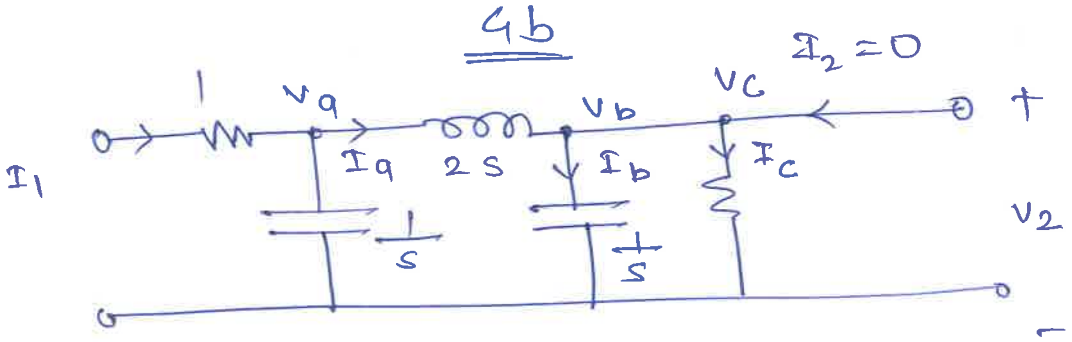
4b



$V_c = V_b = V_2$

$I_a = sV_2 + V_2 = (s+1)V_2$

$V_a = 2s\,I_a + V_2$

$= (2s^2 + 2s + 1)V_2$

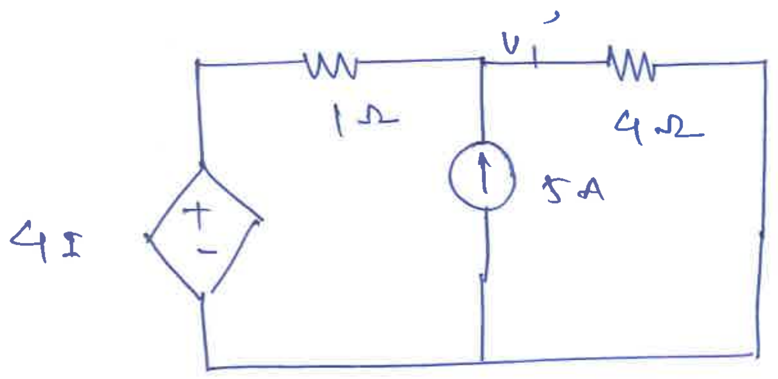
$$I_1 = \frac{V_a}{\frac{1}{s}} + I_a = sV_a + I_a$$

$= (2s^3 + 2s^2 + 2s + 1)V_2$

$V_1 = I_1 + V_a = (2s^3 + 2s^2 + 2s + 1)V_2 + (2s^2 + 2s + 1)V_2$

$$\boxed{\frac{V_2}{V_1} = \frac{1}{2s^3 + 4s^2 + 4s + 2}}$$

4.c



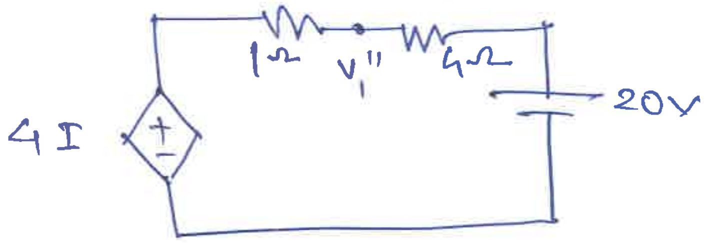
$I = \frac{v_1'}{4}$

$\frac{v_1' - 4I}{1} + \frac{v_1'}{4} = 5$ (kcl at node 1)

$v_1' - 4\left(\frac{v_1'}{4}\right) + \frac{v_1'}{4} = 5$

$v_1' = 20V$

Step - II when 20V source is acting alone



$4I - I - 4I - 20 = 0$

$I = -20 A$

$v_1'' = v_1' + v_1'' = 20 - 60 = -40V$

5a

At $t = 0^-$ no current flows the inductor and hence there is no voltage across the capacitor

$i_L(0^-) = 0 \quad v(0^-) = 0$

At $t = 0+$, $i_L(0+) = 0 \quad v(0+) = 0$

for $t > 0$

$$\frac{V}{2} + \int_0^t v\,dt + 0.5\times10^{-6}\frac{dv}{dt} = 10 \quad — ①$$

At $t = 0+$

$$\frac{v(0+)}{2} + 0 + 0.5 \times 10^{-6} \frac{dv}{dt}(0+) = 10$$

$$\frac{dv}{dt}(0+) = 20 \times 10^{6} \ v|s$$

differentiating eqn ①

$$\frac{1}{2}\frac{dv}{dt} + v + 0.5 \times 10^{-6} \frac{d^2v}{dt^2} = 0$$

At $t = 0+$

$$\frac{1}{2}\frac{dv}{dt}(0+) + v(0+) + 0.5 \times 10^{-6} \frac{d^2v}{dt^2} v(0+) = 0$$

$$\frac{d^2v}{dt^2}(0+) = -20 \times 10^{12} \ v|s^2$$

* * *

5b

$$Z = R + j\omega L = 15.07 \angle 66.54 \ \Omega|km$$

$$Y = G + j\omega C = 0.25 \times 10^{-6} + j 2\pi \times 1000 \times 0.005 \times 10^{-6}$$

$$= 3.14 \times 10^{-5} \angle 89.54^\circ \ \mho|km$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = 692.77 \angle -11.5^\circ \ \Omega$$

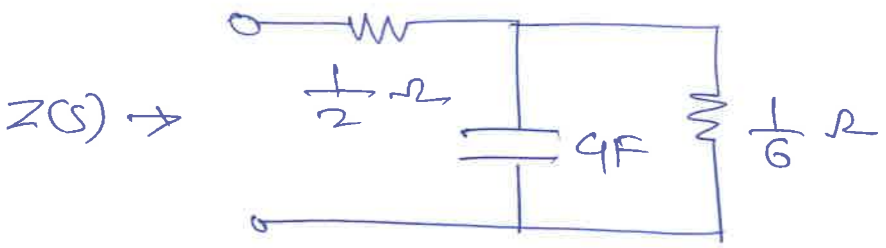
$$\gamma = \sqrt{Z \cdot Y} = (4.55 \times 10^{-3} + j\, 0.022)|km$$

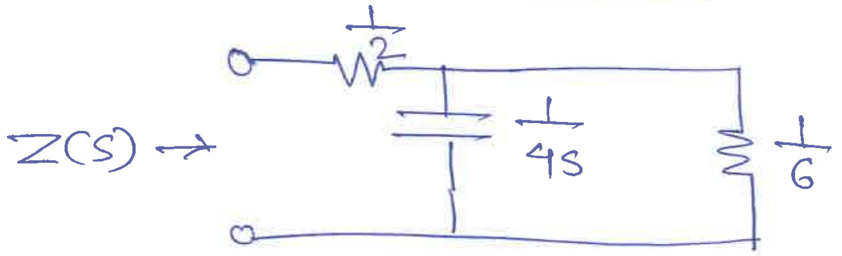
$$\alpha = 4.55 \times 10^{-3} \text{ neper } |km$$

$$\beta = 0.022 \text{ rad } |km$$

* * *

5C





$$Z(s) = \frac{1}{2} + \frac{\frac{1}{4s}\times\frac{1}{6}}{\frac{1}{4s}+\frac{1}{6}}$$

$$= \frac{4s+8}{2(4s+6)} = \frac{s+2}{2s+3}$$

$$= \frac{0\cdot5(s+2)}{s+1\cdot5}$$

The function $Z(s)$ has zeros at $s=-2$, and pole at $s=-1\cdot5$

6a

$Z_0 = 75\Omega$, $Z_L = 150 + j150\,\Omega$

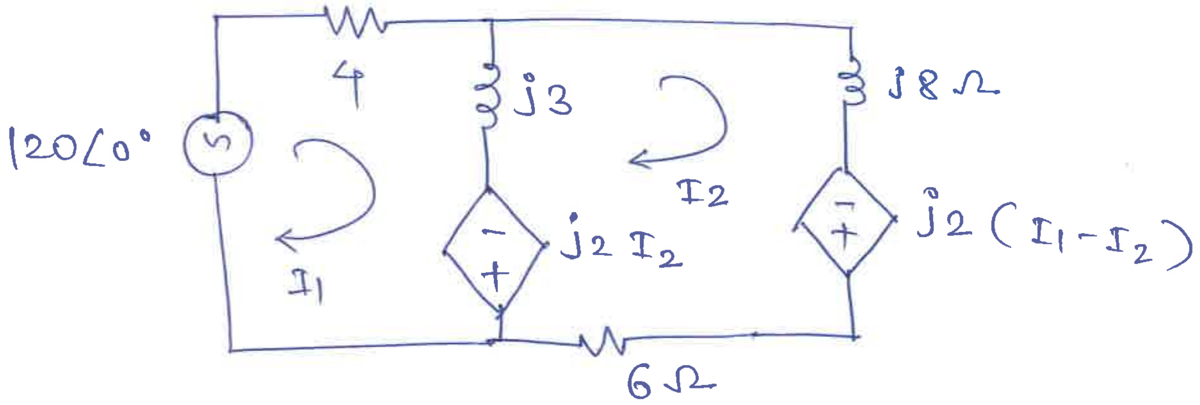
$$\bar{Z}_L = \frac{150+j150}{75} = 2+j2$$

$VSWR = 4\cdot2$

$\Gamma = 0\cdot62 \angle 30^\circ$

6b

The equivalent circuit in terms of dep. sources is shown below



Applying KVL to mesh - I

$$120\angle0 - 4I_1 - j3(I_1-I_2) + j2I_2 = 0$$

$$(4+j3)I_1 - j5I_2 = 120\angle0$$

Applying KVL to mesh 2

$$-j2I_2 - j3(I_2 - I_1) - j8I_2 + j2(I_1 - I_2) - 6I_2 = 0$$

$$-j5I_1 + (6 + j15)I_2 = 0$$

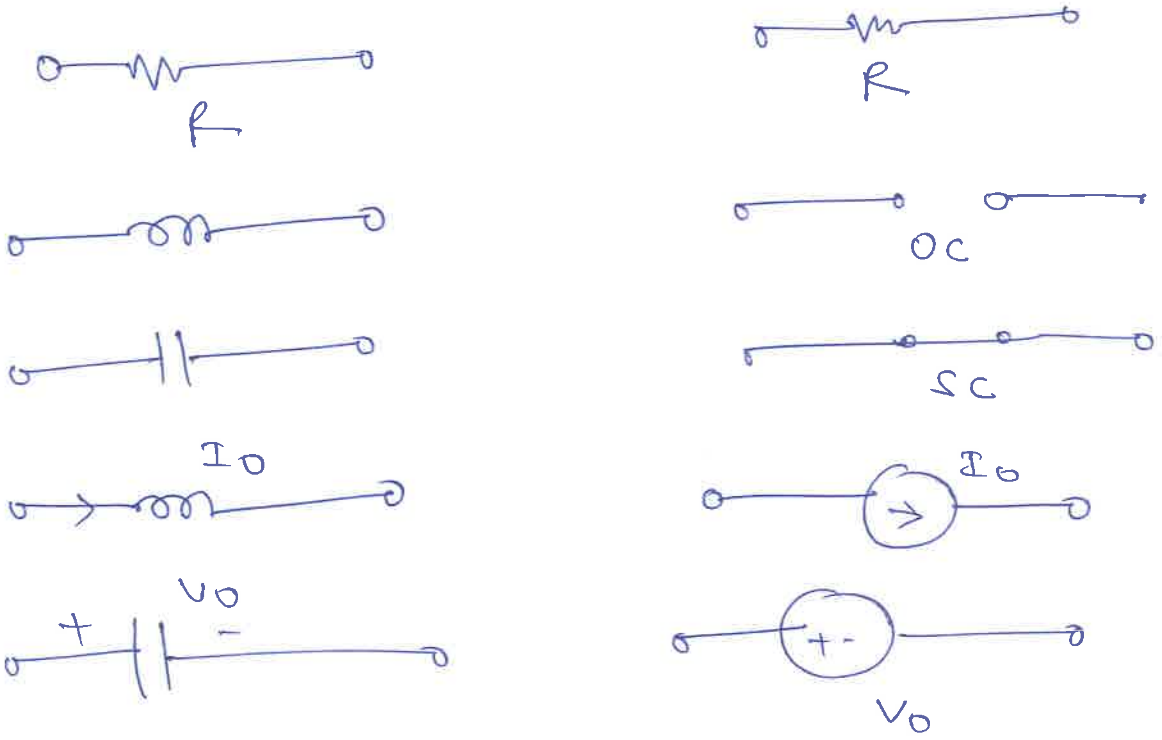
writing equations in matrix form

$$\begin{bmatrix} 4+j3 & -j5 \\ -j5 & 6+j15 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 120\angle 0^\circ \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 4+j3 & 120\angle 0 \\ -j5 & 0 \end{vmatrix}}{\begin{vmatrix} 4+j3 & -j5 \\ -j5 & 6+j15 \end{vmatrix}} = 7.68\angle 2.94^\circ \text{ A}$$

* * *

(c) Elements at initial conditions — circuit at $t = 0^+$



elements with initial conditions

circuit at t=∞