Elementary Signals

Unit step signal

Ramp Signal

Unit impulse function: Amplitude of unit impulse approaches 1 as the width approaches zero and it has zero value at all other values.

Sinusoidal signal: A continuous time sinusoidal signal is given by,

Exponential signal:

b. Find the fundamental period of the signal

$$x(t) = \sin\left(\frac{2\pi}{6}t\right) - \cos(\pi t)$$

Find the fundamental period of the signal

$x(t) = \sin\left(\frac{2\pi}{6}t\right) - \cos(\pi t)$

Ans: $x(t) = x_1(t) + x_2(t)$

$x_1(t) = \sin\left(\frac{2\pi}{6}t\right)$

$x_1(t) = \sin\left(\frac{\pi}{3}t\right)$

$\frac{2\pi}{6} = \omega_1 = 2\pi f_1$

$f_1 = \frac{1}{6}$

$\underline{t_1 = 6 \text{ sec}}$

$x_2(t) = \cos(\pi t)$

$\pi = 2\pi f_2$

$\frac{1}{2} = f_2$

$t_2 = \frac{1}{f_2} = 2$

$\underline{t_2 = 2 \text{ sec}}$

$\frac{t_1}{t_2} = \frac{6}{2} = 3$

It is a rational no, so the resultant waveform is periodic.

∴ The fundamental period LCM[6,2] = 6 sec.

02

f. If system matrix A= [-3, 1 ; -2, 0] find the sate transition matrix.

$$\dot{q} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} q + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x(t)$$

$$(sI - A) = \begin{bmatrix} (s+3) & 1 \\ 2 & s \end{bmatrix}$$

The STM is

$$\phi(s) = [sI - A]^{-1} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$Q(s) = [sI - A]^{-1} q(0) + [sI - A]^{-1} BX(s)$$

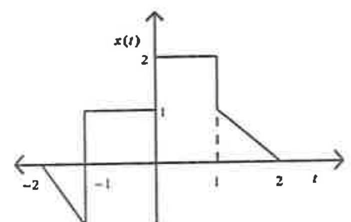
$$(sI - A)^{-1} q(0) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} (s-1) \\ -(s+5) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{(s-1)}{(s+1)(s+2)} \\ \dfrac{(s+5)}{(s+1)(s+2)} \end{bmatrix}$$

Q2.a. Sketch the following signals for the given signal shown in Fig. 2 10

a) x(-t) b) x(2t+5) c) x(2t) d) x(t/2) e) -2x(t)



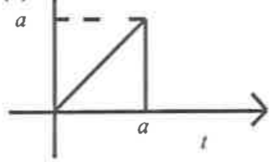


Fig. 1

find $x(-2t)$ and $x(3t+2)$

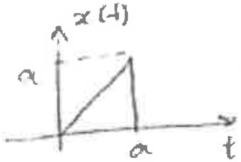
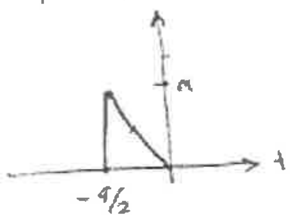


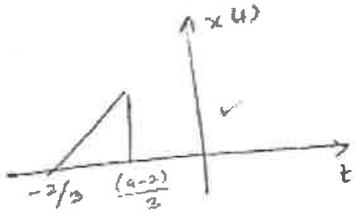
fig 1

Ans -

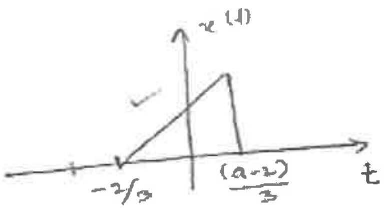
for $x(-2t)$



case 1 $a < 3$



case 2 $a > 3$



e. Test the given system for linearity, causality, stability, memory and time variant.

$$y(t) = x(t^2)$$

$y(t) = x(t^2)$

Linear

Non causal

Stable

Dynamic

Time Variant

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$$(s+1)(s+2)\begin{bmatrix} -2 & s+3 \end{bmatrix}$$

$$Q(s) = [sI - A]^{-1}q(0) + [sI - A]^{-1}BX(s)$$

$$(sI - A)^{-1}q(0) = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)}\begin{bmatrix} (s-1) \\ -(s+5) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{(s-1)}{(s+1)(s+2)} \\ -\dfrac{(s+5)}{(s+1)(s+2)} \end{bmatrix}$$

Q2.a. Sketch the following signals for the given signal shown in Fig. 2 — 10

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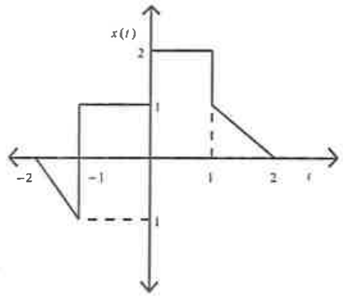

Fig. 2

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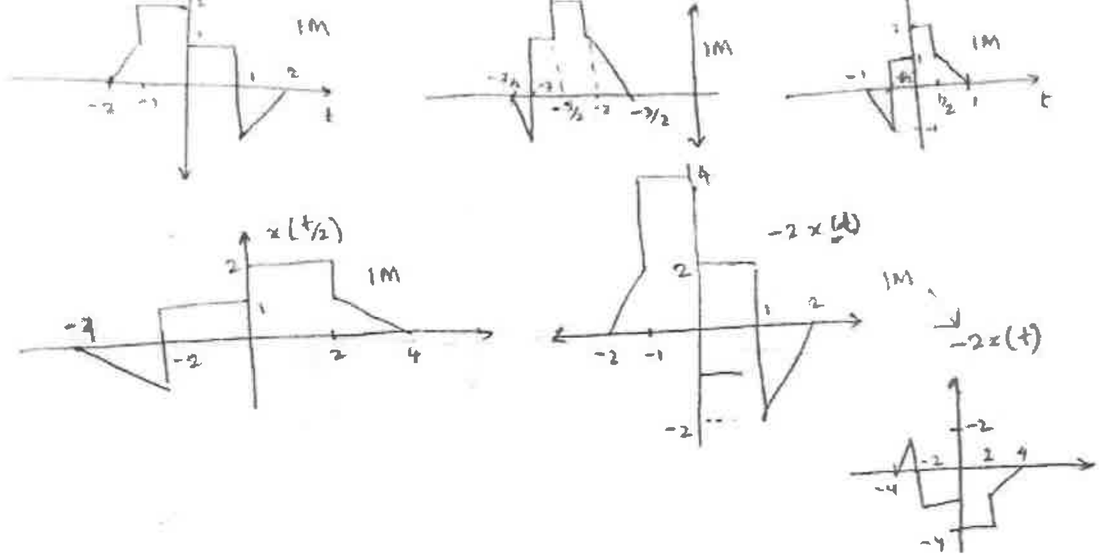

b. Find Inverse Laplace transform using convolution 10

$$L^{-1}=\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\}$$

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Now by applying convolution Theorem,

$$L^{-1}\{F_1(s) * F_2(s)\} = \int_0^t F_1(u)\, F_2(t-u)\,du$$

$$= \int_0^t \cos au \cos b(t-u)\,du$$

$$= \int_0^t \cos au \cdot \cos(bt - bu)\,du$$

$$= \int_0^t \frac{1}{2}[\cos(au + bt - bu) + \cos(au - bt + bu)]\,du$$

$$= \frac{1}{2}\int_0^t [\cos\{(a-b)u + bt\} + \cos\{(a+b)u - bt\}]\,du$$

$$= \frac{1}{2}\left[\frac{\sin\{(a-b)u + bt\}}{a-b} + \frac{\sin\{(a+b)u - bt\}}{a+b}\right]_0^t$$

$$= \frac{1}{2}\left[\left[\frac{\sin(at - bt + bt)}{a-b} + \frac{\sin(at + bt - bt)}{a+b}\right] - \left[\frac{\sin bt}{a-b} + \frac{\sin(-bt)}{a+b}\right]\right]$$

$$= \frac{1}{2}\left[\frac{\sin at}{a-b} + \frac{\sin(+at)}{a+b} - \frac{\sin bt}{a-b} + \frac{\sin bt}{a+b}\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{a-b} + \frac{1}{a+b}\right)\sin at - \left(\frac{1}{a-b} - \frac{1}{a+b}\right)\sin bt\right]$$

$$= \frac{1}{2}\left[\frac{2a}{a^2-b^2}\sin at - \left(\frac{2b}{a^2-b^2}\sin bt\right)\right]$$

$$= \frac{2}{2(a^2-b^2)}[a\sin at - b\sin bt]$$

$$\therefore\ f(t) = \frac{1}{a^2-b^2}(a\sin at - b\sin bt)$$

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$$= 1$$

Residue of $G(z)$ at $(z = e^{-a}) = R_{z=e^{-a}} = (z - e^{-a})G(z)|z = e^{-a}$

$$= \frac{(1-e^{-a})z^n}{(z-1)}|z = e^{-a}$$

$$= -e^{-an}$$

Therefore, $x[n] = 1 - e^{-an}; n \geq 0$

b. State and Prove Parseval's Theorem with respect to DTFT. 10

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Interchanging the order of summation and integration

$$\sum_{n=-\infty}^{\infty} x_1(n) \cdot \overline{x_2(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\{ \overline{X_2(e^{j\omega})} \left[\sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n} \right] \right\} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \overline{X_2(e^{j\omega})}\, X_1(e^{j\omega})\, d\omega$$

Thus. $$\sum_{n=-\infty}^{\infty} x_1(n) \cdot \overline{x_2(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\omega}) \cdot \overline{X_2(e^{j\omega})}\, d\omega \;\ldots\ldots\ldots\ldots (6.12)$$

For the special case when $x_2(n) = x_1(n)$ for all n, we may write

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega \;\ldots\ldots\ldots\ldots\ldots (6.13)$$

We know that LHS of the above equation is the energy E_x of the discrete-time signal $x(n)$. Hence analogous to the way we interpreted $|X(f)|^2$ in the case of Parseval's theorem, for continuous-time Fourier transform, here also, we call $\left|X(e^{j\omega})\right|^2$ it as Energy spectral density of the signal $x(n)$ and denote it by $S_{xx}(\omega)$. Therefore,

$$E_x = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(\omega)\, d\omega \;\ldots\ldots\ldots\ldots (6.14)$$

$S_{xx}(\omega)$, is the Energy spectral density, which contains the information about how the energy of $x(t)$ is distributed with respect to frequency.

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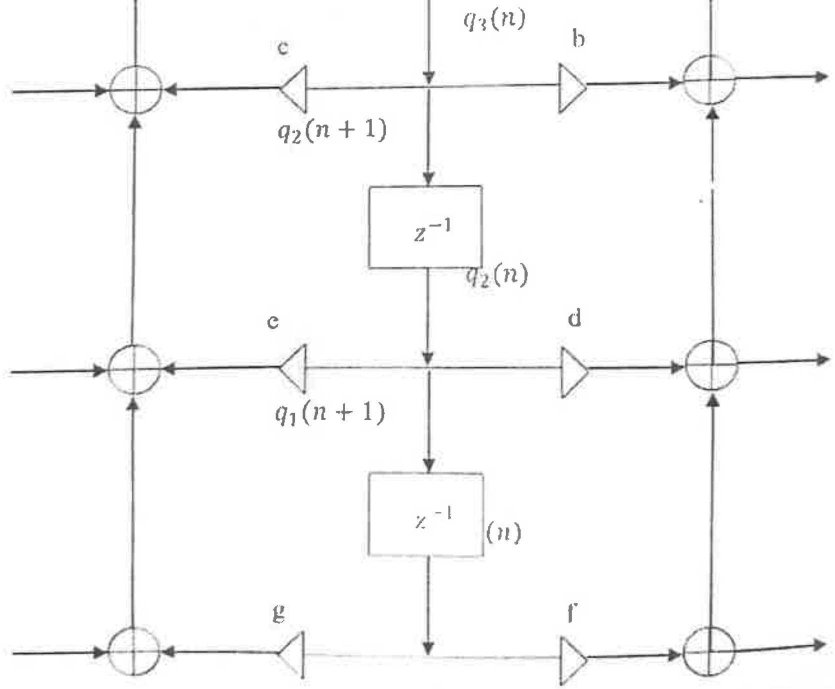

Where, $a = 2$, $b = 1.5$, $c = -2$, $d = 2.5$, $e = 3$, $f = 4$ and $g = 0.5$

$$q_3(n+1) = X(n) + 0.5q_1(n) + 3q_2(n) - 2q_3(n)$$

Arranging equations in matrix form,

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \\ q_3(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 3 & -2 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \\ q_3(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x(n) \ldots\ldots\ldots$$

Output equation $y[n]$ is formed by equating incoming signals of output node point as shown below:

$$y(n) = 4q_1(n) + 2.5q_2(n) + 1.5q_3(n) + 2q_3(n+1)$$

We have $q_3(n+1)$ from

$$y(n) = 4q_1(n) + 2.5q_2(n) + 1.5q_3(n) + 2(x(n)$$

$$+0.5q_1(n) + 3q_2(n) - 2q_3(n)$$

$$y(n) = 5q_1(n) + 8.5q_2(n) - 2.5q_3(n) + 2X(n)$$

Writing in the matrix form,

$$y(n) = \begin{bmatrix} 5 & 8.5 & -2.5 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \\ q_3(n) \end{bmatrix} + [2]X(n)$$

State equations: $Q(n+1) = AQ(n) + BX(n)$

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \\ q_3(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 3 & -2 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \\ q_3(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x(n)$$

Where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Output equations: $Y(n) = C\,Q(n) + D\,X(n)$

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$$= \frac{2}{\pi}\left[\frac{-x^3 \cos nx}{n} + \frac{6x \cos nx}{n^3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-\pi^3 \cos n\pi}{n} + \frac{6\pi \cos n\pi}{n^3} - 0\right]$$

$$= \frac{2}{\pi}\frac{\pi \cos n\pi}{n}\left[-\pi^2 + \frac{6}{n^2} - 0\right]$$

$$= \frac{2(-1)^n}{n}\left[-\pi^2 + \frac{6}{n^2}\right]$$

Hence Fourier series is as follows.

$$f(x) = 0 + \sum_{n=1}^{\infty}\left[0 + \frac{2(-1)^n}{n}\left(-\pi^2 + \frac{6}{n^2}\right)\sin nx\right]$$

$$\therefore x^3 = \sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\left(-\pi^2 + \frac{6}{n^2}\right)\sin nx$$

Q5.a Determine DTFS for the sequence x(n)= $\cos^2((\pi/8)n)$ 8

Solution:

$$x(n) = \frac{1 + cos^2\left(\frac{\pi}{4}n\right)}{2}$$

$$x(n) = \frac{1}{2} + \frac{1}{2}cos\left(\frac{\pi}{4}n\right)$$

Here fundamental period is 8

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2}\right)$$

$$a_0 = \frac{1}{2}, \quad a_1 = \frac{1}{4}, \quad a_{-1} = \frac{1}{4},$$

$$a_0 = \frac{1}{2}, \quad a_{-1} = a_{-1+8} = a_7 = \frac{1}{4}, \quad a_1 = \frac{1}{4},$$

Rest

$$a_k = 0, \qquad 0 \le k \le 7$$

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The symmetry property of a linear-phase FIR filter can be used to reduce the number of the required multiplications.

The main advantage of the cascade structure is its smaller sensitivity to the coefficient quantization.

We can use the *tf2sos* command to rewrite the system function of a FIR filter as the product of second-order sections; however, these second-order sections are not necessarily linear-phase. To have a cascade of linear-phase elements, we need to group the zeros of the transfer function appropriately.

Q6. Write short note on any two:

a. Relation of ESD, PSD with auto-correlation 5

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2. **Energy Spectral Density (ESD)**

- Defined as $\Psi_x(f) = |X(f)|^2$.
- Measures the distribution of signal energy $E = \int |x(t)|^2 dt = \int \Psi_x(f) df$ over frequency.
- Properties of ESD include $\Psi_x(f) \geq 0$, $\Psi_x(-f) = \Psi_x(f)$ for $x(t)$ real, and for $x(t)$ input to a filter with frequency response $H(f)$, the filter output $y(t)$ has ESD $\Psi_y(f) = |H(f)|^2 \Psi_x(f)$.

3. **Autocorrelation of Energy Signals**

- Defined for real signals as $R_x(\tau) = \int x(t)x(t-\tau)dt = x(\tau) * x(-\tau)$.
- Measures the similarity of a signal with a delayed version of itself.
- Autocorrelation defines signal energy: $E = R_x(0)$.
- Since $|R_x(\tau)| \leq R_x(0)$, can use autocorrelation for signal synchronization.
- The autocorrelation is symmetric: $R_x(\tau) = R_x(-\tau)$.
- The autocorrelation and ESD are Fourier Transform pairs: $R_x(\tau) \Leftrightarrow \Psi_x(f)$.

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4. Power Spectral Density (PSD)

- Power signals have infinite energy: Fourier transform and ESD may not exist.
- Power signals need alternate spectral density definition with similar properties as ESD.
- Can obtain ESD for a power signal $x(t)$ that is time windowed with window size $2T$.
- PSD defined as the normalized limit of the ESD for the windowed signal $x_T(t)$: $S_x(f) = \lim_{T\to\infty} \frac{1}{2T}|X_T(f)|^2$.
- PSD measures the distribution of signal power $P = \lim_{T\to\infty} \frac{1}{2T} \int |x_T(t)|^2 dt = \int S_x(f) df$ over frequency domain.

5. **Properties of PSD**

- $S_x(f) \geq 0$
- $S_x(-f) = S_x(f)$

6. **Filtering and Modulation of Power Signals:**

- Let $x(t)$ be a power signal with PSD $S_x(f)$.
- If $x(t)$ is input to a filter with frequency response $H(f)$, then the filter output $y(t)$ has PSD $S_y(f) = |H(f)|^2 S_x(f)$.
- If $S_x(f)$ is bandlimited with bandwidth $B << f_c$, then for $z(t) = x(t)\cos(2\pi f_c t)$, $S_z(f) = .25[S_x(f-f_c) + S_x(f+f_c)]$.

b. ROC in Z-Transform and Laplace Transform

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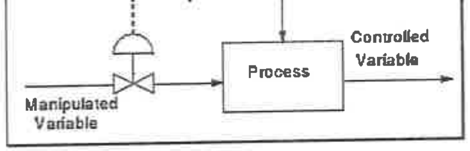

Using feedforward control the performance of control systems can be enhanced greatly.

Process variables such as pressure, level, flow, temperature are interrelated and so one variable may affect another as a disturbance in the process. Feedforward system measure important disturbance variables and take corrective action before they upset the process.

Here the setpoint is fixed in the feedforward controller after doing little complex mathematical derivations. The feedforward controller determines the needed change in the manipulated variable, so that, when the effect of the disturbance is combined with the effect of the change in the manipulated variable, there will be no change in the controlled variable at all. The disturbance is measured at the input side of the process and the manipulating variable also, so the controlling process is done before a disturbance affects the process.