① Solution Paper – I Subject: Signal & System (KT.

Q.P. Code: 39055

Q.1 ⓑ (i) $x(t) = 3 + 2t + 5t^2$

3 mark

even part :- $3 + 5t^2$ , odd part :- $2t$.

(ii) $x(t) = e^t$

2 mark

even part :- $\frac{1}{2}[e^t + e^{-t}]$ odd part: $-\frac{1}{2}(e^t - e^{-t})$

ⓔ $s(t) = u(t) + e^{-2t}u(t)$

$S(s) = \frac{2s+2}{s(s+2)}$

5 mark

T.F. = $H(s) = \frac{S(s)}{U(s)}$

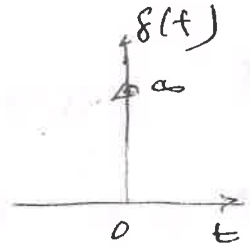
$U(s) = \frac{1}{s}$

$= \frac{2s+2}{s+2}$

Q.2 ⓐ i) Impulse signal $\delta(t) = \infty \quad t = 0$

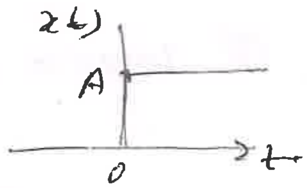
$= 0 \quad t \neq 0$

$\int_{-\infty}^{\infty} \delta(t)\, dt = A$



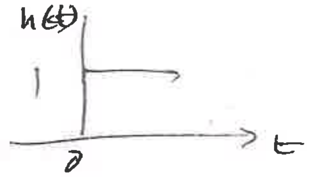

ii) Step signal $x(t) = A \quad t \geq 0$

$= 0 \quad t < 0$



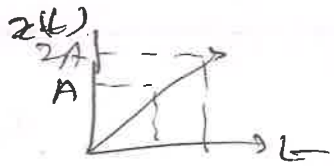

$x(t)\ u(t) = 1 \quad t \geq 0$

$0 \quad t < 0$



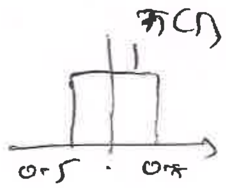

iii) Ramp signal $x(t) = At \quad t \geq 0$

$= 0 \quad t < 0$




(iv) Unit pulse signal

$x(t) = \Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$



Q.2 b (03)

$a_0 = 0,$

$a_n = 0$

$b_n = \frac{4}{T} \int_0^{T/2} x(t) \sin \Omega_0 t \, dt$

$x(t) = A$ : for $t = 0$ to $T/2$

$-A$ for $t = T/2$ to $T$

$b_n = 0$ for even value of $n$

$b_n = \frac{4A}{T}\left[\frac{T}{2n\pi} + \frac{T}{2n\pi}\right] = \frac{4A}{n\pi}$ for odd values of $n$.

Q.4 a

2½ for each

i) $x(t) = \sin \Omega_0 t\, u(t)$

$= \mathcal{L}\{\sin \Omega_0 t\, u(t)\} = \frac{\Omega_0}{s^2 + \Omega_0^2}$

ii) $x(t) = \cos \Omega_0 t\, u(t) = \frac{s}{s^2 + \Omega_0^2}$

Q.4 b

$X(s) = \frac{1}{s+2}$ $Y(s) = \frac{-2}{s+1} + \frac{2}{s+3}$

(i) T.F. $H(s) = \frac{Y(s)}{X(s)} = \frac{-4s-8}{(s+1)(s+3)}$

(ii) Impulse Response :-

$H(s) = \frac{A_1}{s+1} + \frac{A_2}{s+3}$

$A_1 = -2, \quad A_2 = -3$

$A_1 = (s+1)(H(s)) = (s+1)\left(\frac{-4s-8}{(s+1)(s+3)}\right)\Big|_{s=-1} = -2$

$\therefore H(s) = \frac{-2}{s+1} - \frac{2}{s+3}$

Taking inverse Laplace

$h(t) = -2e^{-t}u(t) - 2e^{-3t}u(t)$

Q.5 (a) $y(n) = x(n) + 0.8x(n-1) + 0.8x(n-2) - 0.49y(n-2)$ (04)

taking Z-transform

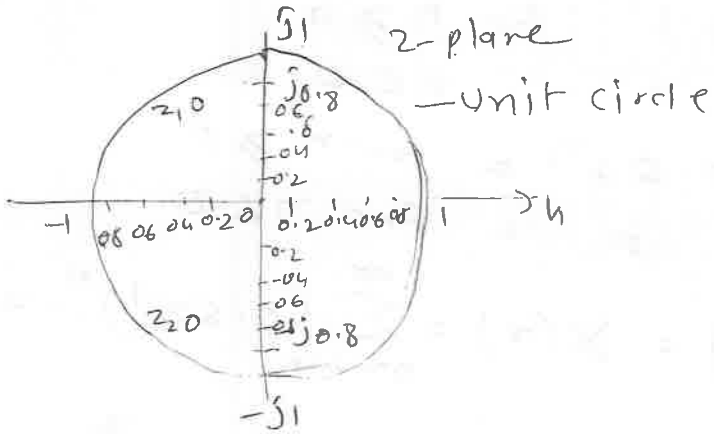
$$Y(z) = X(z) + 0.8z^{-1}X(z) + 0.8z^{-2}X(z) - 0.49z^{-2}Y(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1+0.8z^{-1}+0.8z^{-2}}{1+0.49z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2+0.8z+0.8}{z^2+0.49}$$

Poles $P_1 = j0.7 \quad P_2 = -j0.7$

Zeros $z_1 = -0.4 + j0.8 \quad z_2 = -0.4 - j0.8$




Q.6 (a) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$

$h(n) = \{1, -2, -3, \underset{\uparrow}{4}, 0\}$

| | | x(n) | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 1 | 0 | 1 | 1 |
| h(n) | 1 | | | | | |
| | -2 | | | | | |
| | -3 | | | | | |
| | 4 | | | | | |
| | 0 | | | | | |

$y(n) = x(n) * h(n) = \{1, -1, -5, 2, 3, \underset{\uparrow}{-5}, 1, 4, 0\}$

Q.6 b $y(n) - 4y(n-1) + 4y(n-2) = x(n-1)$

Take z transfor

$$Y(z) - 4z^{-1}Y(z) + 4z^{-2}Y(z) = z^{-1}X(z)$$

$$Y(z)(1 - 4z^{-1} + 4z^{-2}) = z^{-1}X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-4z^{-1}+4z^{-2}}$$

$$H(z) = \frac{z^{-1}}{z^{-2}(z^2+4z+4)} = \frac{z}{(z-2)^2} = \frac{1}{2}\left(\frac{2z}{z-2}\right)$$

$h(n) = z^{-1}\left\{ \frac{1}{2} \, \frac{2z}{(z-2)^2} \right\}$ (04)

$h(n) = \left(\frac{1}{2}\right) n 2^n u(n) = \underline{n 2^{(n-1)} u(n)}$

$h(n) = n 2^{n-1} u(n)$

<u>Q.5b</u> $x(n) = 0.5^n u(n)$

$u(n) = 1$ for $n \geq 0$
$\quad\;\; 0 \quad n < 0$

$\therefore x(n) = 0.5^n$ ; for $n \geq 0$
$\quad\quad 0$ ; for $n < 0$

$z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} (0.5z^{-1})^n = \frac{1}{1-0.5z^{-1}}$

$\therefore X(z) = \frac{1}{1-0.5\frac{1}{z}} = \frac{z}{z-0.5}$

$0 < |0.5z^{-1}| < 1$

$\therefore |0.5z^{-1}| < 1 \Rightarrow \frac{0.5}{|z|} < 1 \Rightarrow |z| = 0.5$