Answer Key to WTP - May '18

Q. P. Code :- 36047

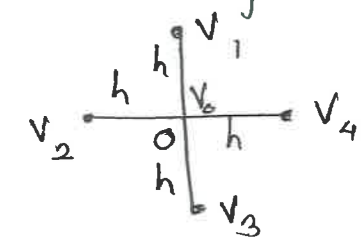
Q1b. $Q = \int_{Vol} \rho_v \, dv = \int_0^1 \int_0^1 \int_0^1 120 \, x^2 y \, dx \, dy \, dz$

$$= 120 \left[\frac{x^3}{3}\right]_0^1 \left[\frac{y^2}{2}\right]_0^1 [z]_0^1 = 20 \mu C$$

Q1d.

$\cancel{\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times 10^8}}$

Hint: a. Division of solution into grid points



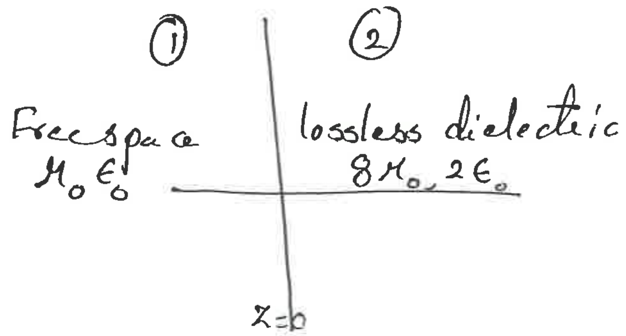
Using the formula.

$$V_0 = \frac{1}{4}(V_1 + V_2 + V_3 + V_4)$$

2b. $\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times 10^8} = \frac{1}{3}$; $\eta_1 = \eta_0 = 120\pi$

$$\beta_2 = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r} = \frac{\omega}{c}\cdot 4 = 4\beta_1 = \frac{4}{3}$$

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = 2\eta_0$$



Given $H_1 = 10 \cos(10^8 t - \beta_1 z) a_x$ mA/m,

$$E_i = E_{io} \cos(10^8 t - \beta_1 z) a_{Ei}$$

where $a_{Ei} = a_{Hi} \times a_{ki} = a_x \times a_z = -a_y$

$$E_{io} = \eta_1 H_{io} = 10 \eta_o$$

$$E_i = -10 \eta_o \cos(10^8 t - \beta_1 z) a_y \text{ mV/m}$$

Now $\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1}{3}$ $\therefore E_{ro} = \frac{1}{3} E_{io}$

$$E_r = -\frac{10}{3} \eta_o \cos\left(10^8 t + \frac{1}{3} z\right) a_y \text{ mV/m}.$$

from which H_r is obtained as

$$H_r = -\frac{10}{3} \cos\left(10^8 t + \frac{1}{3} z\right) a_x \text{ mA/m}$$

$||^{ly}$ $E_{to} = \frac{4}{3} E_{io}$

$$\therefore E_t = E_{to} \cos(10^8 t - \beta_2 z) a_y \text{ mV/m}$$

$$H_t = \frac{20}{3} \cos\left(10^8 t - \frac{4}{3} z\right) a_x \text{ mA/m}.$$

3a. phase constant $\beta = \omega\sqrt{\mu\epsilon} = 2\pi \times 300 \times 10^6 \sqrt{78 \mu_o \epsilon_o} = 55.5$ rad/m

$$v = \frac{\omega}{\beta} = 0.339 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{2\pi}{\beta} = 0.113 \text{ m}$$

$$\eta = \eta_o \sqrt{\frac{\mu_r}{\epsilon_r}} = 42.7\,\Omega$$

03

$$E_x = E_0 \cos(\omega t - \beta z) = 0.1 \cos(6\pi 10^8 t - 55.5z)\ V/m$$

$$H_y = \frac{E_0}{\eta} \cos(\omega t - \beta z) = 2.34 \times 10^{-3} \cos(6\pi \times 10^8 t - 55.z)\ A/m$$

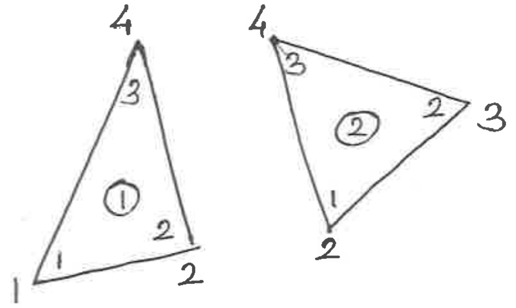
Q3c. $P = 35W$ $f = 90 MHz$ $\lambda = 10/3$ mts

$h_t = 40$ mts $h_r = 25$ mts

$$d = (2R_E h_t)^{1/2} + (2R_E h_r)^{1/2}$$

$$d = 40.366 \text{ kms}$$

Q4b.



$P_1 = -1.3$ $P_2 = 0.9$ $P_3 = 0.4$

$Q_1 = -0.2$ $Q_2 = -0.4$ $Q_3 = 0.6$

$A = \frac{1}{2}(0.54 + 0.16) = 0.35$

$$[C^{(1)}] = \begin{bmatrix} 1.236 & -0.7786 & -0.4571 \\ -0.7786 & 0.6929 & 0.0857 \\ -0.4571 & 0.0857 & 0.3714 \end{bmatrix}$$

$||^{ly}$ $$[C^{(2)}] = \begin{bmatrix} 0.5571 & -0.4571 & -0.1 \\ -0.4571 & 0.8238 & -0.3667 \\ -0.1 & -0.3667 & 0.4667 \end{bmatrix}$$

$$\begin{bmatrix} C_{22} & C_{24} \\ C_{42} & C_{44} \end{bmatrix} \begin{bmatrix} V_2 \\ V_4 \end{bmatrix} = - \begin{bmatrix} C_{21} & C_{23} \\ C_{41} & C_{43} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix}$$

Using $[C][V] = [B]$

$$[C] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1.25 & 0 & -0.0143 \\ 0 & 0 & 1 & 0 \\ 0 & -0.0143 & 0 & 0.8381 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 \\ 4.571 \\ 10 \\ 3.667 \end{bmatrix}$$

By inverting matrix

$$[V] = [C]^{-1}[B] = \begin{bmatrix} 0 \\ 3.708 \\ 10 \\ 4.438 \end{bmatrix}$$

$\therefore V_1 = 0, \quad V_2 = 3.708 \quad V_3 = 10 \quad V_4 = 4.438.$

Q5c. $r_1 = a_x + 2a_y + 3a_z \quad r_2 = a_x + 2a_y + 10a_z$

$R_{12} = 7a_z \quad |R_{12}| = 7$

a. Force on Q_1

$$F_{21} = \frac{Q_1 Q_2 R_{12}}{4\pi\epsilon_0 |R_{12}|^3} = -9.18 a_z \text{ mN}$$

b. $r_3 = a_x + 2a_y + z a_z$

$R_{1z} = r_3 - r_1 = (z-3)a_z$

$R_{23} = (z-10)a_z$

$$\frac{Q_1 Q_3 R_{13}}{4\pi\epsilon_0 |R_{13}|^3} + \frac{Q_2 Q_3 R_{23}}{4\pi\epsilon_0 |R_{23}|^3} = 0$$

$z = 26.89 \ \& \ 7.10$

$\therefore P_3$ is at $(1, 2, 26.9)$

05

Q6a.

$$r_{AO} = -2a_x + a_y + 3a_z$$

$$E_o = \frac{Q}{4\pi\epsilon_0 |r_{AO}|^2} \cdot \frac{r_{AO}}{|r_{AO}|}$$

$$= -1.717a_x + 0.859a_y + 2.576a_z \ V/m.$$

Q6b.

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 2.45\times10^9 \times 600 \times 4\pi\times10^{-7} \times 1.1\times10^6}}$$

$$= 3.97\times10^{-7} \ m$$