@ IV

Q.P. code 38368

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Solution

Electromagnetic Fields and Waves
Sem-IV (Choice based)

1a) $\nabla\cdot\bar{E} = \frac{1}{r}\frac{\partial}{\partial r}(r E_r) + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z} = 16 r \sin\phi$

$= 16y$

$= 80$ at $P(5,5,5)$

2b) $\bar{D} = \frac{r}{4}\bar{a_r}$ nC/m^2

a) $\bar{E} = \frac{D}{\epsilon_0} = 7.06\ \bar{a_r}$ V/m

b) $Q = 4\pi r^2 D_r = 49.1\ pC$

c) $D_r = 4\pi r^2 \times \frac{r}{4} \times 10^{-9} = 1347\ pC$

3b) i) $V = 882\ V$

ii) $\bar{E} = -\nabla V = -\left(\frac{\partial}{\partial x}V\bar{a_x} + \frac{\partial}{\partial y}V\bar{a_y} + \frac{\partial}{\partial z}V\bar{a_z}\right)$

$= -4(x+1)^2(y+2)^2(z+3)\bar{a_z}$
$-4(x+1)(y+2)^2(z+3)^2\bar{a_x}$
$-4(x+1)^2(y+2)(z+3)^2\bar{a_y}$

$= -588\bar{a_x} - 1764\bar{a_y} - 252\bar{a_z}$

iii) $\bar{D} = \epsilon_0\bar{E} = -5.21\bar{a_x} - 15.62\bar{a_y} - 2.23\bar{a_z}$ nC/m^2

iv) $\rho_v = \nabla\cdot\bar{D} = 8.854\times10^{-12}(-196-1764-36) = -17.67\ nC/m^3$

$\bar{a_n} = \frac{588\bar{a_x} + 1764\bar{a_y} + 252\bar{a_z}}{1876.4} = 0.313\bar{a_x} + 0.940\bar{a_y} + 0.1343\bar{a_z}$

4a) a) Normal componet of D_2

$\bar{D}_{n2} = \bar{D}_{n1} = 70\times2.5\times8.854\times10^{-12} = 1.549\times10^{-9}$ C/m^2

b) $\bar{E}_{t1} = \bar{E}_{t2} = -30\bar{a_x} + 5\bar{a_y}$

$D_{t2} = -1.062\times10^{-9}\bar{a_x} + 1.771\times10^{-9}\bar{a_y}$ C/m^2

c) $\bar{D_2} = \bar{D}_{t2} + \bar{D}_{n2}$

$\bar{P_2} = \epsilon_0(\epsilon_{r2}-1)\bar{E_2}$

$= (-0.797\bar{a_x} + 1.328\bar{a_y} + 1.162\bar{a_z})\times10^{-9}$ C/m^2

d) $|\bar{D_2}|\cos\alpha_2 = D_{n2}$

$\cos\alpha_2 = \frac{1.549\times10^{-9}}{2.581\times10^{-9}}$ $\alpha_2 = 53.1°$

(b) IV

4b) $\bar{B} = 0.04\ \bar{a}_y$ T

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$V = 2.5 \sin 10^3 t\ \bar{a}_z$ m/s

$\bar{E} = \bar{V} \times \bar{B} = 0.1 \sin 10^3 t\ (-\bar{a}_x)$ V/m

$V_e = \int (\bar{V} \times \bar{B}) \cdot d\bar{l}$

$= -0.2 \sin 10^3 t$ V

Conducter first moves in z direction the $x = 0.2$ m end is negative wrt the end on the z axis for first half cycle

5b) $i = C \frac{dv}{dt} = C V_0 \omega \cos \omega t$

$C = \epsilon \frac{A}{d}$

$E = \frac{V}{d}$

$D = \epsilon E = \epsilon \frac{V_0}{d} \sin \omega t$

$i_d = \int_s \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s} = C V_0 \omega \cos \omega t$

$\therefore \boxed{i = i_d}$

6b) $E(x,t) = -10^3 \sin \beta (x - V_0 t)\ \bar{a}_y$

$\beta = \frac{\omega}{V_0} = 2$ rad/m

$\lambda = \frac{2\pi}{\beta} = \pi$ m

$\frac{\lambda}{4} = \frac{\pi}{4}$

$T = \frac{1}{f} = \frac{2\pi}{\omega}$

t_1 = time for $\frac{\lambda}{4} = \frac{T}{4} = 2.62 \times 10^{-9}$ s.