Q.P.Code 36412

Q1 a)

(1)

For this case the function that we're going to be working with is,

$$F(x,y,z) = x^2 + y^2 + z^2$$

and note that we don't have to have a zero on one side of the equal sign. All that we need is a constant. To finish this problem out we simply need the gradient evaluated at the point.

$$\nabla F = \langle 2x, 2y, 2z \rangle$$

$$\nabla F(1,-2,5) = \langle 2,-4,10 \rangle$$

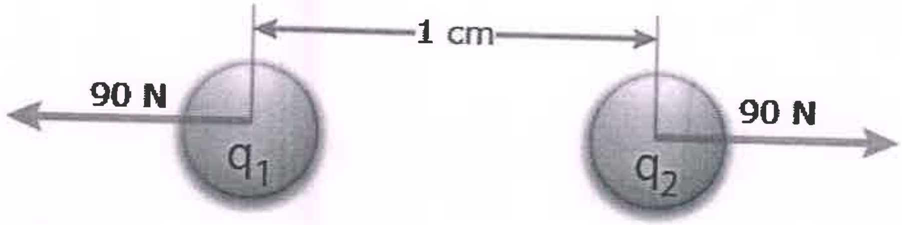
The tangent plane is then,

$$2(x-1) - 4(y+2) + 10(z-5) = 0$$

The normal line is,

$$\vec{r}(t) = \langle 1,-2,5 \rangle + t\langle 2,-4,10 \rangle = \langle 1+2t, -2-4t, 5+10t \rangle$$

Q1b)
First, draw a force diagram of the problem.



Define the variables:
F = 90 N
q_1 = charge of first body
q_2 = charge of second body
r = 1 cm
Use the Coulomb's Law equation

$$F = k\frac{q_1 q_2}{r^2}$$

The problem says the two charges are identical, so
$q_1 = q_2 = q$

Substitute this into the equation

$$F = k\frac{q^2}{r^2}$$

Since we want the charges, solve for q

$$q^2 = \frac{Fr^2}{k}$$

$$q = \sqrt{\frac{Fr^2}{k}}$$

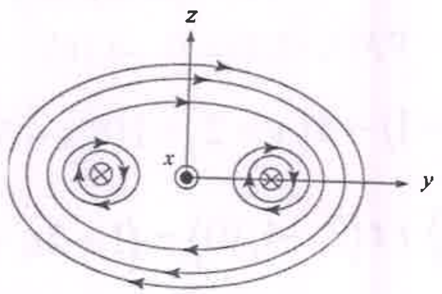
Enter the values for the variables. Remember to convert 1 cm to 0.01 meters to keep the units consistent.

$$q = \sqrt{\frac{(90\ \text{N})(0.01\ \text{m})^2}{8.99\times 109\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

D2

$q = \pm 1.00 \times 10^{-6}$ Coulomb
Since the charges are identical, they are either both positive or both negative. This force will be repulsive.

Q 2a



The magnetic field at (0, 0, z) due to wire 1 on the left is, using Ampere's law:

Q3

Adding up the contributions from both wires, the z-components cancel (as required by symmetry), and we arrive at

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 I \sin\theta}{\pi\sqrt{a^2+z^2}}\hat{\mathbf{j}} = \frac{\mu_0 I z}{\pi(a^2+z^2)}\hat{\mathbf{j}} \tag{9.11.26}$$

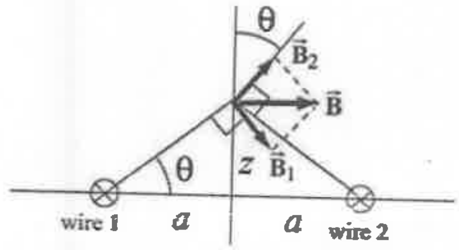


Figure 9.11.7 Superposition of magnetic fields due to two current sources

To locate the maximum of B, we set $dB/dz = 0$ and find

$$\frac{dB}{dz} = \frac{\mu_0 I}{\pi}\left(\frac{1}{a^2+z^2} - \frac{2z^2}{(a^2+z^2)^2}\right) = \frac{\mu_0 I}{\pi}\frac{a^2-z^2}{\left(a^2+z^2\right)^2} = 0 \tag{9.11.27}$$

which gives

$$z = a \tag{9.11.28}$$

Thus, at $z=a$, the magnetic field strength is a maximum, with a magnitude

$$B_{max} = \frac{\mu_0 I}{2\pi a} \tag{9.11.29}$$

9.11.6 Non-Uniform Current Density

Consider an infinitely long, cylindrical conductor of radius R carrying a current I with a non-uniform current density

$$J = \alpha r \tag{9.11.30}$$

where α is a constant. Find the magnetic field everywhere.

Q5a) F= $\frac{4}{\sqrt{2}}$ x $\frac{q^2}{4\pi\epsilon d^2} = 4 \times 10^{-4} N$

Q6b)

i) E at P(1,2,3) =57 **ay**-28.8 **az** V/m

(ii)E =23**ay**-46**az**