1/8

STRUCTURAL ANALYSIS-I — Q. P Code 25897

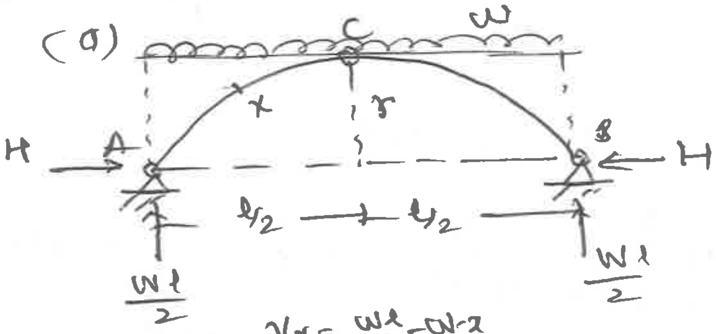
SE CIVIL SEM-IV CBSGS — DOE 05-12-2017

Answer Key (Model Answer) — MAX MARKS- 80

By Prof D.M. Joshi SCOE, (9324866015) — (3 hours)

Q 1 Compulsory (Any four 5×4=20)

(a)

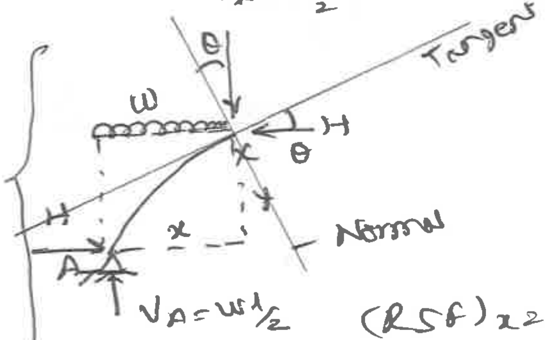


$\Sigma M = 0$ about C

$H \times r + w \frac{l}{2} \cdot \frac{l}{4} = \frac{wl}{2} \times l/2$

$\boxed{H = \frac{wl^2}{8r}}$ — 2 marks

$V_x = \frac{wl}{2} - wx$



$H = \frac{wl^2}{8r} \leftarrow$

$V_x = \frac{w}{2}(l - 2x) \downarrow$

$(RSF)_x = H\sin\theta - V_x\cos\theta$

$(RSF)_x = \cos\theta\left[H\tan\theta - V_x\right]$ — (I)

$y = \frac{4r}{l^2} \cdot x(l-x)$

$\tan\theta = \frac{dy}{dx} = \frac{4r}{l^2}(l-2x)$

3 marks

$H\tan\theta = \frac{wl^2}{8r} \times \frac{4r}{l^2}(l-2x) = \frac{w}{2}(l-2x)$

Putting in (I) $(RSF)_x = \cos\theta\left[\frac{w}{2}(l-2x) - \frac{w}{2}(l-2x)\right]$

$(RSF)_x = 0$ Hence proved

Total $\underbrace{2+3=5 \text{ marks}}$

Q1(b) Prof Perry's formula

$$\boxed{\left(\frac{\sigma_{max}}{\sigma_a} - 1\right)\left(1 - \frac{\sigma_a}{\sigma_e}\right) = \frac{1.2 \cdot e\, y_c}{k^2}}$$ — 1 mark

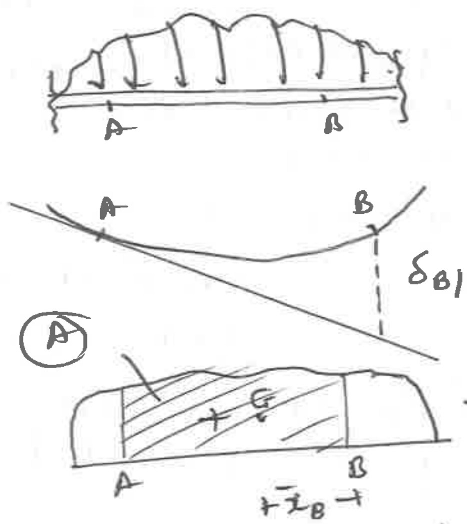
Term Explanation — 2 mark

Importance: P can be determined, if σ_{max} is ~~are~~ given — 2m

Not possible with Secant formula

5 marks

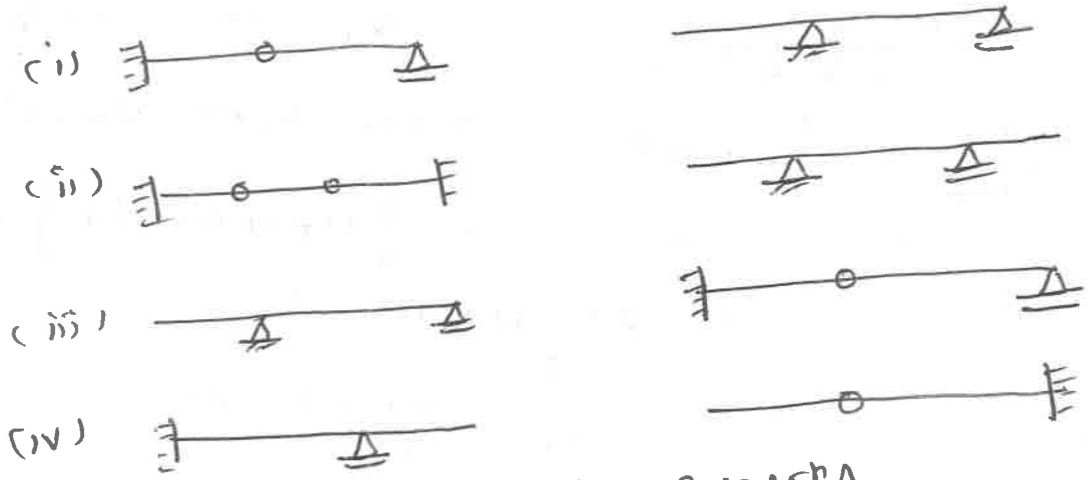
28 1(c) 5 marks Moment Area II theorem



$$\delta_{B/A} = \sum_{A}^{B} (A \bar{x}_B) \frac{M}{EI} \text{ diagram}$$

Statement + Explanation (1 + 2) marks

Real Beam Conjugate Beam



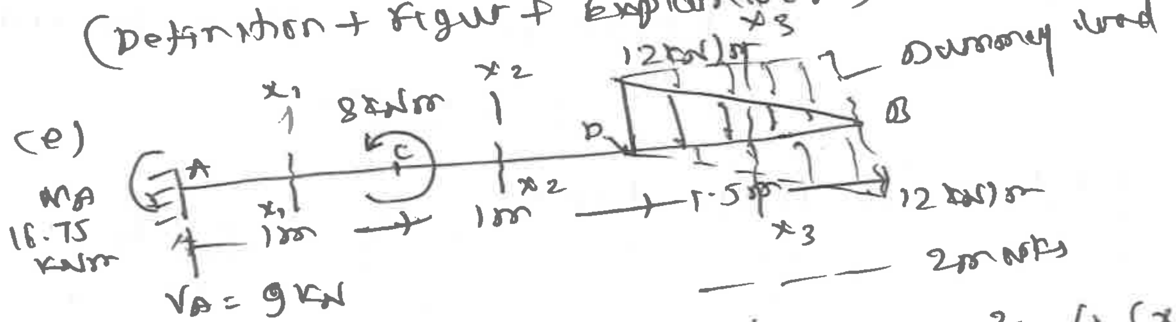
0.5 × 4 = 2 marks

(d) Unsymmetrical Bending -- 3 marks

Shear Centre — -- 2 marks

(Definition + figure + Explaination)

(e)



— — — 2 marks

$$M_x = 9x - 16.75 \mid - 8(x-1)^0 \mid - 6(x-2)^2 + \frac{4}{3}(x-2)^3$$

— — — 3 marks

05 marks

(f) (i) Principle of superposition — 2 m

(ii) Castigliano's theorem ——— 3 m

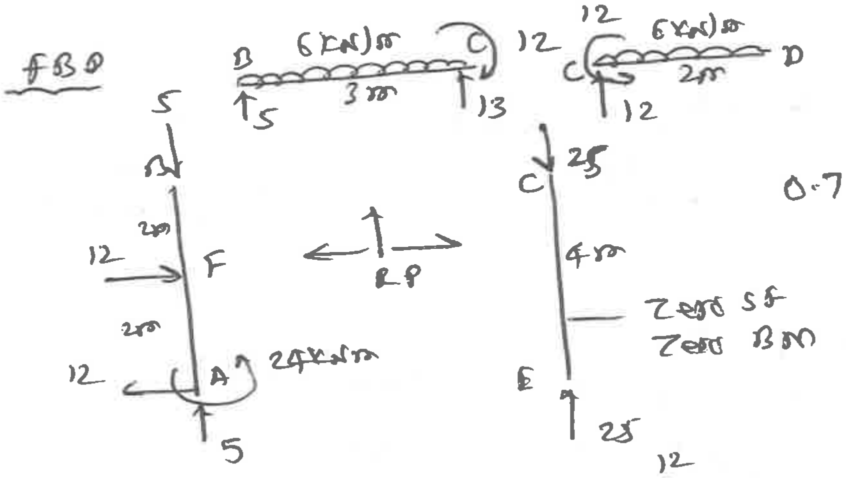
(~~statement~~ statement + figure + explanation)

Q 2(a) 10 marks

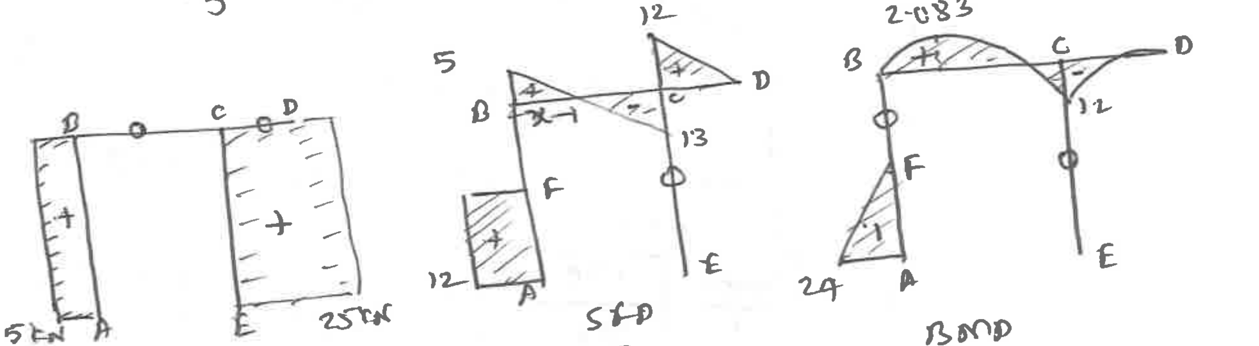
$V_E = 25\,kN \uparrow$ $V_A = 5\,kN\,(\uparrow)$

$H_A = 12\,kN\,(\leftarrow)$ $M_A = 24\,kNm$ — 2 marks

FBD



$0.75 \times 4 = 3$ marks



AFD Compressive — 1 mark

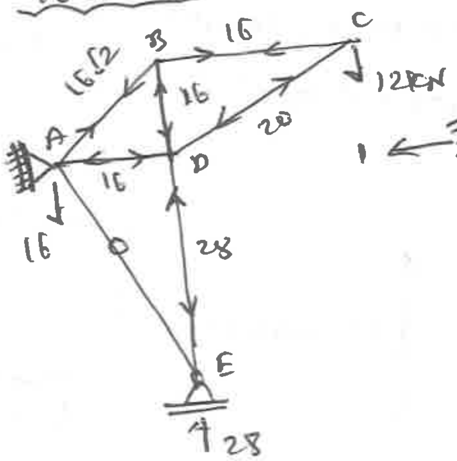
SFD $x = \frac{5}{6}$ m — 2 mark

BMD — 2 mark = 5 mark

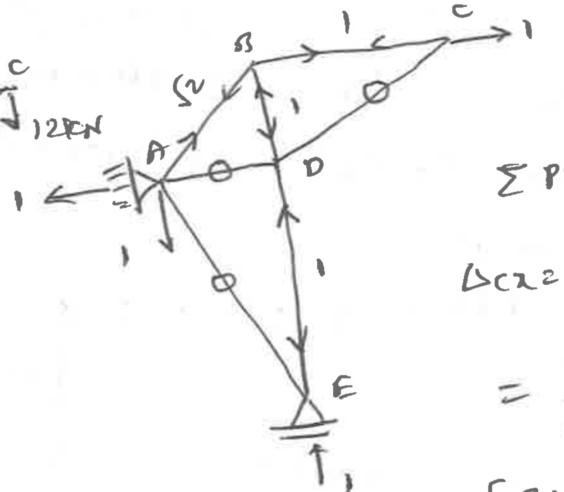
$2 + 3 + 5 = 10$ marks

Q 2(b) 10 marks

$\Delta_{cx} = ?\ (\rightarrow)$ $AE = 30\,000$ kN



P-force diagram 3 mark



u (or k) force diagram 3 mark

Table — 3 mark

$\Sigma PuL = 359.765$

$\Delta_{cx} = \Sigma \frac{PuL}{AE}$

$= \underline{11.992\text{ mm}\ (\rightarrow)}$ Ans — 1 mark

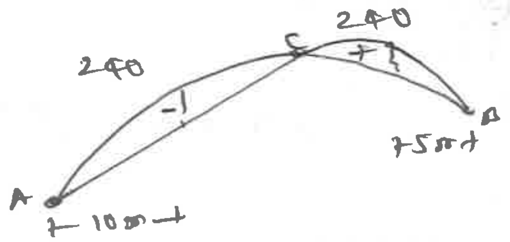
$[3+3+3+1 = 10]$

Q 3(a) (i) $y = kx^2$ (C-origin)
10 marks

$$\frac{h_1}{h_2} = \frac{l_1^2}{l_2^2} \therefore h = 1.5\,m \quad \text{--- } 1m$$

(ii) $V_A = 48$ kN $V_B = 84$ kN (↑) & $H = 160$ kN (→ ←) --- 3m

(iii) for AC portion $x = 10\,m$ $M_{max} = -240$ kNm
for BC portion $x = 5\,m$ $M_{max} = 240$ kNm --- 3m



BMD --- 2m

10 marks

Q 3(b) 10 marks

$l_e = 0.5L = 3\,m$

$A = \frac{\pi}{4}(200^2 - 150^2) = 13.744 \times 10^3\ mm^2$

$I = \frac{\pi}{64}(200^4 - 150^4) = 53.689 \times 10^6\ mm^4$

$Z = \frac{I}{y_c} = 536.89 \times 10^3\ mm^3$ — 3 marks

$\sigma_a = \frac{180 \times 10^3}{13.744 \times 10^3} = 13.097\ N/mm^2$ (Comp^n)

$$\theta = \left[\frac{3000}{2}\sqrt{\frac{180 \times 10^3}{96 \times 10^3 \times 53.689 \times 10^6}}\right] \times \frac{180}{\pi} \quad \text{2 marks}$$

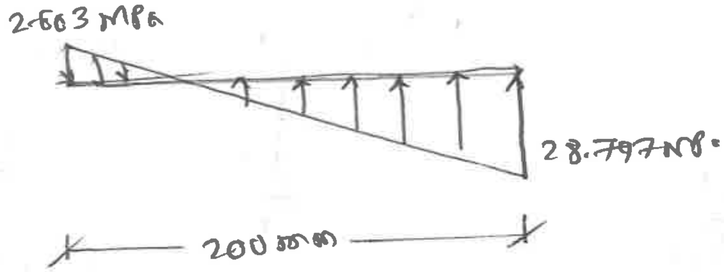
$\theta = 16.06)^\circ$

$M = P \cdot e \sec\theta = 180 \times 10^3 \times 45 \times \sec\theta$

$M = 8.429 \times 10^6$ Nmm

$\sigma_b = \pm \frac{8.429 \times 10^6}{536.89 \times 10^3} = \pm 15.700\ N/mm^2$ — 2m

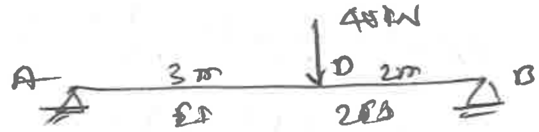
$\sigma_{max} = \sigma_a + \sigma_b = 28.797$ MPa (Comp^n)
$\sigma_{min} = \sigma_a - \sigma_b = 2.603$ MPa (Tensile) — 2m

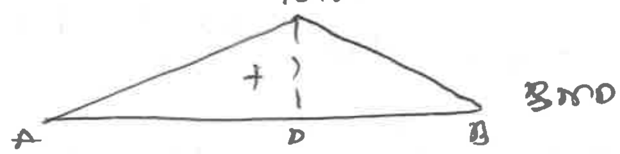


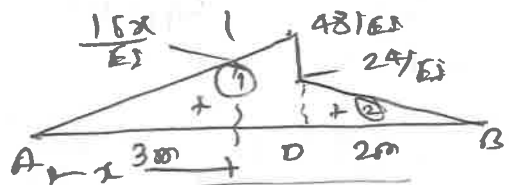
— 1m

10 marks

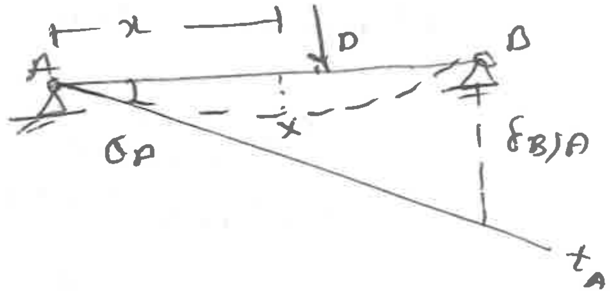
Q 4(a) Moment Area method







M/EI diagram



Area	$\bar{x}_A$	$\bar{x}_B$
$A_1 = \frac{72}{EI}$	2	3
$A_2 = \frac{24}{EI}$	$\frac{11}{3}$	$\frac{4}{3}$

$\cancel{\delta_{B/A} = \frac{72}{EI} \times 3}$

Theorem II $\quad \delta_{B/A} = \sum_A^B [A\bar{x}_B]_{\frac{M}{EI}} = \frac{72}{EI}\times 3 + \frac{24}{EI}\times\frac{4}{3} = \frac{248}{EI}$

$\tan\theta_A \simeq \theta_A = \frac{\delta_{B/A}}{BA} = \frac{49.6}{EI}$ (2) --- 5 marks

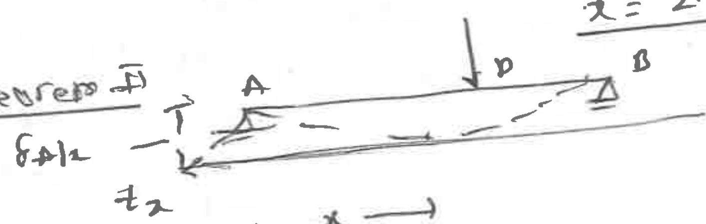
Slope at x is zero $\quad$ theorem I $\quad \delta_{A/x} = \sum_A^x [A]_{\frac{M}{EI}}$

$\theta_A - \theta_x = \frac{1}{2}\cdot x \cdot \frac{16x}{EI} \qquad 8x^2 = 49.6$

$x = 2.49\ m$ — Ans

--- 2 marks

y_{max} Theorem II

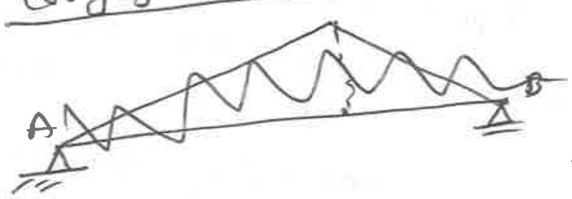


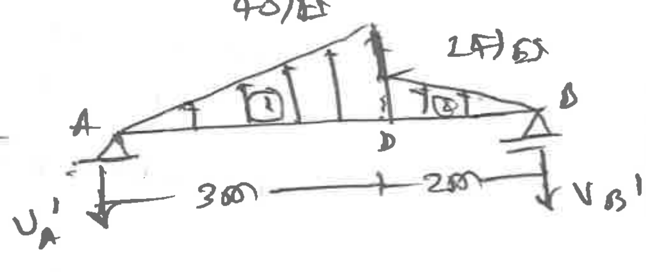
$y_{max} = \delta_{A/x} = \sum_A^x (A\bar{x}_A)_{\frac{M}{EI}} = \frac{82.337}{EI}$ m — Ans

--- 2 marks

10 marks

Conjugate beam method





Loads	$\bar{x}_A$	$\bar{x}_B$
$P_1 = \frac{72}{EI}$ ↑	2 m	3 m
$P_2 = \frac{24}{EI}$ ↑	$\frac{11}{3}$	$\frac{4}{3}$

$\Sigma P = \frac{96}{EI}$ (↑)

$\Sigma M = 0$ about B

$V_A' \times 5 = \frac{72}{EI} \times 3 + \frac{24}{EI} \times \frac{4}{3}$

$V_A' = \frac{49.6}{EI}$ (↓)

$\Sigma V = 0 \quad V_B' = \frac{46.4}{EI}$ (↓)

$(\theta_A)_{Real} = (SF_A)_{conj} = -\frac{49.6}{EI}$

$\theta_A = \frac{49.6}{EI}$ (↻) — — 4 mark

$(\theta_D)_{Real} = (SF_D)_L)_{conj} = -\frac{49.6}{EI} + \frac{72}{EI} = \frac{22.4}{EI}$ (↺) — — 2 mark

$\therefore$ $\underline{\theta_{Real} = 0 \text{ betn } A \text{ \& } D}$

$(\theta_x)_{Real} = (SF_x)_{conj} = -\frac{49.6}{EI} \cdot x + \frac{8x^2}{EI} = 0$

$\therefore x = 2.49$ m — Ans — — 2 mark

$\therefore (y_{max})_{Real} = (BM_x)_{conj} = -\frac{49.6}{EI} \times 2.49 + \frac{8}{EI}(2.49)^2 \times \frac{2.49}{3}$

$y_{max} = -\frac{82.335}{EI}$

$y_{max} = \frac{82.335}{EI}$ m (↓) — Ans

— 2 mark

10 mark

Q 4(b) 10 marks $R = \Sigma W = 90$ kN $\quad l_W = 10$ m

$\bar{x}_L = \frac{(20 \times 3 + 20 \times 5 + 25 \times 7 + 15 \times 10)}{90} = 5.389$ m

or 4.611 m from right

middle 20 kN load is critical load 'P'

$d = x_W - x_P = 5.389 - 5 = 0.389$ m — — 5 mark

$\therefore x = \frac{d}{2} = 0.195$ m

critical section $x_C = 12 - 0.195 = 11.805$ — Ans — — 1 mark

from left $x_W = 12 + 0.195 = 12.195$

for above placement $V_A = 44.269$ kN, $V_B = 45.731$ kN } — 2 m

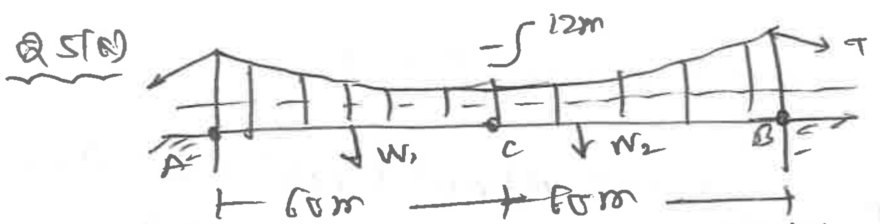
Ans

$(BM)_{absolute} = \frac{432.596 \text{ kNm}}{\text{Central load}}$ under CL P (= 20 kN)

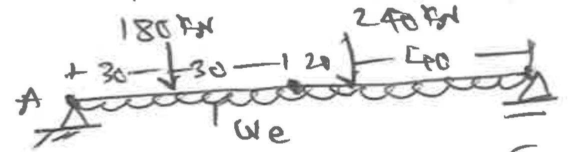
2 m

10 marks

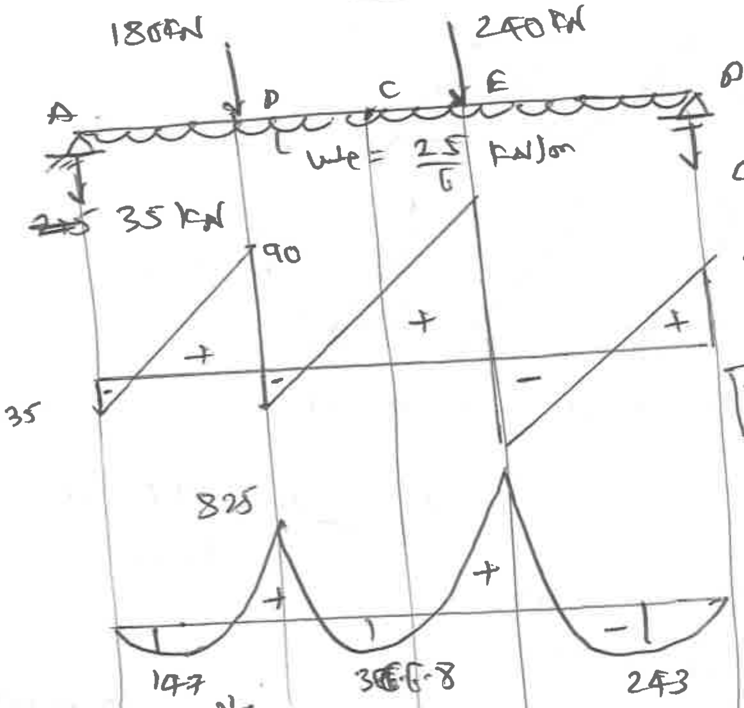
Q 5(a)



FBD of Girder



$W_e = 4.167$ kN/m $= \frac{25}{6}$ kN/m — 2 mark

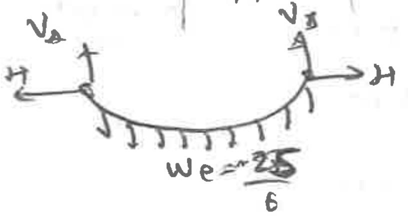


— 4 marks

* Calculate ordinate

— 4 mark

calculate ordinates



$V_A = V_B = V = \frac{\frac{25}{6} \times 120}{2} = 250$ kN

$H = \frac{W_e l^2}{8d} = 750$ kN $= T_{min} = T_C$

$T_{max} = \sqrt{V^2 + H^2} = 790.569$ kN

— 2 marks

Q 5(b) T-section

8 marks

$M_{max} = \frac{Wl}{4}$

$= 40$ kNm

$\bar{y} = y_{top} = 70$ mm $y_{bottom} = 130$ mm

$I_{uu} = 24.5 \times 10^6$ mm^4 $(= I_{xx})$

$I_{vv} = 3 \times 10^6$ mm^4 $(= I_{yy})$

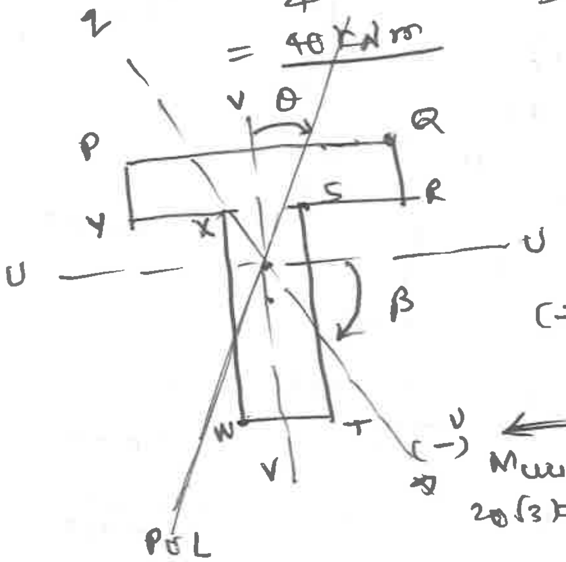
$\tan\beta = \frac{I_{uu}}{I_{vv}} \tan\theta$

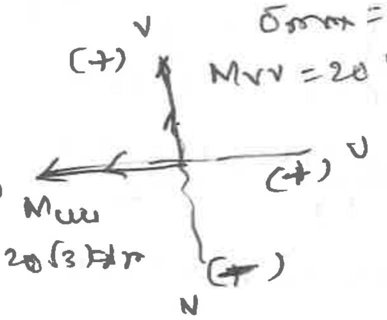
$\beta = 77.882°$

$\sigma_{max} = \sigma_Q$ (comp)

$\sigma_{max} = \sigma_W$ (tensile)

— 4 mark





$\sigma_Q = 500.281$ N/mm^2

(comp)

Ans

2 mark

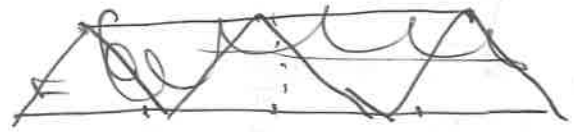
$\sigma_w = -252.755$ N/mm² (Compⁿ)

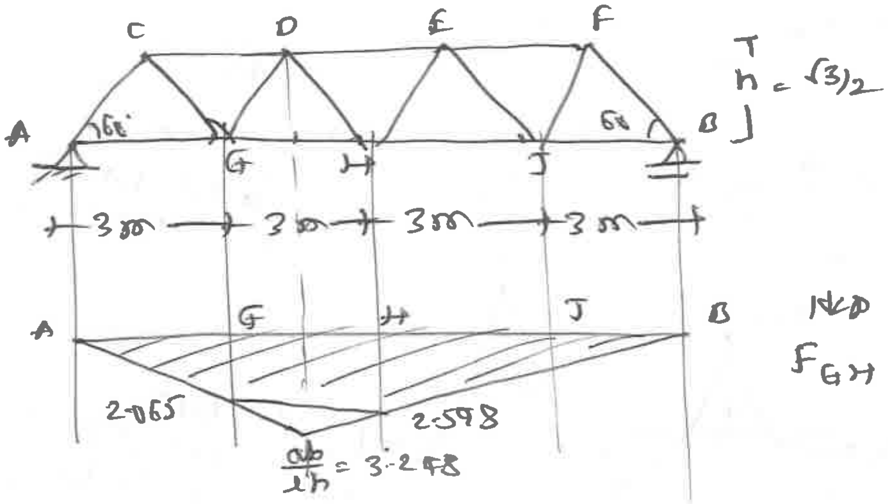
or $\sigma_w = 252.755$ N/mm² (Tensile) — Ans

— 2 marks

8 marks

Q 6(a)
4 mark





Calculation 2 marks + diagram 2 marks = 4 marks

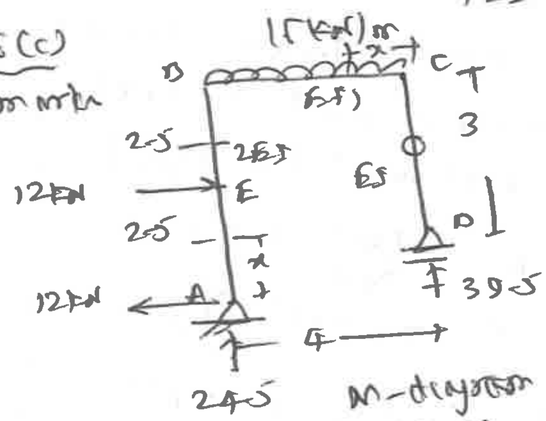
Q 6(b)
6 marks

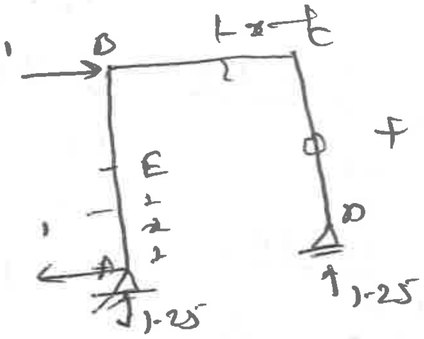
$U = U_{bending} = \frac{\pi W^2 R^3}{8 EI}$ — 4m

$\Delta_{By} = \frac{\pi W R^3}{4 EI}$ m (↓) — 2m

6 marks

Q 6(c)
10 marks





+ Table — 5 mark

$$\Delta_{Bx} = \int \frac{M m\, dx}{EI} = \int_{A(0)}^{E(2.5)} + \int_{2.5(E)}^{5(B)} + \int_{C(0)}^{B(4)}$$ — 4 mark

$$= \frac{31.25}{EI} + \frac{140.625}{EI} + \frac{413.333}{EI} = \frac{585.208}{EI}$$ — 1 mark

$\delta_{Bx} = 14.63$ mm (→) — Ans

10 marks

— END —