

1) a) Along $y = x^2$, $dy = 2x dx$

$$\begin{aligned} \therefore I &= \int_0^3 (x + x^2) dx + x^2 x^2 (2x dx) \\ &= \int_0^3 (x + x^2 + 2x^5) dx = \left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^6}{3} \right]_0^3 = \boxed{256 \frac{1}{2}} \# \end{aligned}$$

b) $a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx = \frac{1}{\pi} \int_0^{2\pi} x dx = 2\pi$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_0^{2\pi} x \cos nx dx = 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx = \frac{1}{\pi} \int_0^{2\pi} x \sin nx dx = -\frac{2}{\pi}$$

$$\therefore x = \pi - 2 \left[\sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x + \dots \right] \#$$

c) $F\{f(x)\} = \int_{-1}^1 (1-x^2) e^{-isx} dx$

$$= 2 \int_0^1 (1-x^2) \cos sx dx$$

$$= 2 \left[(1-x^2) \frac{\sin sx}{s} - 2x \frac{\cos sx}{s^2} + 2 \frac{\sin sx}{s^3} \right]_0^1$$

$$= \frac{4}{s^3} (\sin s - s \cos s) \#$$

d) $E[f(x)] = f(x+h)$, $E(e^x) = e^{x+h}$ — (1)

$$\Delta e^x = e^{x+h} - e^x = e^x [e^h - 1]$$

$$\Delta^2 e^x = \Delta \cdot \Delta(e^x) = \Delta e^x [e^h - 1]$$

$$= (e^h - 1) \Delta e^x = (e^h - 1) e^x (e^h - 1)$$

$$= (e^h - 1)^2 e^x \quad \text{--- (2)}$$

$$\left(\frac{\Delta^2}{E}\right) e^x = (\Delta^2 E^{-1}) e^x = \Delta^2 e^{x-h} = e^{-h} \Delta^2 e^x = e^{-h} (e^h - 1)^2 e^x \quad \text{--- (3)}$$

$$\therefore \text{LHS} = e^{-h} (e^h - 1)^2 e^x \cdot \frac{e^{x+h}}{(e^h - 1)^2 e^x} = e^x = \underline{\underline{\text{RHS}}}$$

a) $f(x) = \frac{1}{2} - x$

$$a_0 = \frac{1}{L} \int_0^L f(x) dx = \frac{2}{L} \int_0^L \left(\frac{1}{2} - x\right) dx = \frac{2}{L} \left[\frac{x}{2} - \frac{x^2}{2} \right]_0^L = 0$$

$$a_n = \frac{1}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx = 0$$

$$b_n = \frac{1}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx = \frac{2}{L} \int_0^L \left(\frac{1}{2} - x\right) \sin \frac{n\pi x}{L} dx$$

$$= \frac{1}{n\pi} \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$\frac{1}{2} - x = \frac{1}{\pi} \sum \frac{1}{n} \sin \frac{2n\pi x}{L} \quad \#$$

b) $L(x_1, x_2, \lambda) = (4x_1 + 8x_2 - x_1^2 - x_2^2) - \lambda(x_1 + x_2 - 4)$

$$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - \lambda, \quad \frac{\partial L}{\partial x_2} = 8 - 2x_2 - \lambda, \quad \frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 4)$$

Solving, $\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \frac{\partial L}{\partial \lambda} = 0$, we get

$$4 - 2x_1 - \lambda = 0, \quad 8 - 2x_2 - \lambda = 0, \quad x_1 + x_2 = 4$$

\therefore Solving, $\lambda = 2, \quad x_1 = 1, \quad x_2 = 3$

$\therefore X_0$ is $(1, 3)$

Now $h(x_1, x_2) = x_1 + x_2 - 4 = 0$

$$\frac{\partial h}{\partial x_1} = 1, \quad \frac{\partial h}{\partial x_2} = 1$$

$$f(x_1, x_2) = 4x_1 + 8x_2 - x_1^2 - x_2^2$$

$$\frac{\partial f}{\partial x_1} = 4 - 2x_1, \quad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0, \quad \frac{\partial^2 f}{\partial x_1^2} = -2$$

$$\frac{\partial f}{\partial x_2} = 8 - 2x_2, \quad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0, \quad \frac{\partial^2 f}{\partial x_2^2} = -2$$

$$\Delta = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \\ \frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda \frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_1 \partial x_2} \\ \frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda \frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda \frac{\partial^2 h}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 4 - 2x_1 - \lambda & -2 & 0 \\ 8 - 2x_2 - \lambda & 0 & -2 \end{vmatrix} = 4$$

$\therefore \Delta$ is positive, X_0 is maxima

$\therefore x_1 = 1, x_2 = 3 \quad Z_{max} = 18$

2. c) $f(z) = \frac{a}{z} + \frac{b}{z^2} + \frac{c}{z-1} + \frac{d}{z-2}$

Solving,

$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3(z-1)} - \frac{1}{12(z+2)}$$

Case 1. $0 < |z| < 1 \Rightarrow |z| < 2 \Rightarrow \left|\frac{z}{2}\right| < 1$

~~$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3} (1-z)^{-1} - \frac{1}{24} \left(1 + \frac{z}{2}\right)^{-1}$$

$$= -\frac{1}{4z} - \frac{1}{2z^2} - \frac{1}{3} (1+z+z^2+\dots) - \frac{1}{24} \left(1 - \frac{z}{2} + \frac{z^2}{4} - \dots\right)$$~~

Case 2. $1 < |z| < 2$
 $1 < |z| \Rightarrow \left|\frac{1}{z}\right| < 1$, $|z| < 2 \Rightarrow \left|\frac{z}{2}\right| < 1$

$$\therefore f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z} \left(1 - \frac{1}{z}\right)^{-1} - \frac{1}{12 \cdot 2} \left(1 + \frac{z}{2}\right)^{-1}$$

$$= -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots\right) - \frac{1}{24} \left(1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \dots\right)$$

Case 3 $|z| > 2 \Rightarrow |z| > 1$
 ~~$\left|\frac{z}{2}\right| < 1$~~ , $\left|\frac{1}{z}\right| < 1$

$$\therefore f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z} \left(1 - \frac{1}{z}\right)^{-1} - \frac{1}{12z} \left(1 + \frac{z}{2}\right)^{-1}$$

$$= -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots\right) - \frac{1}{12z} \left(1 - \frac{z}{2} + \frac{z^2}{2^2} - \dots\right)$$

3. a) we have $x_0=1, x_1=2, x_2=7, x_3=8$
 $y_0=4, y_1=4, y_2=5, y_3=4$
 and $x=5$.

$$y = y_0 \cdot \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \cdot \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} + y_2 \cdot \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3 \cdot \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$y = 4 \cdot \frac{(5-2)(5-7)(5-8)}{(1-2)(1-7)(1-8)} + 4 \cdot \frac{(5-1)(5-7)(5-8)}{(2-1)(2-7)(2-8)} + 5 \cdot \frac{(5-1)(5-2)(5-8)}{(7-1)(7-2)(7-8)} + 4 \cdot \frac{(5-1)(5-2)(5-1)}{(8-1)(8-2)(8-7)}$$

$$= \frac{26}{5}$$

b) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$
 $\Rightarrow u = (c_1 \cos pt + c_2 \sin pt) (c_3 \cos px + c_4 \sin px)$ ①

$x=0, u=0 \therefore c_3=0$

$\therefore u = (c_1 \cos pt + c_2 \sin pt) (c_4 \sin px)$

put $x=\pi, u=0$

$c_4 \sin p\pi = 0 \Rightarrow \sin p\pi = 0 = \sin n\pi$
 $p\pi = n\pi \Rightarrow p=n, \quad n=1, 2, 3, \dots$

$\therefore u = (c_1 \cos nt + c_2 \sin nt) c_4 \sin nx$

$\frac{du}{dt} = (-c_1 n \sin nt + c_2 n \cos nt) c_4 \sin nx$

$\frac{du}{dt} = 0, t=0 \Rightarrow 0 = (c_2 n) (c_4 \sin nx)$
 $\Rightarrow c_2 = 0$

$\therefore u = c_1 c_4 \cos nt \sin nx$

3.6) contd

(5)

$$u(x,0) = 2(\sin x + \sin 3x)$$

$$t=0, u(x,0) = \sum C_n \sin nx$$

$$2(\sin x + \sin 3x) = \sum C_n \sin nx$$

$$4 \sin 2x \cos x = \sum C_n \sin nx$$

$$C_n C_n = 4 \cos x, \quad \sin nx = \sin 2x$$

$n=2$

$$u(x,t) = 4 \cos x \cos at \sin 2x \neq$$

e) $f(x_1, x_2) = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

$$h(x_1, x_2) = x_1 + 3x_2 - 6$$

$$w(x_1, x_2) = 5x_1 + 2x_2 - 10$$

Kuhn Tucker conditions for maxima are

$$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h}{\partial x_1} - \lambda_2 \frac{\partial w}{\partial x_1} = 0 \quad \therefore 2 - 2x_1 - \lambda_1 - 5\lambda_2 = 0$$

$$\frac{\partial f}{\partial x_2} - \lambda_1 \frac{\partial h}{\partial x_2} - \lambda_2 \frac{\partial w}{\partial x_2} = 0 \quad \therefore 3 - 4x_2 - 3\lambda_1 - 2\lambda_2 = 0$$

$$\lambda_1 (x_1 + 3x_2 - 6) = 0$$

$$\lambda_2 (5x_1 + 2x_2 - 10) = 0$$

$$x_1 + 3x_2 - 6 \leq 0$$

$$5x_1 + 2x_2 - 10 \leq 0$$

$$x_1, x_2 \geq 0 \quad \lambda_1, \lambda_2 \geq 0$$

Case 1 $\lambda_1 = 0$ & $\lambda_2 = 0$

$$2 - 2x_1 = 0, \quad x_1 = 1, \quad 3 - 4x_2 = 0 \quad \therefore x_2 = 3/4$$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$$

$$A_1 = |a_{11}| = -2 \quad \& \quad A_2 = \begin{vmatrix} -2 & 0 \\ 0 & -4 \end{vmatrix} = 8$$

Since the principal minors are alternatively ^{positive} negative

$x_1 = 1, x_2 = 3/4$ gives a maxima.

$$Z_{\max} = 2 + 3\left(\frac{3}{4}\right) - 1 - 2\left(\frac{9}{16}\right) = \frac{17}{8}$$

Case 2 $\lambda_1 = 0 \quad \& \quad \lambda_2 \neq 0$

$$2 - 2x_1 - 5\lambda_2 = 0$$

$$3 - 4x_2 - 2\lambda_2 = 0$$

Solving $x_1 = \frac{89}{54} = 1.648, \quad x_2 = \frac{95}{108} = 0.880$

$$5\lambda_2 = -\frac{70}{54} \quad \text{-ve}$$

reject the pair.

Case 3. $\lambda_1 \neq 0$ and $\lambda_2 = 0$

$$2 - 2x_1 - \lambda_1 = 0, \quad 3 - 4x_2 - 3\lambda_1 = 0$$

$$\therefore \text{ solving, } x_1 = 3/2, \quad x_2 = 3/2$$

But x_1, x_2 do not satisfy the eqns.

reject the pair.

Case 4. $\lambda_1 \neq 0, \lambda_2 \neq 0$

$$x_1 + 3x_2 = 6 \quad \& \quad 5x_1 + 2x_2 = 10$$

$$x_1 = \frac{20}{13}, \quad x_2 = \frac{18}{13}$$

$$\lambda_1 + 5\lambda_2 = -\frac{14}{13}, \quad 3\lambda_1 + 2\lambda_2 = -\frac{33}{13}$$

$$\lambda_1 = -\frac{137}{169} \quad \& \quad \lambda_2 = -\frac{9}{169}$$

negative

reject the pair.

$$\boxed{Z_{\max} = \frac{17}{8}}$$

4. a) Half range cosine series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$

Here $d = \pi$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x(\pi-x) dx = \frac{\pi^2}{3}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x(\pi-x) \cos nx dx = \frac{2}{\pi} \left(\frac{\pi}{n^2}\right) \left[-(-1)^n - 1\right]$$

$$= \begin{cases} -\frac{4}{n^2} & \text{when } n \text{ is even} \\ 0 & \text{when } n \text{ is odd} \end{cases}$$

Hence $x(\pi-x) = \frac{\pi^2}{6} - 4 \left[\frac{\cos 2x}{2^2} + \frac{\cos 4x}{4^2} + \dots \right] \neq$

b) $f(z)$ is not analytic at

$$z(z-1)(z-2) = 0 \Rightarrow z=0, z=1, z=2.$$

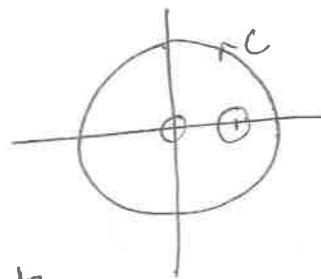
$z=0, 1$ lies inside C but $z=2$ lies outside C .

$$\int_C \frac{4-3z}{z(z-1)} dz$$

$$= \int_C \frac{4-3z}{(z-2)(z-1)} dz + \int_C \frac{4-3z}{z(z-2)} dz$$

$$= 2\pi i \left[\frac{4-3z}{(z-1)(z-2)} \right]_{z=0} + 2\pi i \left[\frac{4-3z}{z(z-2)} \right]_{z=1}$$

$$= 2\pi i \frac{4}{(-1)(-2)} + 2\pi i \frac{4-3}{1(1-2)} = 2\pi i (2-1) = 2\pi i \neq$$



c) we have $\frac{\partial u}{\partial t} = \frac{1}{h^2} \frac{\partial^2 u}{\partial x^2}$

(08)

$$u = C_1 e^{-\frac{p^2 t}{h^2}} (C_2 \cos px + C_3 \sin px)$$

$$u=0, x=0 \quad \therefore C_1 C_2 e^{-\frac{p^2 t}{h^2}} = 0$$

$$C_1 \neq 0, \quad C_2 = 0$$

$$\therefore u = C_3 \sin px e^{-\frac{p^2 t}{h^2}}$$

$$x=l, u=0$$

$$0 = C_3 \sin pl e^{-\frac{p^2 t}{h^2}}$$

$$C_3 \neq 0 \quad \therefore \sin pl = 0 = \sin n\pi$$

$$\text{or } p = \frac{n\pi}{l}$$

$$\therefore u = C_3 \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 t}{h^2}}$$

$$t=0, u = \frac{\sin \pi x}{l}$$

$$\therefore \frac{\sin \pi x}{l} = C_3 \sin \frac{n\pi x}{l}$$

$$\Rightarrow n=1, \quad C_3=1$$

$$\therefore u = \frac{\sin \pi x}{l} e^{-\frac{\pi^2 t}{h^2}} \quad \#$$

5. a) Let $f(5) = u_5, f(6) = u_6, f(7) = u_7$

	x	$u(x)$	$u(x)$
0	3	-2	4
1	1	2	10
2	3	12	16
3	15	28	$u_5 - 71$
4	43	$u_5 - 43$	$u_6 - 2u_5 + 43$
5	u_5	$u_6 - u_5$	$u_7 - 2u_6 + u_5$
6	u_6	$u_7 - 6u_6$	
7	u_7		

b)

$$f(x) = \sum_{-\infty}^{\infty} C_n e^{in\pi x/l}$$

$$C_n = \frac{1}{2l} \int_{-l}^l f(x) e^{-in\pi x/l} dx$$

$$= \frac{1}{2l} \int_{-l}^l e^{ax} e^{-in\pi x/l} dx$$

$$= \frac{1}{2l} \int_{-l}^l e^{(a - \frac{in\pi}{l})x} dx$$

$$= \frac{1}{2l} \left[\frac{e^{(a - \frac{in\pi}{l})x}}{a - \frac{in\pi}{l}} \right]_{-l}^l$$

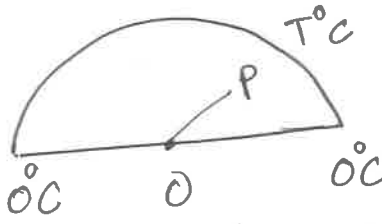
$$= \frac{(al + in\pi) \cdot l}{2l (a^2 l^2 + n^2 \pi^2)} \left[e^{al} e^{-in\pi} - e^{-al} e^{in\pi} \right]$$

$$= \frac{1}{2} \frac{(al + in\pi)}{a^2 l^2 + n^2 \pi^2} e^{(-1)^n} \left[e^{al} - e^{-al} \right]$$

$$= \frac{al + in\pi}{a^2 l^2 + n^2 \pi^2} (-1)^n \sinh al$$

$\therefore e^{ax} = \sum_{-\infty}^{\infty} (-1)^n \frac{\sinh al (al + in\pi)}{a^2 l^2 + n^2 \pi^2} e^{in\pi x/l}$

c)



Let the centre O of the semicircular plate be the pole and the bounding diameter be as the initial line. Let $u(r, \theta)$ be the steady state temperature at any point $P(r, \theta)$ and u satisfies the equation

$$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$$

(10)

The boundary conditions are

$$i) u(r, 0) = 0 \quad 0 \leq r \leq a$$

$$ii) u(r, \pi) = 0 \quad 0 \leq r \leq a$$

$$iii) u(a, \theta) = T$$

From (ii) & (iii), we have $u \rightarrow 0$ as $r \rightarrow 0$.

Hence the appropriate solution of (i) is

$$u = (C_1 r^p + C_2 r^{-p}) (C_3 \cos p\theta + C_4 \sin p\theta) \quad (2)$$

$$u(r, 0) = 0 \text{ in (2)}$$

$$0 = (C_1 r^p + C_2 r^{-p}) C_3 \rightarrow C_3 = 0$$

$$\therefore u = (C_1 r^p + C_2 r^{-p}) C_4 \sin p\theta$$

putting $u(r, \pi) = 0$ in (3),

$$0 = (C_1 r^p + C_2 r^{-p}) C_4 \sin p\pi$$

$$\Rightarrow \sin p\pi = 0 = \sin n\pi$$

$$\Rightarrow p\pi = n\pi \Rightarrow p = n$$

$$\therefore u = (C_1 r^n + C_2 r^{-n}) C_4 \sin n\theta$$

$$u = 0 \text{ when } r = 0, \quad 0 = C_2$$

$$u = C_4 C_1 r^n \sin n\theta$$

$$\therefore u(r, \theta) = \sum_{n=1}^{\infty} b_n r^n \sin n\theta$$

$$r = a, \quad u = T$$

$$T = \sum_{n=1}^{\infty} b_n a^n \sin n\theta$$

By Fourier half range series we get

(11)

$$b_n a^n = \frac{2}{\pi} \int_0^{\pi} T \sin n\theta \, d\theta$$

$$= \frac{2}{\pi} T \left(-\frac{\cos n\theta}{n} \right)_0^{\pi}$$

$$= \frac{2T}{n\pi} [-(-1)^n + 1]$$

$$= \begin{cases} 0, & \text{if } n \text{ is even} \\ \frac{4T}{n\pi}, & n \text{ is odd.} \end{cases}$$

$$\therefore b_n = \frac{4T}{a^n n\pi}$$

$$\therefore u(r, \theta) = \frac{4T}{\pi} \left[\frac{ra}{1} \sin \theta + \frac{(ra)^3}{3} \sin 3\theta + \frac{(ra)^5}{5} \sin 5\theta + \dots \right]$$

6. a) $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$

Let $u = X(x)T(t)$ where X is the ~~sub~~ function of x only and T , function of T only.

$$\therefore \frac{\partial (X.T)}{\partial x} = 2 \frac{\partial}{\partial t} (X.T) + X.T$$

$$T \frac{dX}{dx} = 2X \frac{dT}{dt} + X.T$$

or $T \cdot X' = 2X T' + X \cdot T$

or $\frac{X'}{X} = 2 \frac{T'}{T} + 1 = c$ (say)

$\frac{X'}{X} = c$ or $\frac{1}{X} \frac{dX}{dx} = c$

~~$\frac{dX}{dx}$~~ $\frac{dX}{X} = c dx$

$\log X = cx + \log a$

$\log \frac{X}{a} = cx$

$\frac{X}{a} = e^{cx}$ or $X = a e^{cx}$

$2 \frac{T'}{T} + 1 = c$ or $\frac{T'}{T} = \frac{1}{2}(c-1)$

$\frac{1}{T} \frac{dT}{dt} = \frac{1}{2}(c-1)$

$\frac{dT}{T} = \frac{1}{2}(c-1) dt$

$\log T = \frac{1}{2}(c-1)t + \log b$

$\log T/b = \frac{1}{2}(c-1)t$

$T/b = e^{\frac{1}{2}(c-1)t}$
 $T = b e^{\frac{1}{2}(c-1)t}$

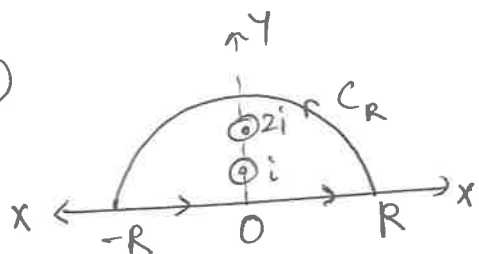
$\therefore u = X \cdot T = a e^{cx} \cdot b e^{\frac{1}{2}(c-1)t}$
 $u = ab e^{cx + \frac{1}{2}(c-1)t}$

$u(x, 0) = ab e^{cx} = 6 e^{-3x}$

$\Rightarrow ab = 6, c = -3$

$\therefore u = 6 e^{-3x + \frac{1}{2}(-4)t} = \underline{\underline{6 e^{-3x - 2t}}}$

6. b)



$$\text{Consider } \int_C \frac{z^2 dz}{(z^2+1)(z^2+4)}$$

$$= \int_C f(z) dz$$

where C is the contour consisting of the semi circle C_R of radius R together with the part of the real axis from $-R$ to $+R$.

The integral has simple poles at $z = \pm i$, $z = \pm 2i$ of which $z = i$, $2i$ only lie inside C .

$$\text{The residue (at } z = i) = \lim_{z \rightarrow i} (z-i) \frac{z^2}{(z-i)(z+i)(z^2+4)}$$

$$= \lim_{z \rightarrow i} \frac{z^2}{(z+i)(z^2+4)} = -\frac{1}{6i}$$

$$\text{The residue (at } z = 2i) = \lim_{z \rightarrow 2i} (z-2i) \frac{z^2}{(z^2+1)(z+2i)(z-2i)}$$

$$= \lim_{z \rightarrow 2i} \frac{z^2}{(z^2+1)(z+2i)} = \frac{1}{3i}$$

By Cauchy's Residue Theorem

$$\int_C f(z) dz = 2\pi i [\text{Res } f(i) + \text{Res } f(2i)]$$

$$= 2\pi i \left(-\frac{1}{6i} + \frac{1}{3i} \right) = \frac{\pi}{3}$$

$$\text{i.e. } \int_{-R}^R f(x) dx + \int_{C_R} f(z) dz = \frac{\pi}{3}$$

for any pt on C_R as $|z| \rightarrow \infty$, $f(z) = 0$

$$\therefore \lim_{|z| \rightarrow \infty} \int_{C_R} f(z) dz = 0$$

$\therefore R \rightarrow \infty,$

$$\int_{-\infty}^{\infty} f(x) dx = \frac{\pi}{3}$$

$$\therefore \int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3} \neq$$

c) Consider Newton's Backward difference formula.

$x_0 = 90, a = 90, h = 10$

$\therefore a - kh = 84$ ie $90 - 10k = 84$

$\therefore k = 0.6$

x	u	Δu	$\Delta^2 u$	$\Delta^3 u$
$x_3 = 60$	$u_3 = 226$	24	2	
$x_2 = 70$	$u_2 = 250$	26	2	0
$x_1 = 80$	$u_1 = 276$	28		
$x_0 = 90$	$u_0 = 304$			

$$\begin{aligned}
 y_{-k} &= y_{-0.6} = (E^{-1})^{0.6} y_0 = (1 - \nabla)^{0.6} u_0 \\
 &= \left(1 - 0.6 \nabla - \frac{(0.6)(0.4)}{2!} \nabla^2 - \dots \right) u_0 \\
 &= u_0 - 0.6 \nabla u_0 - \frac{(0.6)(0.4)}{2!} \nabla^2 u_0 - \dots \\
 &= 304 - (0.6)(28) - \frac{(0.6)(0.4)}{2!} \cdot 2 = \boxed{286.96}
 \end{aligned}$$

6.

c)

$$(ii) f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$$

$$= ax(x-1)(x-2)(x-3) + bx(x-1)(x-2) + cx(x-1) + dx + e$$

$$= ax^{(4)} + bx^{(3)} + cx^{(2)} + dx^{(1)} + e$$

When $x=0$, $9=e$

$x=1$, $10 = d + e \quad \therefore d = 1$

$x=2$, $37 = 2c + 2d + e \quad \therefore c = 13$

$x=3$, $54 = 6b + 6c + 3d + e$, $b = -6$

Comparing the coeff of x^4 , $a=1$

$$\therefore f(x) = x^{(4)} - 6x^{(3)} + 13x^{(2)} + x^{(1)} + 9$$

